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# Laser Cooling

List of Topics:

- Why Laser Cooling?
- Classical Description of Light Forces
- Opical Bloch-Equation for Two-Level Atoms
- Light Shift and Dressed States
- Cooling with Radiation Pressure, Doppler Limit
- Magneto-optic Trap
- Dipole Forces
- Interference Effects in Multiple Beam Geometries
- Polarization Gradient Cooling
- Optical Lattices
- Cooling below the Recoil Limit: VSCPT, Raman Cooling

# Textbooks & Reviews

Laser Cooling and Trapping
H. Metcalf, P. van der Straten, Springer Verlag (1999)

Atoms and Molecules Interacting with Light
P. van der Straten, H. Metcalf, Cambridge University Press (2016)

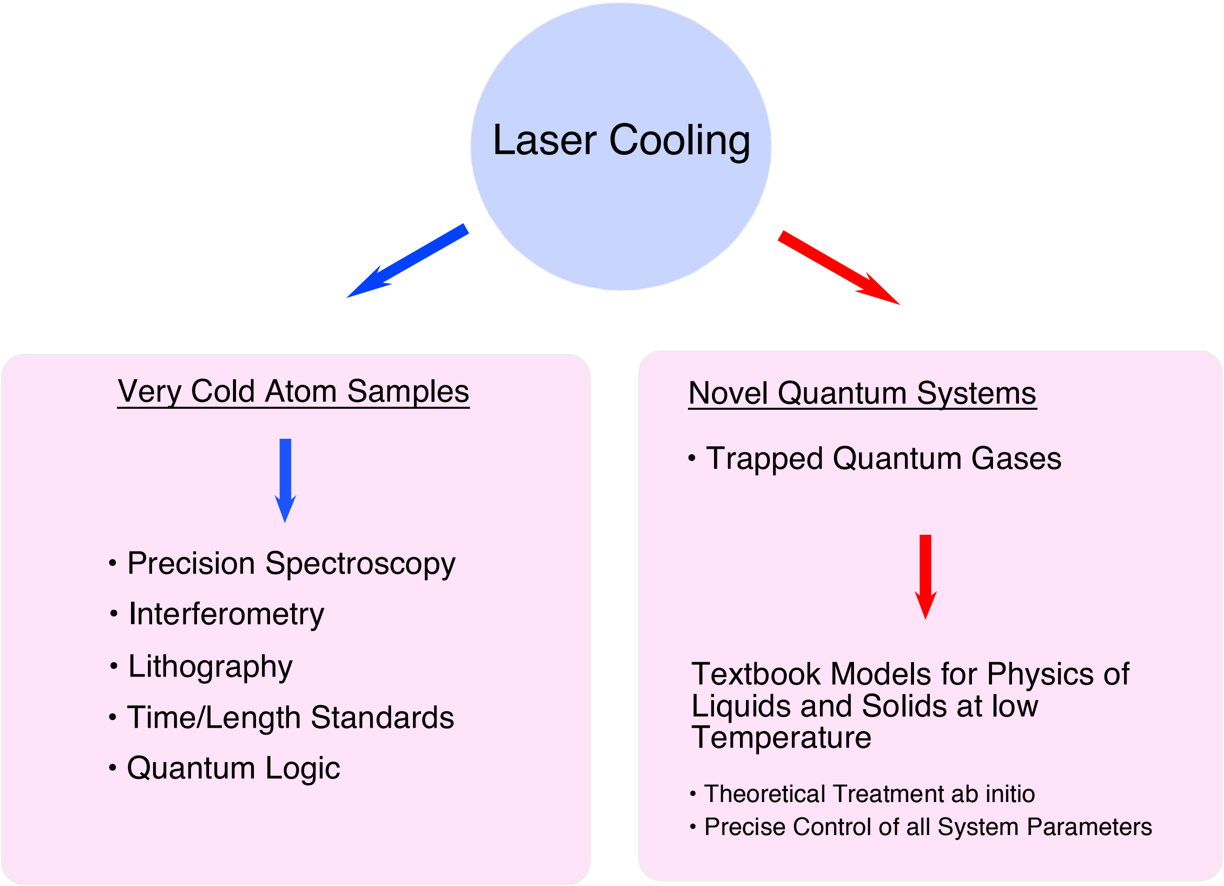
Atomic Physics
M. Inguscio, L. Fallani, Oxford University Press (2013)

Laser Cooling and Trapping of neutral Atoms
C. S. Adams, E. Riis Prog. Quonr. Elecr, Vol. 21, No. 1, pp. 1-79 (1997)

Manipulating atoms with photons
Claude N. Cohen-Tannoudji, Reviews of Modern Physics, Vol. 70, No. 3, (1998)

Electromagnetic trapping of cold atoms
V I Balykin, V G Minogin and V S Letokhov, Rep. Prog. Phys. 63, 1429–1510 (2000)

Cold Atoms in Dissipative Optical Lattices
G. Grynberg, C. Robilliard, Physics Reports 355, 335–451 (2001)



Laser Cooling

Very Cold Atom Samples

- Precision Spectroscopy
- Interferometry
- Lithography
- Time/Length Standards
- Quantum Logic

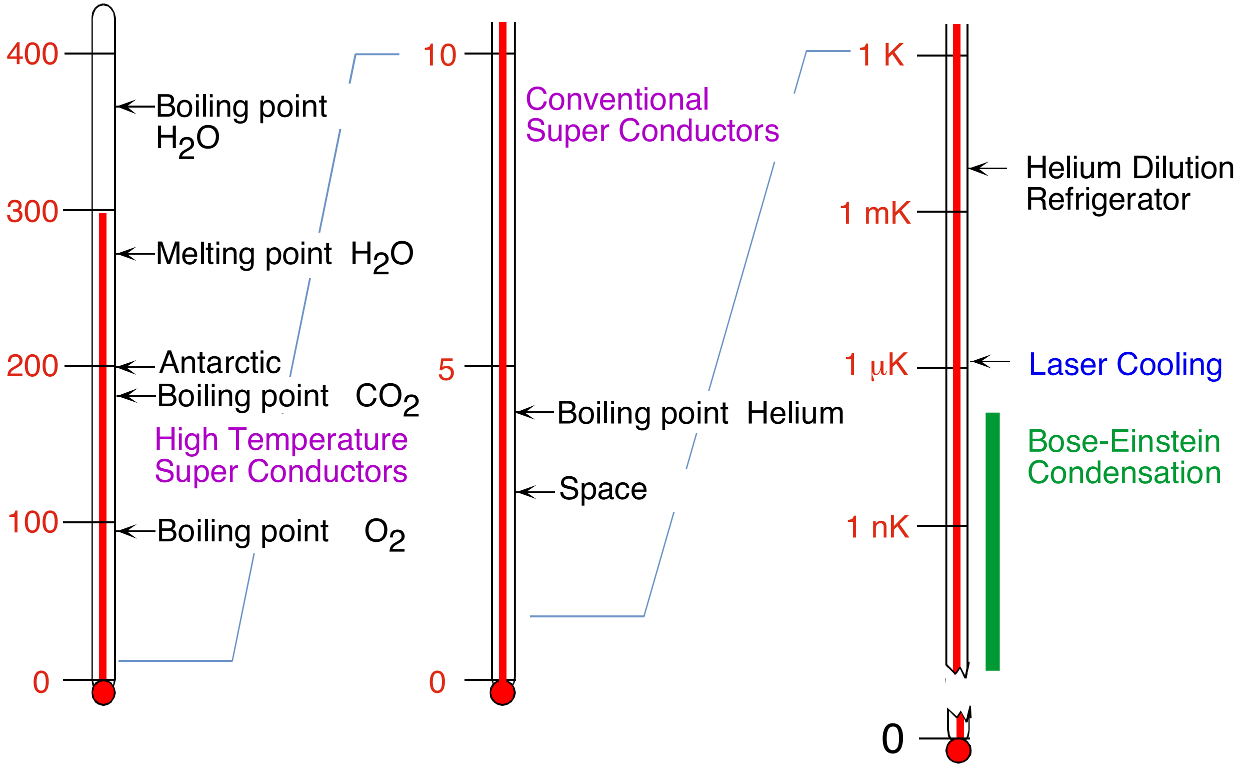
Novel Quantum Systems

- Trapped Quantum Gases

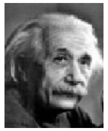
Textbook Models for Physics of Liquids and Solids at low Temperature

- Theoretical Treatment ab initio
- Precise Control of all System Parameters

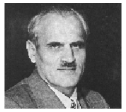
# Approaching Zero Temperature



# Discovery of Light Forces



Albert Einstein 1917: Atomic (molecular) gases thermalize in thermal light fields



Arthur H. Compton 1923: Significance of recoil in photon electron scattering

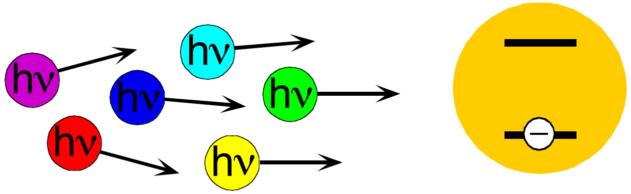
Otto R. Frisch 1933: First deflection of atomic beam by light

These experiments were performed in Hamburg, Jungiusstr. 9a, and had to be cut off, when Frisch and Stern (because of their jewish denomination) were expelled from the university.

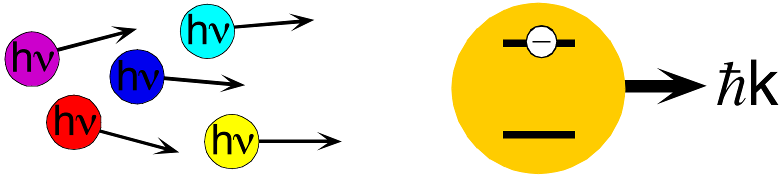
Theodore Maiman 1960: First laser

| | |
|---|---|
| 1975 | Proposals of laser cooling: T. Hänsch, A. Schalow, D. Wineland, H. Dehmelt |
| 1980-1990 | Experimental realization |
| 1997 | Nobelprize laser cooling: S. Chu, C. Cohen-Tannoudji, W. Phillips |
| 1995 | First Bose-Einstein Condensates: E. Cornell, C. Wieman, R. Hulet, W. Ketterle |
| 2001 | Nobelprize Bose-Einstein-Condensation: E. Cornell, W. Ketterle, C. Wieman |

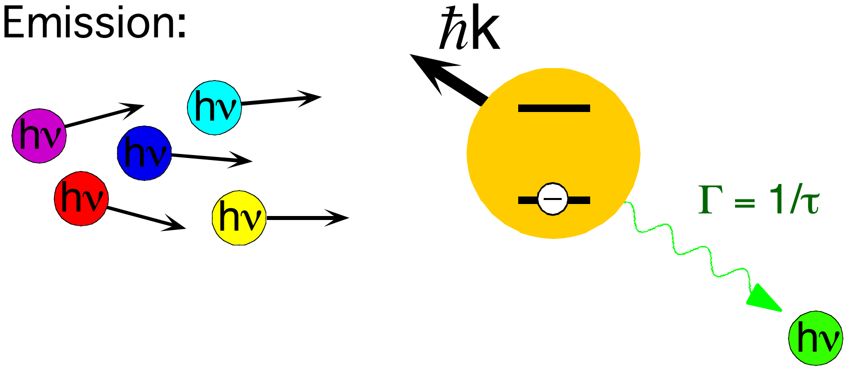
# Radiation Pressure



Absorption:



Emission:

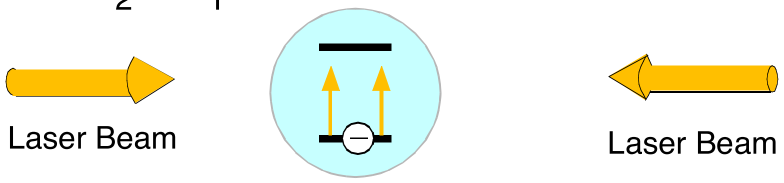


Hale-Bopp: 5. April 1997

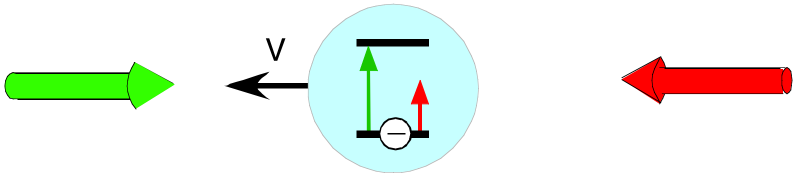
$$a = \frac{F}{m} = \frac{\Pi_e}{m} \hbar k \, \Gamma = 10^5 g$$

# Cooling with Radiation Pressure

$h\nu < E_2 - E_1$



Resting Atom: radiation pressure cancels



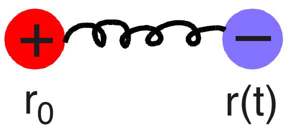
Moving Atom:  atoms tunes into resonance with counter-propagating beam

→ force decelerates atom proportional to its velocity

→ faster atom experience stronger force: velocity spread is reduced

# Classical Description of Light Forces

Lorentz Model:



dipole moment: $P(t) = e\,(r(t) - r_0)$

Oscillating electron at position $r(t) = r_0 + e^{-1}\,P(t)$ and proton at position $r_0$ experience time-averaged Coulomb-force:

$$F_C = \Big\langle e\,E(r(t),t) - e\,E(r_0,t) \Big\rangle \approx \Big\langle (P\nabla)\,E \Big\rangle \qquad \langle A \rangle \equiv \frac{1}{T}\int_0^T A(t)\,dt$$

$$\vec{E}(\vec{r} + \vec{\delta r}) = \vec{E}(\vec{r}) + (\vec{\delta r}\,\vec{\nabla})\vec{E} + O(\delta r^2)$$

Induced dipole $P(t) = (r(t) - r_0)\,e$ yields time-averaged Lorentz-force:

∇ acts on E only

$$F_L = \Big\langle \frac{\partial}{\partial t} P \times B \Big\rangle = - \Big\langle P \times \frac{\partial}{\partial t} B \Big\rangle = \Big\langle P \times (\nabla \times E) \Big\rangle = \Big\langle \nabla(PE) - (P\nabla)\,E \Big\rangle$$

integration by parts, bondary terms vanish because P,B periodic in t

a x (b x c) = b(ac) - c(ab)

Total Force: $F = F_C + F_L = \Big\langle \nabla(PE) \Big\rangle$

⇒ Same expression as known for static dipoles in static electric fields

Consider Harmonic Field: $E(r,t) = \frac{1}{\sqrt{2}} \left( E(r)\, e^{i\omega t} + E(r)^*\, e^{-i\omega t} \right)$

$P(t) = \frac{1}{\sqrt{2}} \left( P\, e^{i\omega t} + P^*\, e^{-i\omega t} \right)$

$E(r), P$ = complex, time-independent vectors

∇ acts on E only

$$\Rightarrow \qquad F = \frac{1}{2} \left( \nabla(PE^*) + \nabla(P^*E) \right)$$

Express complex polarization $P$ by means of polarizability tensor $\alpha(E)$ : $P = \varepsilon_0\, \alpha(E)\, E$

$\alpha(E)$ = complex 3x3 Matrix

Choose basis such that $\alpha(E)$ diagonal, with $\alpha_{\nu\nu} \equiv \alpha_\nu + i\, \beta_\nu$ , $E_\nu \equiv \sqrt{\frac{I_\nu}{\varepsilon_0}}\; e^{-i\psi_\nu}$

$$F = \frac{1}{2} \sum_{\nu=1}^{3} \alpha_\nu \nabla I_\nu - \sum_{\nu=1}^{3} \beta_\nu I_\nu \nabla \psi_\nu$$

dipole force

radiation pressure

# detailed calculation of force:

$\nabla$ acts on $E$ only

$$F = \frac{1}{2}\left(\overset{\curvearrowright}{\nabla(PE^*)} + \overset{\curvearrowright}{\nabla(P^*E)}\right) = \frac{1}{2}\sum_{n=1}^{3} P_n\nabla E_n^* + P_n^*\nabla E_n = \frac{\varepsilon_0}{2}\sum_{n=1}^{3}\sum_{m=1}^{3} \alpha_{nm}E_m\nabla E_n^* + \alpha_{nm}^* E_m^* \nabla E_n$$

$$= \frac{\varepsilon_0}{2}\sum_{n=1}^{3} \alpha_{nn}E_n\nabla E_n^* + \alpha_{nn}^* E_n^* \nabla E_n = \frac{\varepsilon_0}{2}\sum_{n=1}^{3} \left(\alpha_n + i\beta_n\right)E_n\nabla E_n^* + \left(\alpha_n - i\beta_n\right)E_n^* \nabla E_n$$

$$= \frac{\varepsilon_0}{2}\sum_{n=1}^{3} \alpha_n \left(E_n\nabla E_n^* + E_n^*\nabla E_n\right) + i\,\beta_n\left(E_n\nabla E_n^* - E_n^* \nabla E_n\right) = \frac{\varepsilon_0}{2}\sum_{n=1}^{3} \alpha_n\nabla\left(E_nE_n^*\right) + i\,\beta_n\left(E_n\nabla E_n^* - E_n^* \nabla E_n\right)$$

$$E_n \equiv \sqrt{I_n/\varepsilon_0}\; e^{-i\psi_n} \quad\Rightarrow\quad \begin{aligned} E_n\nabla E_n^* + E_n^*\nabla E_n &= \frac{1}{\varepsilon_0}\nabla I_n \\ E_n\nabla E_n^* - E_n^*\nabla E_n &= 2i\ \nabla\psi_n\,\frac{I_n}{\varepsilon_0} \end{aligned}$$

$$\Rightarrow\quad F = \frac{1}{2}\sum_{n=1}^{3} \alpha_n\nabla I_n - \sum_{n=1}^{3} \beta_n\, I_n\nabla\psi_n$$

## Example: linear polarization along z-axis

$$E \equiv \hat{z} \sqrt{\frac{I(x,y,z)}{\varepsilon_0}} \, e^{-i\psi(x,y,z)}$$

$I(x,y,z)$ energy density, $\psi(x,y,z)$ local phase

$$\Rightarrow \quad F = \frac{1}{2}\alpha_z \nabla I - \beta_z I \nabla\psi$$

plane travelling wave:

$$I = I_0 = \text{constant}, \quad \psi = k r \quad \Rightarrow \quad \nabla I = 0\,, \quad \nabla\psi = k$$

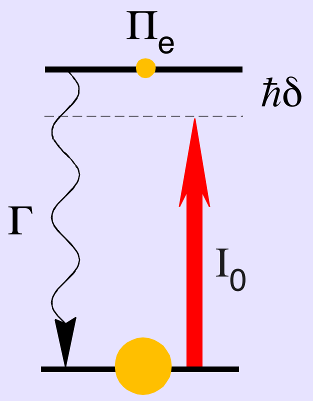
$$\Rightarrow \quad F = -\beta_z I_0 \nabla\psi = -\beta_z I_0 k$$
$$= \hbar k \, \Gamma \, \Pi_e$$

plane standing wave:

$$I = I_0 \cos^2(kr)\,, \quad \psi = \text{constant} \quad \Rightarrow \quad \nabla I = -k I_0 \sin(2kr), \qquad \nabla\psi = 0$$

$$\Rightarrow \quad F = \frac{1}{2}\alpha_z \nabla I = -\frac{1}{2}\alpha_z I_0 \, k \sin(2kr)$$
$$= -\hbar k \, \delta \, \Pi_e \sin(2kr)$$

$$-\beta_z I_0 = \Pi_e \hbar \Gamma$$
$$\alpha_z I_0/2 = \Pi_e \hbar \delta$$
$$\Pi_e = \Pi_e(I_0,\Gamma,\delta)$$



**Conclusion:** general structure of light forces follows from classical treatment of the light, however, we need to treat internal atomic degrees of freedom quantum mechanically in order to calculate polarizability tensor $\alpha_{\nu\nu} \equiv \alpha_\nu + i\,\beta_\nu$

# Concept of density matrix

Physical states are described by quantum mechanics as elements $|\psi\rangle$ of a Hilbert-space $\mathcal{H}$.

Physical quantities are implemented as self-adjoint operators (Observables) $A \in O(\mathcal{H})$.

The most significant property of quantum states is the superposition principle, i.e., we may compose any state via basis states $|\nu\rangle$ , $\nu = 0,1,\ldots.$

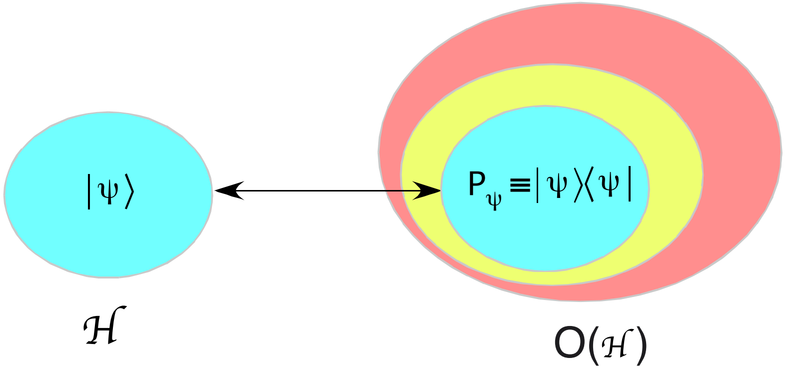
$$|\psi\rangle = \sum_{\nu=1}^{N} \psi_\nu \, |\nu\rangle \qquad \text{with complex numbers } \psi_\nu$$

Any state $|\psi\rangle$ is fully determined by knowing the statistical weights $|\psi_\nu|^2$ of the states $|\nu\rangle$

and the relative phases $\dfrac{\psi_\nu{}^* \psi_\mu}{|\psi_\nu{}^* \psi_\mu|}$ between states $|\nu\rangle$ and $|\mu\rangle$.

Can we describe a physical state with these phases not entirely fixed or not known, for example, because we consider a statistical ensemble ?

Extension of the concept of a quantum mechanical state:



$P_\psi \equiv |\psi\rangle\langle\psi|$ projector with respect to $|\psi\rangle$ :

$$P_\psi \, P_\psi = P_\psi$$

$$P_\psi \, |\psi\rangle = |\psi\rangle$$

$$P_\psi \, |\phi\rangle = 0 \quad \text{if} \quad |\phi\rangle \perp |\psi\rangle$$

$$A \in O(\mathcal{H}) \quad \Rightarrow \quad \langle\psi|A|\psi\rangle = \text{Trace}\,[\, A \;\, P_\psi \,]$$

# pure states and mixed states

pure state: $|\psi\rangle \equiv \sum_{\nu=1}^{N} \psi_\nu \; |\nu\rangle \quad \Rightarrow$ projector $P_\psi \equiv |\psi\rangle\langle\psi| = \sum_{\nu,\mu=1}^{N} \psi_\nu \psi_\mu^* \; |\nu\rangle\langle\mu|$

matrix elements: $\langle\nu|P_\psi|\mu\rangle = \psi_\nu\psi_\mu^* \quad \Rightarrow$ non-zero off-diagonal elements provide complete phase information with respect to any basis

mixed state: density operator $\rho \equiv \sum_{k=1}^{K} \Pi_k \; |\psi^{(k)}\rangle\langle\psi^{(k)}|$ with $\Pi_k \in [0,1]$ and $\sum_{k=1}^{K} \Pi_k = 1$

$\rho = \rho^+$, $\mathrm{Trace}[\rho] = 1$

$$A \in O(\mathcal{H}) \quad \Rightarrow \quad \langle A\rangle_\rho \equiv \sum_{k=1}^{K} \Pi_k \; \langle\psi^{(k)}|A|\psi^{(k)}\rangle = \mathrm{Trace}\,[\,A\,\rho\,]$$

matrix elements: $\langle\nu|\,\rho\,|\mu\rangle = \sum_{k=1}^{K} \Pi_k \; \langle\nu|\psi^{(k)}\rangle\langle\psi^{(k)}|\,\mu\rangle$

off-diagonal elements $\nu \neq \mu$ (coherences): $\langle\nu|\,\rho\,|\mu\rangle$ can be zero, although some $\langle\nu|\psi^{(k)}\rangle\langle\psi^{(k)}|\,\mu\rangle$ are non-zero.
i.e., mixed states are characterized by reduced phase information

diagonal elements (populations):

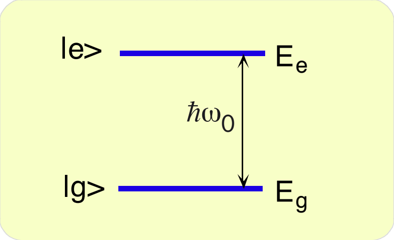
$$\underbrace{\langle\nu|\,\rho\,|\nu\rangle}_{\text{statistical weight of } |\nu\rangle \text{ in mixture } \rho} = \sum_{k=1}^{K} \underbrace{\Pi_k}_{\text{statistical weight of } |\psi^{(k)}\rangle \text{ in mixture } \rho} \; \underbrace{|\langle\nu|\psi^{(k)}\rangle|^2}_{\text{statistical weight of } |\nu\rangle \text{ in state } |\psi^{(k)}\rangle}$$

## Evolution of density matrix (von Neumann)

$$i\hbar \frac{\partial}{\partial t} |\psi\rangle\langle\psi| = i\hbar \left[ |\dot{\psi}\rangle\langle\psi| + |\psi\rangle\langle\dot{\psi}| \right] = H\,|\psi\rangle\langle\psi| - |\psi\rangle\langle\psi|\,H = \left[ H, |\psi\rangle\langle\psi| \right]$$

$$\rho \equiv \sum_{n=1}^{N} \Pi_n \, |\psi^{(n)}\rangle\langle\psi^{(n)}| \quad \Rightarrow \quad \boxed{i\hbar \frac{\partial}{\partial t} \rho = [H, \rho]}$$

## Two-Level Atom:



**Atomic Hamiltonian:** $H_A = \hbar\omega_0 \, b^+ b \quad \Rightarrow \quad H_A |g\rangle = 0 \, |g\rangle \, , \, H_A |e\rangle = \hbar\omega_0 \, |e\rangle$

$b \equiv |g\rangle\langle e|$ anihilation of excitation

**Atomic Dipole Operator:** d must not have diagonal elements → no permanent dipole moment

$$d \equiv \mu \, b + \mu^* \, b^+ \quad \text{with} \quad \mu \equiv \langle g|\, d \,|e\rangle$$

Interaction Operator: $W(t) = d\,E(r,t)\,, \quad E(r,t) = \frac{1}{\sqrt{2}}\left( E(r)\,e^{-i\omega t} + E(r)^{*}\,e^{i\omega t} \right)$

$$\Rightarrow \quad W = v\,b + v^{*}\,b^{+} \quad \text{with} \quad v \equiv \langle g|\,W(t)\,|e\rangle = \frac{1}{\sqrt{2}}\left( \mu\,E\,e^{-i\omega t} + \mu\,E^{*}\,e^{i\omega t} \right)$$

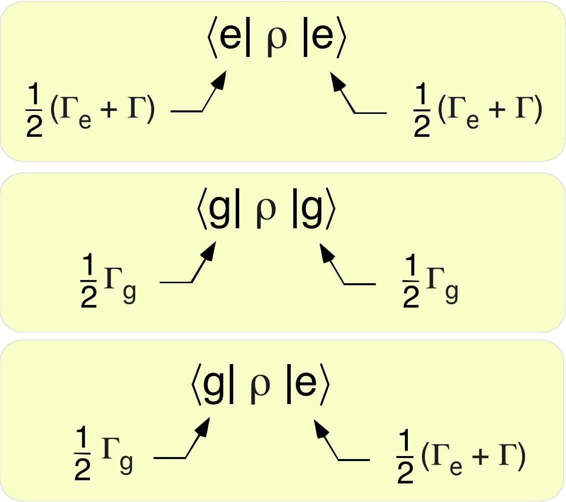
Evaluate evolution equation for $H = H_A + W$ in Basis {|g>,|e>}, $\rho_{nm} \equiv \langle n|\,\rho\,|m\rangle$:

$$\frac{\partial}{\partial t}\rho_{eg} = -i\,\omega_0\,\rho_{eg} - i\,\frac{v^{*}}{\hbar}\,(\rho_{gg} - \rho_{ee})$$

$$\frac{\partial}{\partial t}\rho_{ee} = i\,\frac{v}{\hbar}\,\rho_{eg} - i\,\frac{v^{*}}{\hbar}\,\rho_{ge} \qquad (*)$$

$$\frac{\partial}{\partial t}\rho_{gg} = i\,\frac{v^{*}}{\hbar}\,\rho_{ge} - i\,\frac{v}{\hbar}\,\rho_{eg}$$

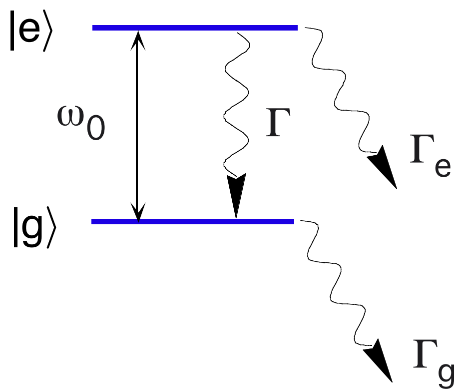
Damping of $\rho_{nm} \equiv \langle n|\,\rho\,|m\rangle$ by spontaneous emission:



$$\frac{\partial}{\partial t}\rho_{eg} = -\gamma\,\rho_{eg}$$

$$\frac{\partial}{\partial t}\rho_{ee} = -(\Gamma_e + \Gamma)\,\rho_{ee}$$

$$\frac{\partial}{\partial t}\rho_{gg} = \Gamma\,\rho_{ee} - \Gamma_g\,\rho_{gg}$$



Damping of coherence: $\gamma = \gamma_{coh} + (\Gamma_g + \Gamma_e + \Gamma)/2$

$\gamma_{coh}$ can result from dephasing by collisions etc.

# Evaluation of evolution equation (*)

$$\partial_t\,\rho = \frac{1}{i\hbar}\left[H,\rho\right]$$

$$\rho = \begin{pmatrix} \rho_{gg} & \rho_{ge} \\ \rho_{eg} & \rho_{ee} \end{pmatrix},\ H = \begin{pmatrix} H_{gg} & H_{ge} \\ H_{eg} & H_{ee} \end{pmatrix} = \begin{pmatrix} 0 & v \\ v^{*} & \hbar\omega_0 \end{pmatrix}$$

$$\left[H,\rho\right] = \begin{pmatrix} H_{gg} & H_{ge} \\ H_{eg} & H_{ee} \end{pmatrix}\begin{pmatrix} \rho_{gg} & \rho_{ge} \\ \rho_{eg} & \rho_{ee} \end{pmatrix} - \begin{pmatrix} \rho_{gg} & \rho_{ge} \\ \rho_{eg} & \rho_{ee} \end{pmatrix}\begin{pmatrix} H_{gg} & H_{ge} \\ H_{eg} & H_{ee} \end{pmatrix} =$$

$$\begin{pmatrix} H_{gg}\rho_{gg} + H_{ge}\rho_{eg} - \rho_{gg}H_{gg} - \rho_{ge}H_{eg} & H_{gg}\rho_{ge} + H_{ge}\rho_{ee} - \rho_{gg}H_{ge} - \rho_{ge}H_{ee} \\ H_{eg}\rho_{gg} + H_{ee}\rho_{eg} - \rho_{eg}H_{gg} - \rho_{ee}H_{eg} & H_{eg}\rho_{ge} + H_{ee}\rho_{ee} - \rho_{eg}H_{ge} - \rho_{ee}H_{ee} \end{pmatrix} =$$

$$\begin{pmatrix} \rho_{eg}H_{ge} - \rho_{ge}H_{eg} & \rho_{ge}\left(H_{gg} - H_{ee}\right) + \left(\rho_{ee} - \rho_{gg}\right)H_{ge} \\ \rho_{eg}\left(H_{ee} - H_{gg}\right) + \left(\rho_{gg} - \rho_{ee}\right)H_{eg} & \rho_{ge}H_{eg} - \rho_{eg}H_{ge} \end{pmatrix} =$$

$$\frac{1}{i\hbar}\left[H,\rho\right] = \begin{pmatrix} i\frac{v^{*}}{\hbar}\rho_{ge} - i\frac{v}{\hbar}\rho_{eg} & i\omega_0\rho_{ge} + i\frac{v}{\hbar}\left(\rho_{gg} - \rho_{ee}\right) \\ -i\omega_0\,\rho_{eg} + i\frac{v^{*}}{\hbar}\left(\rho_{ee} - \rho_{gg}\right) & i\frac{v}{\hbar}\rho_{eg} - i\frac{v^{*}}{\hbar}\rho_{ge} \end{pmatrix}$$

Evolution equation with damping:

$$\frac{\partial}{\partial t}\rho_{eg} = -i\,\omega_0\,\rho_{eg} - i\,\frac{v^*}{\hbar}(\rho_{gg}-\rho_{ee}) - \gamma\,\rho_{eg}$$

$$\frac{\partial}{\partial t}\rho_{ee} = i\,\frac{v}{\hbar}\,\rho_{eg} - i\,\frac{v^*}{\hbar}\,\rho_{ge} - (\Gamma_e+\Gamma)\,\rho_{ee}$$

$$\frac{\partial}{\partial t}\rho_{gg} = i\,\frac{v^*}{\hbar}\,\rho_{ge} - i\,\frac{v}{\hbar}\,\rho_{eg} + \Gamma\,\rho_{ee} - \Gamma_g\,\rho_{gg}$$

Equation is time-dependent via $e^{-i\omega t}$ and $e^{i\omega t}$ terms of v(t)

co-rotating basis:

$$|g\rangle \rightarrow |g\rangle$$

$$|e\rangle \rightarrow |e\rangle\, e^{-i(\omega t+\phi)}$$

equivalent to

$$-\omega_0 \rightarrow \delta \equiv \omega - \omega_0$$

$$v \rightarrow u \equiv v\, e^{-i(\omega t+\phi)}$$

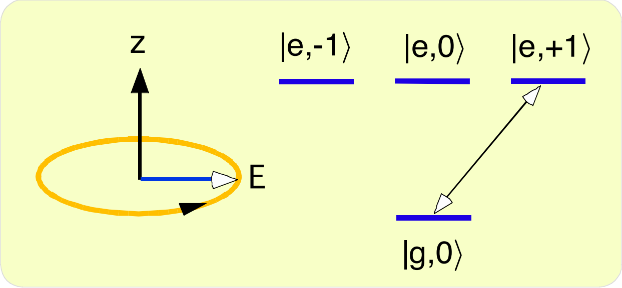
$\Rightarrow$

$$\frac{\partial}{\partial t}\rho_{eg} = i\,\delta\,\rho_{eg} - i\,\frac{u^*}{\hbar}(\rho_{gg}-\rho_{ee}) - \gamma\,\rho_{eg}$$

$$\frac{\partial}{\partial t}\rho_{ee} = i\,\frac{u}{\hbar}\,\rho_{eg} - i\,\frac{u^*}{\hbar}\,\rho_{ge} - (\Gamma_e+\Gamma)\,\rho_{ee}$$

$$\frac{\partial}{\partial t}\rho_{gg} = i\,\frac{u^*}{\hbar}\,\rho_{ge} - i\,\frac{u}{\hbar}\,\rho_{eg} + \Gamma\,\rho_{ee} - \Gamma_g\,\rho_{gg}$$

physical significance of co-rotating basis:



simplest radially symmetric atomic dipole transition: J=0 → J=1 transition

quantization axis = z-axis, light circularly polarized in xy-plane couples Ig,0> → Ie,+1>

angular momentum: $J_z\,|e,\nu\rangle = \nu\,\hbar\,|e,\nu\rangle$ , $\nu = -1,0,1$

$J_z\,|g,0\rangle = 0$

rotation operator with respect to z-axis: $R(z,\alpha) \equiv \exp(\frac{-i}{\hbar}\,\alpha\,J_z)$ $\Rightarrow$

$$R(z,\,\omega t+\phi)\,|g,0\rangle = |g,0\rangle$$

$$R(z,\,\omega t+\phi)\,|e,+1\rangle = e^{-i(\omega t+\phi)}\,|e,+1\rangle$$

rotating wave approximation (RWA):

$$u \equiv v\, e^{-i(\omega t + \phi)} = \frac{1}{\sqrt{2}} \mu\, E\, e^{-i(2\omega t + \phi)} + \frac{1}{\sqrt{2}} \mu\, E^* e^{-i\phi}$$

$$\underset{\text{RWA}}{\approx} \frac{1}{\sqrt{2}} \mu\, E^* e^{-i\phi} = \frac{\hbar}{2} \omega_1$$

cf. Cohen Tannoudji QM II, Chap.XIII, Sec.C

Rabi-frequency: $\omega_1 \equiv \frac{\sqrt{2}}{\hbar} \mu\, E^* e^{-i\phi}$ choose $\phi$ such that $\omega_1$ real & positive

Evolution equation in rotating frame:

$$\Rightarrow \quad \begin{aligned}
\frac{\partial}{\partial t} \rho_{eg} &= i\,\delta\, \rho_{eg} - i\,\frac{\omega_1}{2} (\rho_{gg} - \rho_{ee}) - \gamma\, \rho_{eg} \\
\frac{\partial}{\partial t} \rho_{ee} &= i\,\frac{\omega_1}{2} \rho_{eg} - i\,\frac{\omega_1}{2} \rho_{ge} - (\Gamma_e + \Gamma)\, \rho_{ee} \\
\frac{\partial}{\partial t} \rho_{gg} &= i\,\frac{\omega_1}{2} \rho_{ge} - i\,\frac{\omega_1}{2} \rho_{eg} + \Gamma\, \rho_{ee} - \Gamma_g\, \rho_{gg}
\end{aligned}$$

# Optical Bloch-Equation

define:

$u \equiv \rho_{eg} + \rho_{ge}$ real part of coherence

$v \equiv i\,(\rho_{eg} - \rho_{ge})$ imaginary part of coherence

$w \equiv \rho_{ee} - \rho_{gg}$ inversion

$z \equiv \rho_{ee} + \rho_{gg}$ total population

$(u,v,w) \equiv$ Bloch-vector

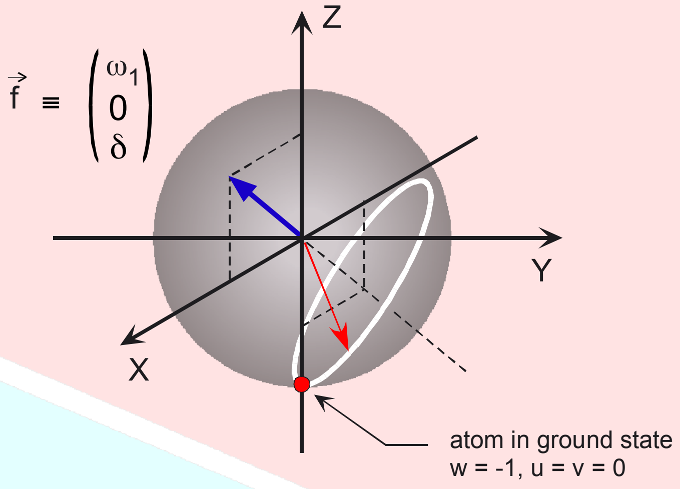
$$\Rightarrow \quad \frac{\partial}{\partial t}\begin{pmatrix} u \\ v \\ w \\ z \end{pmatrix} = \begin{pmatrix} -\gamma & -\delta & 0 & 0 \\ \delta & -\gamma & -\omega_1 & 0 \\ 0 & \omega_1 & -\frac{\Gamma_g+\Gamma_e+2\Gamma}{2} & \frac{\Gamma_g-\Gamma_e-2\Gamma}{2} \\ 0 & 0 & \frac{\Gamma_g-\Gamma_e}{2} & -\frac{\Gamma_g+\Gamma_e}{2} \end{pmatrix} \begin{pmatrix} u \\ v \\ w \\ z \end{pmatrix}$$

Closed Two-Level System:

$\Gamma_g = \Gamma_e = 0,\ \ z = 1$ $\Rightarrow$

$$\frac{\partial}{\partial t}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} -\gamma & -\delta & 0 \\ \delta & -\gamma & -\omega_1 \\ 0 & \omega_1 & -\Gamma \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ \Gamma \end{pmatrix} = \begin{pmatrix} \omega_1 \\ 0 \\ \delta \end{pmatrix} \times \begin{pmatrix} u \\ v \\ w \end{pmatrix} + \begin{pmatrix} -\gamma & 0 & 0 \\ 0 & -\gamma & 0 \\ 0 & 0 & -\Gamma \end{pmatrix}\begin{pmatrix} u \\ v \\ w \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \\ \Gamma \end{pmatrix}$$

# Elementary Solutions of Optical Bloch-Equation



## no damping ($\gamma$, $\Gamma = 0$):

- Bloch-vector precesses around $\vec{f}$ with angular frequency $\Omega = |\vec{f}| = \sqrt{\omega_1^2 + \delta^2}$
- for pure states, Bloch vector has constant length 1 and points onto Bloch-sphere
- for mixed states, constant length of Bloch vector < 1 lies within Bloch-sphere

special cases:

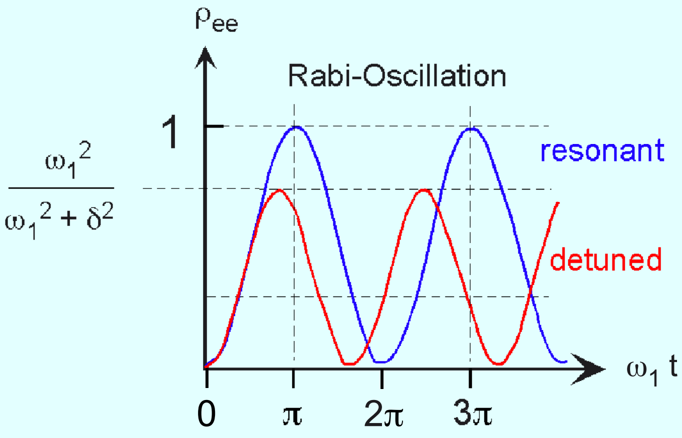
light off, i.e., $\vec{f} = (0,0,\delta)$ $\Rightarrow$ for $W(t_0) = -1$ follows $W(t) = -1$ for all later times t

resonance, i.e., $\vec{f} = (\omega_1,0,0)$ $\Rightarrow$ Bloch-vector travels on great circle within the yz-plane with angular frequency $\omega_1$. System is periodically inverted.

- evolution of excited state population (Initial condition $W(t_0) = -1$):

$$\rho_{ee} = \frac{\omega_1^2}{\omega_1^2 + \delta^2} \sin^2\left(\frac{1}{2}\sqrt{\omega_1^2 + \delta^2}\; t\right)$$

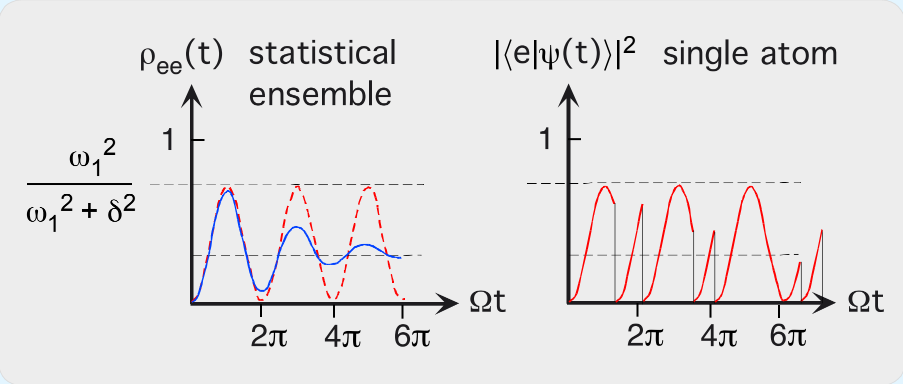
$$= \frac{1}{4}\,\omega_1^2 t^2 + O(t^4)$$



## damping:

Bloch-vector shrinks and approaches steady state:

$$\frac{\partial}{\partial t}\begin{pmatrix} u \\ v \\ w \end{pmatrix} = 0 \quad \Rightarrow \quad \begin{pmatrix} \bar{u} \\ \bar{v} \\ \bar{w} \end{pmatrix} = M^{-1}\begin{pmatrix} 0 \\ 0 \\ \Gamma \end{pmatrix} = \frac{1}{1+s}\begin{pmatrix} -s\,\frac{\Gamma\,\delta}{\gamma\,\omega_1} \\ s\,\frac{\Gamma}{\omega_1} \\ -1 \end{pmatrix}$$



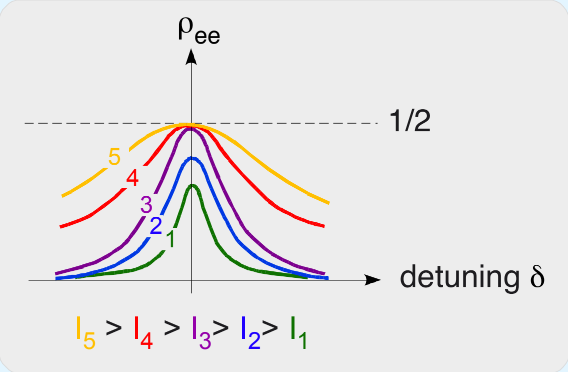
resonant saturation parameter: $s_0 \equiv \frac{\omega_1^2}{\gamma\,\Gamma} = \frac{I}{I_{sat}}$

saturation parameter: $s \equiv s_0 \frac{1}{1 + (\delta/\gamma)^2}$

### cases & comments:

- population of excited state: $\rho_{ee} = \frac{1}{2}(1+\bar{w}) = \frac{1}{2}\frac{s}{1+s} = \frac{1}{2}\frac{s_0\,\gamma^2}{\delta^2 + (\tilde{\Gamma}/2)^2}$

  power broadened linewidth (FWHM): $\tilde{\Gamma} \equiv 2\,\gamma\sqrt{1 + s_0} = 2\,\gamma\sqrt{1 + I/I_{sat}}$



- resonance, i.e., $\delta = 0 \Rightarrow s = s_0$, $\bar{u} = 0$, $\bar{v} = \frac{1}{1+s_0}\frac{\omega_1}{\gamma}$, $\bar{w} = \frac{-1}{1+s_0}$
- light off. i.e., $\omega_1 = 0 \Rightarrow s = 0$, $(\bar{u}, \bar{v}, \bar{w}) = (0,0,-1)$
- steady state Bloch-vector lies within southern hemisphere with length: $\sqrt{\bar{u}^2 + \bar{v}^2 + \bar{w}^2} = \frac{1 + s\,\Gamma/\gamma}{1 + 2s + s^2} < 1$ if $2\gamma > \Gamma$

# stationary polarizability

calculate expectation value of polarization

$$P \;=\; \mathrm{Trace}\,(\;\rho\,[\,\mu\,|g\rangle\langle e| \;+\; \mu^*\,|e\rangle\langle g|\,]\,) \;=\; \rho_{ge}\,d_{eg} \;+\; \rho_{eg}\,d_{ge}$$

$$= \frac{1}{2}\,\mu^*\,e^{i(\omega t+\phi)}\,(\bar{u} + i\bar{v}) \quad + \quad c.c$$

matrix element in co-rotating basis

$$d_{eg} \;\equiv\; \mu^*\;e^{i(\omega t+\phi)}$$

$$= \frac{1}{2}\,\mu^*\,e^{i(\omega t+\phi)}\left[-\frac{\delta\,\omega_1}{\delta^2+\gamma^2} + i\,\frac{\gamma\,\omega_1}{\delta^2+\gamma^2}\right]\frac{1}{1+s} \qquad + \qquad c.c$$

saturation parameter

$$s \;\equiv\; \frac{\omega_1^{\,2}}{\gamma\,\Gamma}\;\frac{1}{1\;+\;\delta/\gamma\;^2}$$

$$= \frac{1}{\sqrt{2}}\,|\mu|^2\,E^*\,e^{i\omega t}\;\frac{1}{\hbar}\left[-\frac{\delta}{\delta^2+\gamma^2} + i\,\frac{\gamma}{\delta^2+\gamma^2}\right]\frac{1}{1+s} \qquad + \qquad c.c$$

Rabi-frequency

$$\omega_1 \;\equiv\; \frac{\sqrt{2}}{\hbar}\;\mu\,E^*\,e^{-i\phi}$$

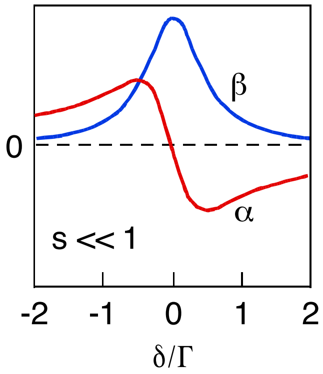
$$= \frac{1}{\sqrt{2}}\;E^*\,e^{i\omega t}\quad(\alpha - i\beta) \quad + \quad c.c$$

$$\alpha \;\equiv\; -\,\frac{|\mu|^2}{\hbar}\;\frac{\delta}{\delta^2+\gamma^2}\;\frac{1}{1+s}$$

$$\beta \;\equiv\; -\,\frac{|\mu|^2}{\hbar}\;\frac{\gamma}{\delta^2+\gamma^2}\;\frac{1}{1+s}$$

resonance

saturation



# Coherent and incoherent scattering rate

Total rate ($\Gamma_{tot}$) of radiation energy emitted by atom in steady state can be split into a coherent ($\Gamma_{coh}$) and an incoherent ($\Gamma_{inc}$) part:

Expand $b \equiv |g\rangle\langle e|$ according to $b = \langle b\rangle + \delta b$ with $\langle \delta b\rangle = 0$ mean value plus fluctuations

$$\Gamma_{tot}/\Gamma \equiv \rho_{ee} = \langle b^+ b\rangle = \langle(\langle b^+\rangle + \delta b^+)(\langle b\rangle + \delta b)\rangle = \langle b^+\rangle\langle b\rangle + \langle \delta b^+ \delta b\rangle \equiv \Gamma_{coh}/\Gamma + \Gamma_{inc}/\Gamma$$

Total rate:

$$\Gamma_{tot}/\Gamma = \frac{s}{2(1+s)}$$

monotonously increases and saturates for large s at 1/2

Coherent rate:

$2\rho_{ge} = u + i\,v$

$$\Gamma_{coh}/\Gamma \equiv \langle b^+\rangle\langle b\rangle = |\rho_{ge}|^2 = \frac{1}{4}(\bar{u}^2 + \bar{v}^2) = \frac{s}{2(1+s)^2}\,\frac{\Gamma}{2\gamma}$$

relative maximum at s=1, zero bandwidth

$\langle b\rangle = \mathrm{Trace}[\,|g\rangle\langle e|\,\rho] = \langle g|g\rangle\langle e|\rho|g\rangle + \langle e|g\rangle\langle e|\rho|e\rangle = \langle e|\rho|g\rangle = \rho_{ge}$

Incoherent rate = total rate - cherent rate:

$$\Gamma_{inc}/\Gamma \equiv \langle \delta b^+ \delta b\rangle = \Gamma_{tot}/\Gamma - \Gamma_{coh}/\Gamma = \frac{s}{2(1+s)} - \frac{s}{(1+s)^2}\,\frac{\Gamma}{4\gamma} = \frac{s}{2(1+s)^2}\left(s+1-\frac{\Gamma}{2\gamma}\right)$$

monotonously increases, saturates for large s at 1/2, bandwidth $\Gamma$

# Significance of coherent rate:

$\mu = |\mu|\, e^{i\xi}\ ,\ \ \chi = \phi - \xi$

$$P = \tfrac{1}{2}\, \mu^*\, e^{i(\omega t + \phi)}\, (\bar{u} + i\bar{v}) \quad + \quad c.c. \quad = \tfrac{1}{2}|\mu|\, e^{i(\omega t + \chi)}\, (\bar{u} + i\bar{v}) \quad + \quad c.c.$$

harmonic addition theorem: $a \sin(x) + b \cos(x) = (a^2+b^2)^{1/2} \cos(x+\theta)$

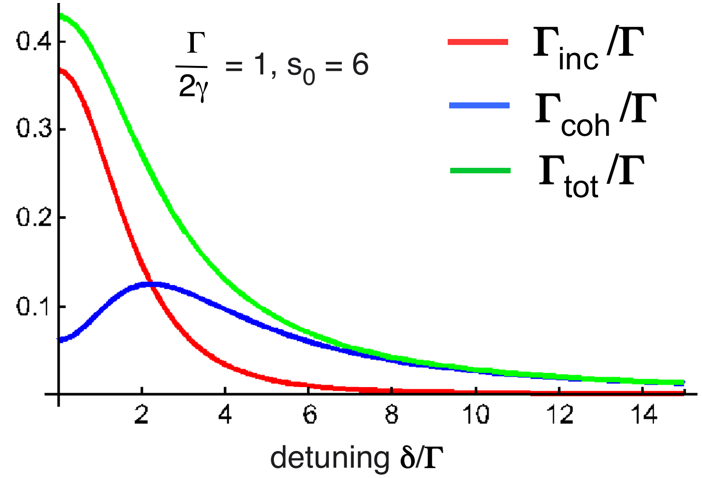
$$= |\mu|\, (\bar{u} \cos(\omega t + \chi) - \bar{v} \sin(\omega t + \chi)) = P_{max} \cos(\omega t + \theta) \quad \text{with} \quad \boxed{P_{max} \equiv |\mu|\, (\bar{u}^2 + \bar{v}^2)^{1/2}}$$

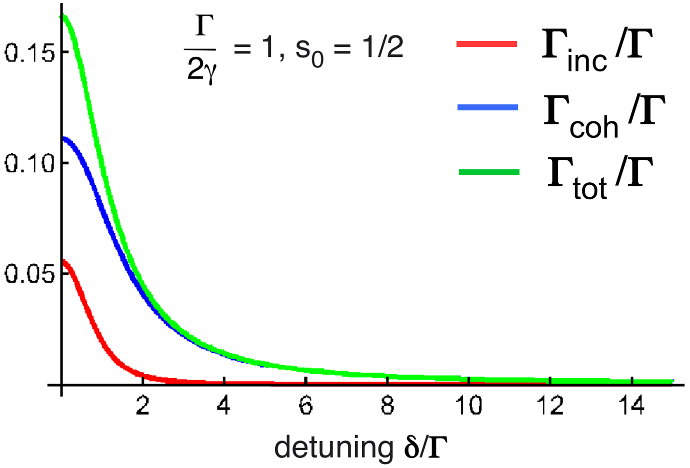
$$\Gamma_{coh} = \tfrac{1}{4}\, (\bar{u}^2 + \bar{v}^2)\, \Gamma = \frac{|P_{max}|^2}{4|\mu|^2}\, \Gamma = \frac{1}{\hbar\omega} \frac{\omega^4 |P_{max}|^2}{12\pi\varepsilon_0 c^3} = \frac{W}{\hbar\omega}$$

use $\Gamma = \dfrac{\omega^3 |\mu|^2}{3\pi\varepsilon_0 c^3 \hbar}$

$$W = \frac{\omega^4 |P_{max}|^2}{12\pi\varepsilon_0 c^3}$$ = power radiated by classical dipole with polarization amplitude $P_{max}$

# coherent and incoherent scattering rate





for $\delta \rightarrow 0$:

$$\Gamma_{inc}/\Gamma = \frac{s_0}{2(1+s_0)^2}\left(1 + s_0 - \frac{\Gamma}{2\gamma}\right)$$

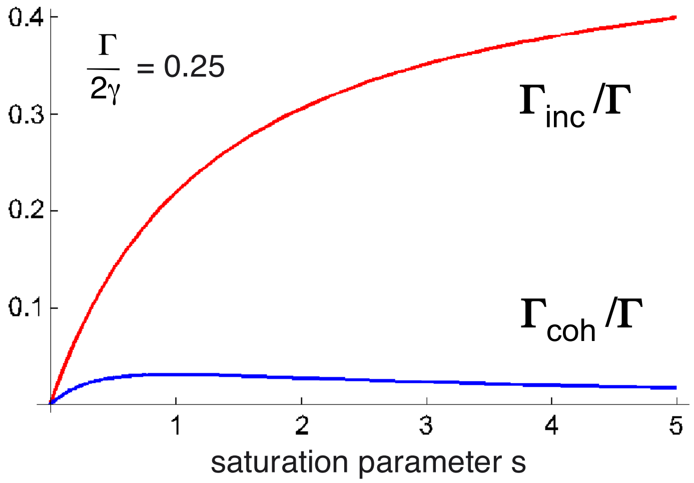
$$\Gamma_{coh}/\Gamma = \frac{s_0}{2(1+s_0)^2}\,\frac{\Gamma}{2\gamma}$$

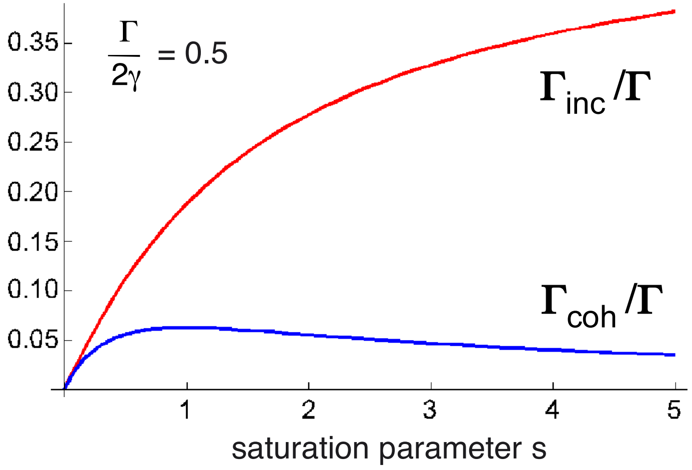
for $\delta \rightarrow \infty$:

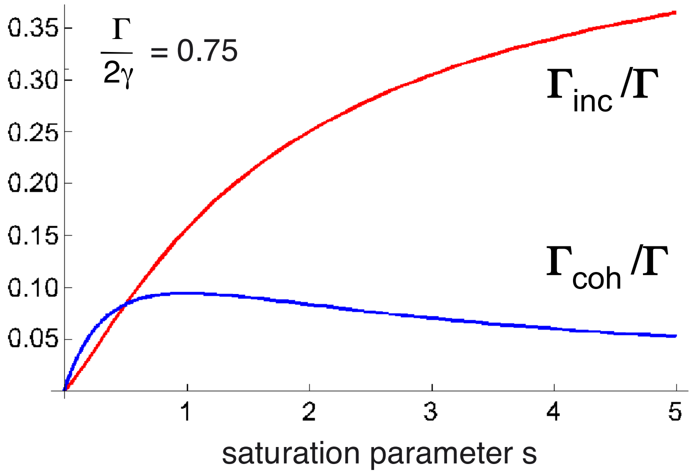
$$\Gamma_{inc}/\Gamma = \frac{s_0}{2\delta^2}\left(1 - \frac{\Gamma}{2\gamma}\right)$$

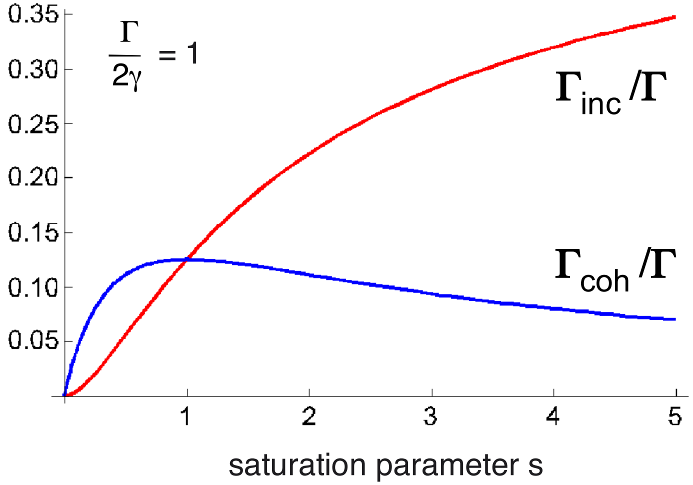
$$\Gamma_{coh}/\Gamma = \frac{s_0}{2\delta^2}\,\frac{\Gamma}{2\gamma}$$

# coherent and incoherent scattering rate









# Quantum mechanical model of two-level-atoms in a monochromatic light field

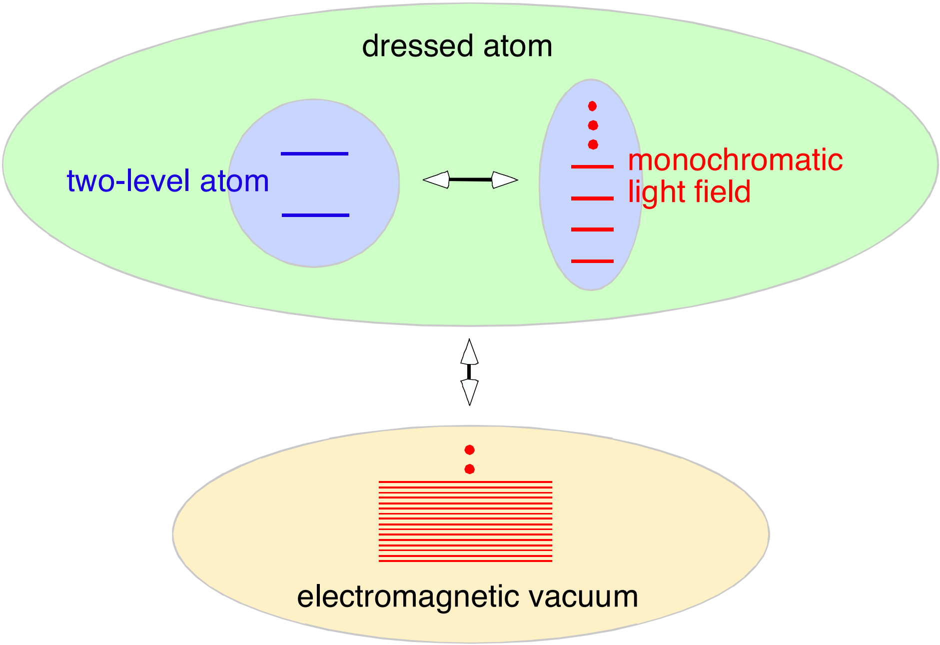
(Jaynes Cummings Model: E. Jaynes and F. Cummings, Proc. IEEE 51, 89 (1963))

## 1) Coupling to electromagnetic field, i.e., laser mode:

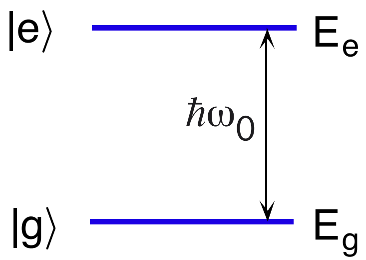
Interaction is conservative. Energy is periodically exchanged between atom and laser mode at characteristic frequencies (Rabi oscillations).

## 2) Coupling to the vacuum modes (Spontaneous decay):

A finite number of discrete states is coupled to infinitely many states with continuous energy spectrum. Dynamics has dissipative character: damping of Rabi-oscillations



## Two-Level Atom:



Atomic Hamiltonian: $H_A = \hbar\omega_0\, b^+ b$

$b \equiv |g\rangle\langle e|$ ground state projector

Atomic Dipole Operator: $d = \mu\, b + \mu^*\, b^+$ with $\mu \equiv \langle g|\, d\, |e\rangle$

# Monochromatic Light Field:

**Classical Electric Field:**

$$E(x,t) = i\sqrt{\frac{\hbar\omega_L}{2\varepsilon_0}}\left[\,\hat{\varepsilon}(x)\,\alpha(t) - \hat{\varepsilon}^*(x)\,\alpha(t)^*\right], \quad \dot{B}(x,t) = -\nabla\times E(x,t)$$

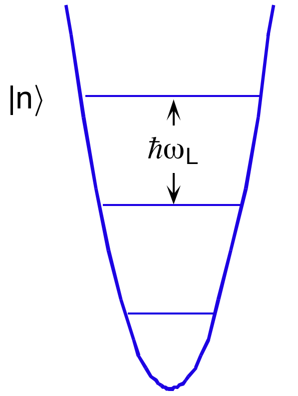
$$\left[\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right] E(x,t) = 0 \quad \begin{cases} \left[\Delta + \left(\frac{\omega_L}{c}\right)^2\right]\hat{\varepsilon}(x) = 0 & \text{Normalization: } 1 = \int \hat{\varepsilon}(x)\,\hat{\varepsilon}^*(x)\; d^3x \\ \left[\frac{\partial^2}{\partial t^2} + \omega_L{}^2\right]\alpha(t) = 0 & \Rightarrow \quad \alpha(t) = \alpha\, e^{-i\omega_L t} \end{cases}$$

$$H = \frac{\varepsilon_0}{2}\int E(x,t)^2\; d^3x + \frac{1}{2\mu_0}\int B(x,t)^2\; d^3x = \hbar\omega_L\,\alpha(t)^*\,\alpha(t) = \hbar\omega_L\,\alpha^*\,\alpha = \frac{1}{2}\hbar\omega_L\,(\alpha^*\alpha + \alpha\,\alpha^*)$$

**Quantization:** $\alpha \to a\,,\ \alpha^* \to a^+\ ,\ [a, a^+] = 1$

Hamiltonian: $\mathbf{H}_L = \hbar\omega_L\,(a^+a + \frac{1}{2})$

$E_n = (\,n + \frac{1}{2}\,)\,\hbar\omega_L$



**Fock-states:** $|n\rangle = \frac{a^{+n}}{\sqrt{n!}}|0\rangle\ ,\quad a|n\rangle = n^{1/2}\,|n{-}1\rangle,\ \ a^+|n\rangle = (n{+}1)^{1/2}\,|n{+}1\rangle$

Electric Field: $\langle n|E|n\rangle = 0$ , phase of Fock-states is undetermined

$$\langle n|E^2|n\rangle = \frac{\hbar\omega_L}{2\varepsilon_0}\,\hat{\varepsilon}(x)\,\hat{\varepsilon}^*(x)\;(2n{+}1) \neq 0$$ even for n = 0 !

( $\langle 0|E^2|0\rangle \neq 0$ → Vacuum Fluctuations )

Energy: $\mathbf{H}_L\,|n\rangle = \hbar\omega_L(n + \frac{1}{2})\,|n\rangle$

# Quasi-Classical States (Coherent States, Glauber States):

Find States such that:

$$\langle\alpha|\mathbf{H}_L|\alpha\rangle = H$$

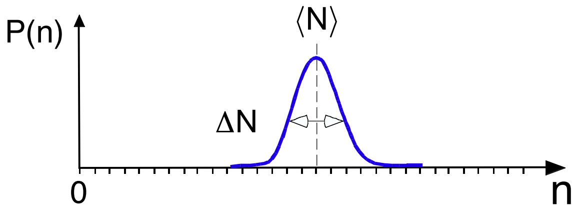
$$\langle\alpha|\mathbf{E}_L|\alpha\rangle = E \quad \text{(Schrödinger Picture)}$$

Solution:

$$a\,|\alpha\rangle = \alpha\,|\alpha\rangle \qquad |\alpha\rangle = \exp(-\frac{|\alpha|^2}{2}) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

Photon Satistics: Probability to measure n photons in state $|\alpha\rangle$ → Poisson Distribution

$$P(n) = |\langle\alpha|n\rangle|^2 = \exp(-|\alpha|^2)\,\frac{|\alpha|^{2n}}{n!}$$



$N \equiv a^{+}a \Rightarrow$

$\langle N\rangle = |\alpha|^2$ mean photon number

$\Delta N = \sqrt{\langle N^2\rangle - \langle N\rangle^2} = \sqrt{\langle N\rangle} = |\alpha|$ shot noise

Completeness:

$$1 = \frac{1}{\pi}\int d^2\alpha\, |\alpha\rangle\langle\alpha|$$

the set of coherent state is over-complete

Orthonormality:

$$\langle\alpha|\beta\rangle = \exp(-|\alpha-\beta|^2)$$

coherent states are nearly orthogonal

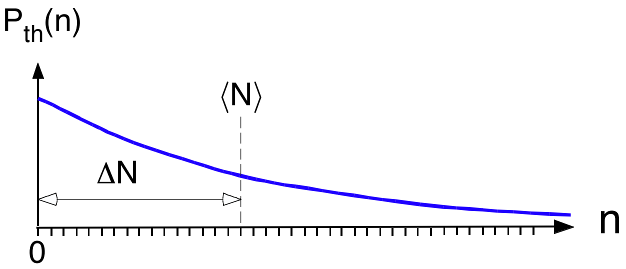
Other Properties: minimum uncertainty states, non-dispersive time-evolution, i.e., $\Delta N = \text{constant}$

# Thermal States:

Find State $\rho_{th}$ such that:

- No ability for interference is maintained, i.e., $\langle E \rangle = 0$
- Probability to measure n photons in state $\rho$ is given by a Boltzmann factor

$$P_{th}(n) \equiv \langle n|\rho|n\rangle = (1 - e^{-\beta})\, e^{-n\beta} \quad , \qquad \beta \equiv \frac{\hbar\omega_L}{k_B T} \quad , \qquad \sum_{n=0}^{\infty} P_{th}(n) = 1$$



Solution:

$$\rho_{th} = \sum_{n=0}^{\infty} P_{th}(n)\; |n\rangle\langle n|$$

Mean Photon Number:

$$\langle N \rangle = \sum_{n=0}^{\infty} n\, P_{th}(n) = \frac{1}{e^{\beta} - 1} \quad \Rightarrow \quad \beta = \ln(1 + \langle N \rangle^{-1})$$

Planck-Distribution !

$$\Delta N = \sqrt{\langle N^2 \rangle - \langle N \rangle^2} = \sqrt{\langle N \rangle + \langle N \rangle^2} \approx \langle N \rangle$$

Electric Field:

$$\langle E \rangle = \mathrm{Trace}(\, \rho_{th}\, E\, ) = 0 \;,\; \langle E^2 \rangle \neq 0$$

# Atom + Laser (without Interaction)

Hamiltonian: $H_{AL} = \hbar\omega_0 \, b^{+}b + \hbar\omega_L \, (a^{+}a + 1/2)$

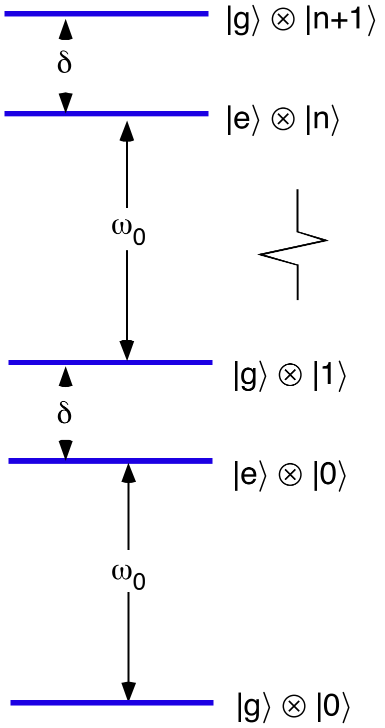
Eigen-Basis of $H_{AL}$ : $\{|g\rangle \otimes |n\rangle, |e\rangle \otimes |n\rangle : n = 0,1...\}$

$E_{g,n+1} = (n+1) \, \hbar\omega_L$

$E_{e,n} = (n+1) \, \hbar\omega_L - \hbar\delta$

$E_{g,n} = 1 \, \hbar\omega_L$

$E_{e,n-1} = 1 \, \hbar\omega_L - \hbar\delta$



# Atom interacting with monochromatic light field

Atom Interacts with Light Field via Dipole Coupling:

$$W \;=\; -\,d\,E \;=\; -\sqrt{\frac{\hbar\omega_L}{2\,\varepsilon_0}}\;i\,\left[\,\mu\hat{\varepsilon}(x)\,b\,a \;-\; \mu^*\hat{\varepsilon}^*(x)\,b^+a^+ \;-\; \mu\hat{\varepsilon}^*(x)\,b\,a^+ \;+\; \mu^*\hat{\varepsilon}(x)\,b^+a\,\right]$$

$$\overset{\text{RWA}}{\approx}\;\sqrt{\frac{\hbar\omega_L}{2\,\varepsilon_0}}\;\left[\,i^*\mu\hat{\varepsilon}^*(x)\,b\,a^+ \;+\; i\,\mu^*\hat{\varepsilon}(x)\,b^+a\,\right]$$

Rotating Wave Approximation: Neglect fast oscillatory terms

Free evolution of $a\,b$ : $\propto$ ~~$\exp(-i\,(\omega_L+\omega_0)t)$~~

Free evolution of $a\,b^+$ : $\propto \exp(-i\,\delta t)$

New Basis: $|e\rangle \rightarrow |e\rangle\, e^{-i\psi}$ , $b \rightarrow b\, e^{i\psi}$ , $\quad e^{i\psi} \equiv \dfrac{-i\,\mu^*\,\hat{\varepsilon}(x)}{|\mu^*\,\hat{\varepsilon}(x)|}$ $\quad\Rightarrow\quad$ $W \;=\; \frac{1}{2}\,\hbar\omega_1\,[\; b\,a^+ \;+\; b^+a\;]$

Rabi-frequency per photon: $\omega_1 \;\equiv\; \sqrt{\dfrac{2\omega_L}{\hbar\varepsilon_0}}\;|\mu^*\hat{\varepsilon}(x)|$

W has non-vanishing matrix elements only within subspaces $\{\, |e\rangle\otimes|n\rangle,\; |g\rangle\otimes|n+1\rangle \,\}$ $\quad\Rightarrow$

Matrix of Hamiltonian $H = H_{AL} + W$ with respect to product basis is composed of 2x2-matrices

$$H \;=\; \begin{pmatrix} E_{g,0} & & & & \\ & H[1] & & & \\ & & H[2] & & \\ & & & \ddots\; H[n] & \\ & & & & \ddots \end{pmatrix} \qquad H[n] \;\equiv\; \begin{pmatrix} E_{e,n\text{-}1} & \frac{1}{2}\hbar\omega_n \\ \frac{1}{2}\hbar\omega_n & E_{g,n} \end{pmatrix}$$

$\omega_n \;\equiv\; \omega_1\,\sqrt{n}$ $\qquad n = 1,2,\ldots$ $\qquad$ n-photon Rabi-frequencies

# Diagonalization of H → Dressed States

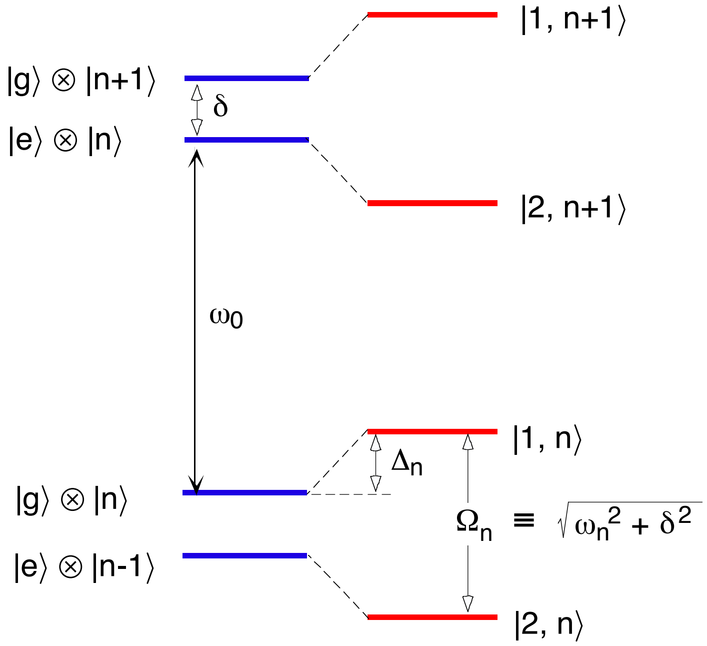
**New Eigen-States: (Dressed States)** photon states and atomic states are entangled

$$|2,n\rangle = \cos(\theta_n)\,|e\rangle\otimes|n-1\rangle - \sin(\theta_n)\,|g\rangle\otimes|n\rangle$$

$$|1,n\rangle = \sin(\theta_n)\,|e\rangle\otimes|n-1\rangle + \cos(\theta_n)\,|g\rangle\otimes|n\rangle$$

n = 1,2,....

Interaction Angle: $\theta_n \equiv \frac{1}{2}\arctan[\frac{\omega_n}{\delta}]$



**New Eigen-Energies:**

$$E_{2,n} = E_{e,n-1} - \hbar\Delta_n$$

$$E_{1,n} = E_{g,n} + \hbar\Delta_n$$

Light Shift: $\Delta_n = \frac{\delta}{2}\left[\sqrt{1+\frac{\omega_n^2}{\delta^2}} - 1\right]$

$$\approx \frac{\omega_n^2}{4\delta} \quad \text{if} \quad \omega_n << |\delta|$$

**Case of Resonance:** In Resonance ($\delta$=0) mixing becomes maximal: $\theta_n = \pi/4$

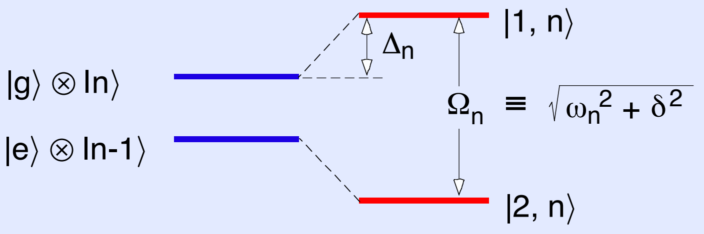
$$\Rightarrow \qquad \cos(\theta_n) = \sin(\theta_n) = \frac{1}{\sqrt{2}}$$

$$\Delta_n = \frac{\omega_n}{2}$$

**Selection rules:** Matrix Elements of Dipole-Operator for Dressed Atom: $d_{ij} = \langle i,n\text{-}1|\, d\, |\, j,n\rangle \neq 0$ for all i,j

# Excitation probability for Fock-state $|g\rangle \otimes |n\rangle$ :

(stimulated absorption and emission)



$$|2,n\rangle = \cos(\theta_n)\, |e\rangle\otimes|n-1\rangle - \sin(\theta_n)\, |g\rangle\otimes|n\rangle$$

$$|1,n\rangle = \sin(\theta_n)\, |e\rangle\otimes|n-1\rangle + \cos(\theta_n)\, |g\rangle\otimes|n\rangle$$

General solution of Schrödinger equation within n-th family

$$|\psi(t)\rangle \equiv A_1 \exp(-i\, E_{1,n}\, t/\hbar)\, |1,n\rangle + A_2 \exp(-i\, E_{2,n}\, t/\hbar)\, |2,n\rangle$$

special solution with $|\psi(0)\rangle = |g\rangle\otimes|n\rangle$ :

$$|\psi(t)\rangle \equiv \cos(\theta_n)\, |1,n\rangle - \exp(-\,i\Omega_n\, t) \sin(\theta_n)\, |2,n\rangle$$

after half of a Rabi-cycle:

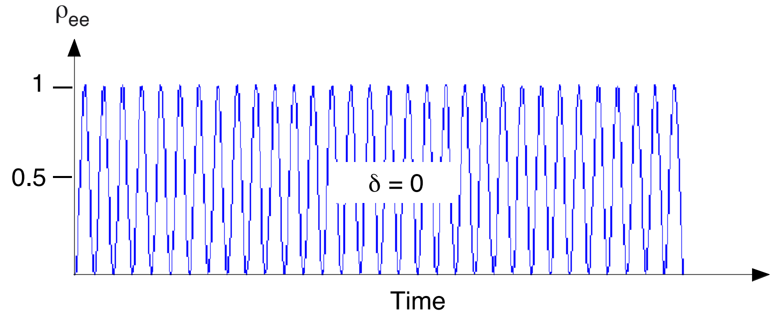
$$|\psi(\Omega_n t = \pi)\rangle = \sin(2\theta_n)\, |e\rangle\otimes|n-1\rangle + \cos(2\theta_n)\, |g\rangle\otimes|n\rangle = \frac{\omega_n}{\Omega_n}\, |e\rangle\otimes|n-1\rangle - \frac{\delta}{\Omega_n}\, |g\rangle\otimes|n\rangle$$

(resonance → complete inversion)

population of excited state:

$$\rho_{ee}(n) = |\langle e,n-1|\psi(t)\rangle|^2 = \left| \sin(\theta_n)\cos(\theta_n)\, \left(1 - \exp(-i\Omega_n\, t)\right) \right|^2 = \frac{\omega_n^2}{\Omega_n^2} \sin^2\left(\frac{1}{2}\, \Omega_n\, t\right)$$

$$= \frac{n\,\omega_1^2}{n\,\omega_1^2 + \delta^2} \sin^2\left(\frac{1}{2}\sqrt{n\,\omega_1^2 + \delta^2}\;\; t\right)$$

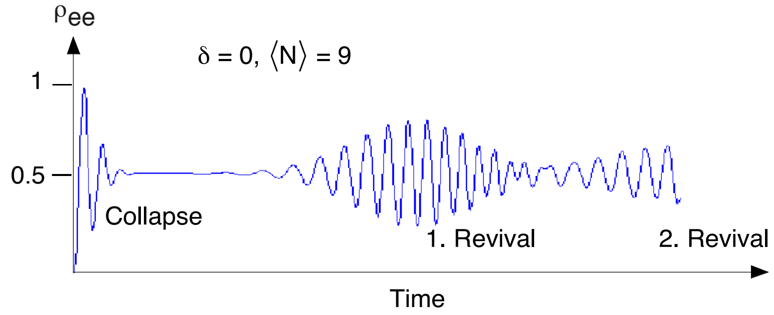


# Atoms dressed by a coherent state or a thermal state:

Excitation probability for coherent state: $|g\rangle \otimes |\alpha\rangle = \exp(-\frac{|\alpha|^2}{2}) \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |g\rangle \otimes |n\rangle$ :

$$\rho_{ee} = \sum_{n=0}^{\infty} P_{coh}(n)\, \rho_{ee}(n) \ , \quad P_{coh}(n) = \exp(-|\alpha|^2) \frac{\alpha^{2n}}{n!}$$

$$= \exp(-|\alpha|^2) \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} \frac{n\omega_1^2}{n\omega_1^2 + \delta^2} \sin^2\left(\frac{1}{2}\sqrt{n\omega_1^2 + \delta^2}\ t\right)$$

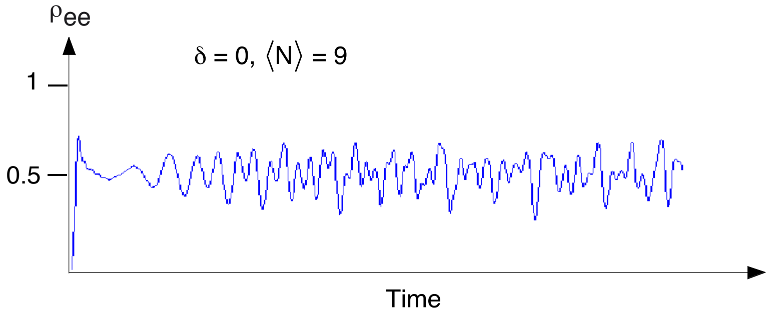


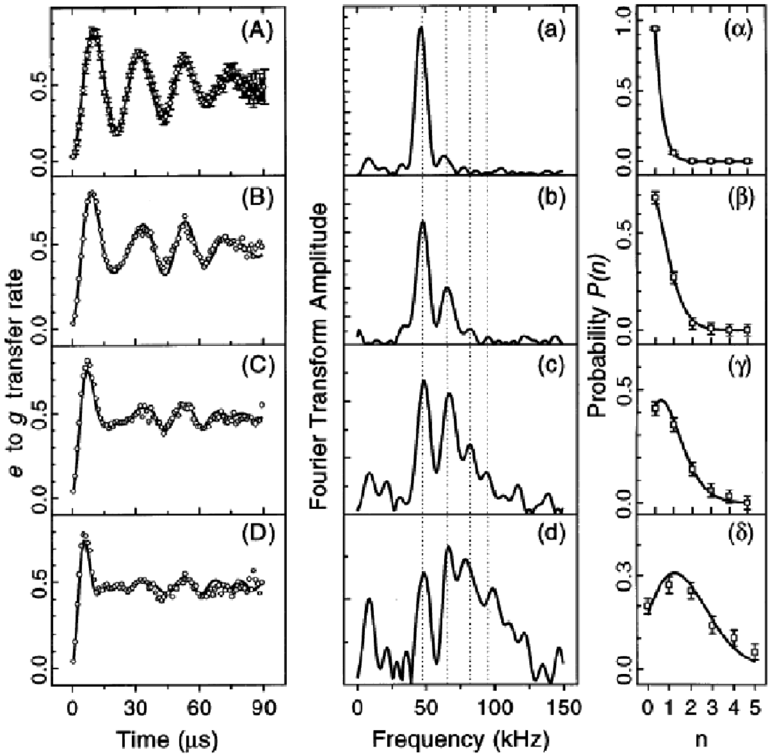
Revivals in $\rho_{ee}$ are a signature of field quantization.
They can only occur because the sum over $\rho_{ee}(n)$ is discrete.

Excitation probability for thermal state: $|g\rangle\langle g| \otimes \rho_{th} = |g\rangle\langle g| \otimes \sum_{n=0}^{\infty} P_{th}(n)\, |n\rangle\langle n|$

$$\rho_{ee} = \sum_{n=0}^{\infty} P_{th}(n)\, \rho_{ee}(n) \ , \quad P_{th}(n) = (1 - e^{-\beta})\ e^{-n\beta} \ , \quad \beta = \ln(1 + \langle N\rangle^{-1})$$

$$= (1 - e^{-\beta}) \sum_{n=0}^{\infty} e^{-n\beta} \frac{n\omega_1^2}{n\omega_1^2 + \delta^2} \sin^2\left(\frac{1}{2}\sqrt{n\omega_1^2 + \delta^2}\ t\right)$$

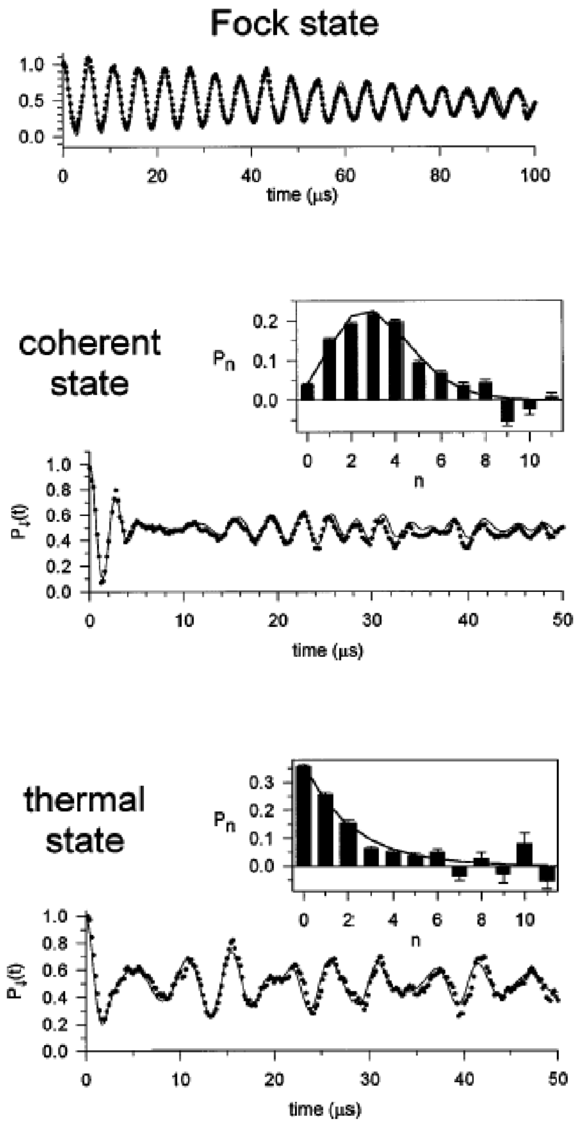




M. Brune et al., Phys. Rev. Lett. 76, 1800 (1996).

Two-level system = two electronic Rydberg levels of Rubidium

Harmonic Oscillator = single mode of RF cavity



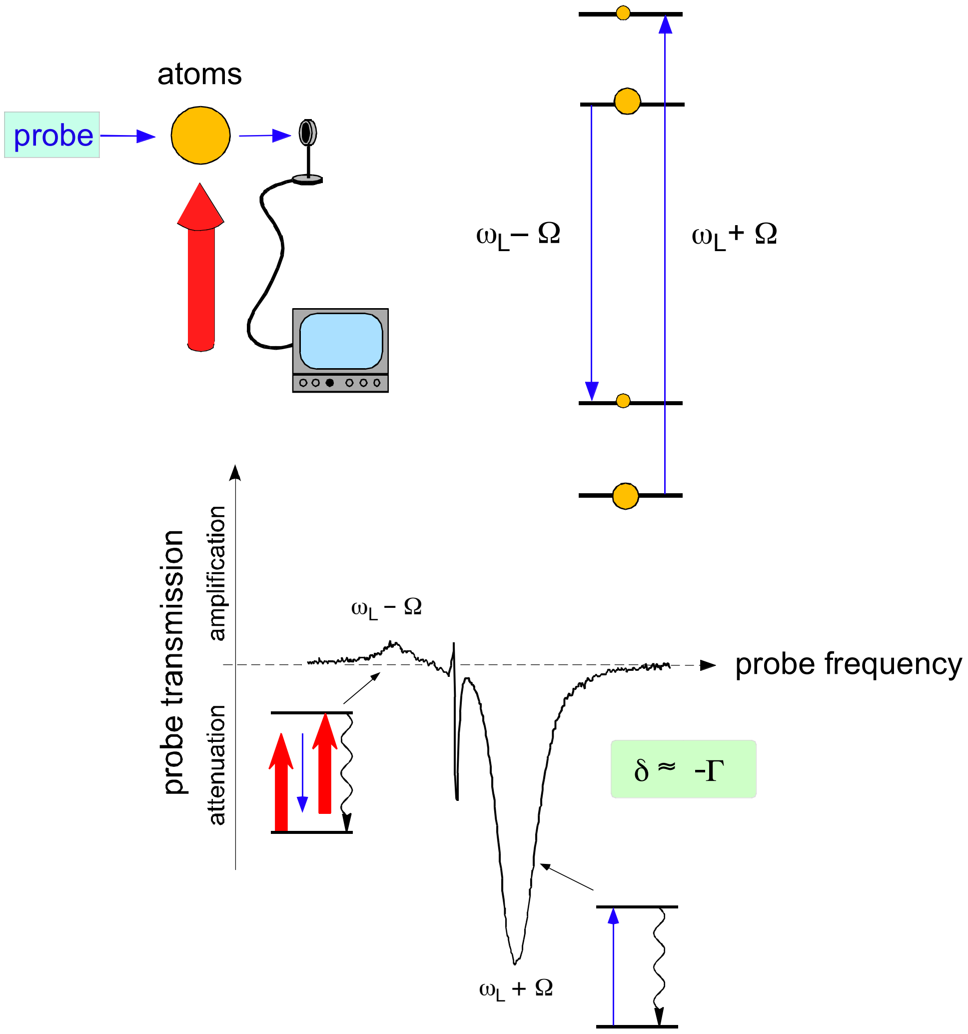
D. Meekhof et al., Phys. Rev. Lett. 76, 1796 (1996).

Two-level system = two electronic levels of an ion

Harmonic Oscillator = vibrational mode in RF-trap

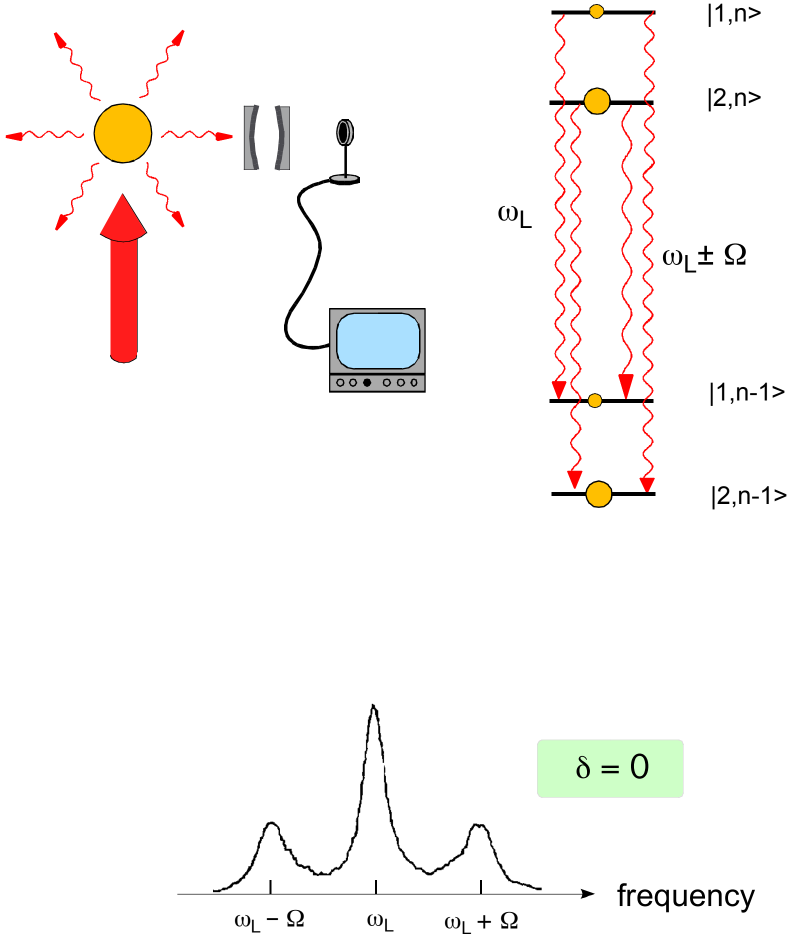
# Absorption and Fluorescence Spectrum of Atoms coupled to a Laser Field

## Probe Transmission



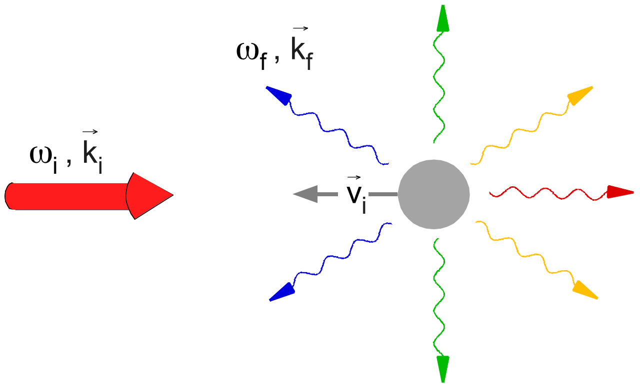
D. Grison et al., Europhys. Lett. 15, 149 (1991).

## Fluorescence



Grove et al., Phys. Rev. A15, 227 (1977).

# Radiation Pressure: Energy-Momentum Budget



single absorption emission cycle:

$$\hbar\omega_f - \hbar\omega_i = -\frac{\hbar^2(\vec{k}_f - \vec{k}_i)^2}{2\,m} + \hbar\,(\vec{k}_f - \vec{k}_i)\,\vec{v}_i$$

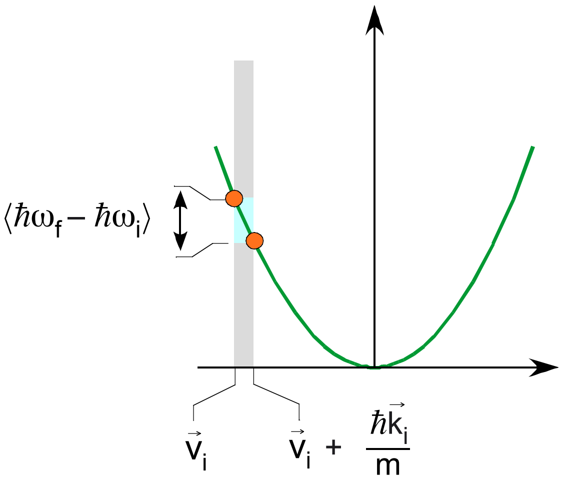
$$\vec{v}_f - \vec{v}_i = -\frac{\hbar\,(\vec{k}_f - \vec{k}_i)}{m}$$

time-averaged for $|\vec{k}_i| \approx |\vec{k}_f|$ :

$$\langle \hbar\omega_f - \hbar\omega_i \rangle = -\frac{\hbar^2 \vec{k}_i^{\,2}}{m} - \hbar\,\vec{k}_i\,\vec{v}_i$$

$$\langle \vec{v}_f - \vec{v}_i \rangle = \frac{\hbar\vec{k}_i}{m}$$

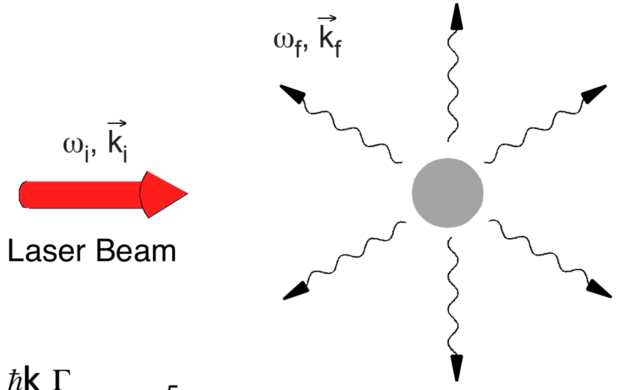
recoil term Doppler term

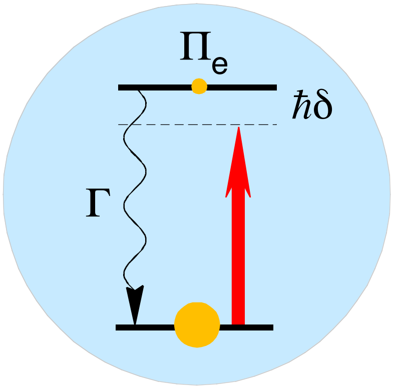


$\langle \hbar\omega_f - \hbar\omega_i \rangle > 0$ if atomic velocity exceeds recoil-velocity & laser beam counterpropagates atomic motion

# Radiation Pressure Cooling

## Resting Atom:

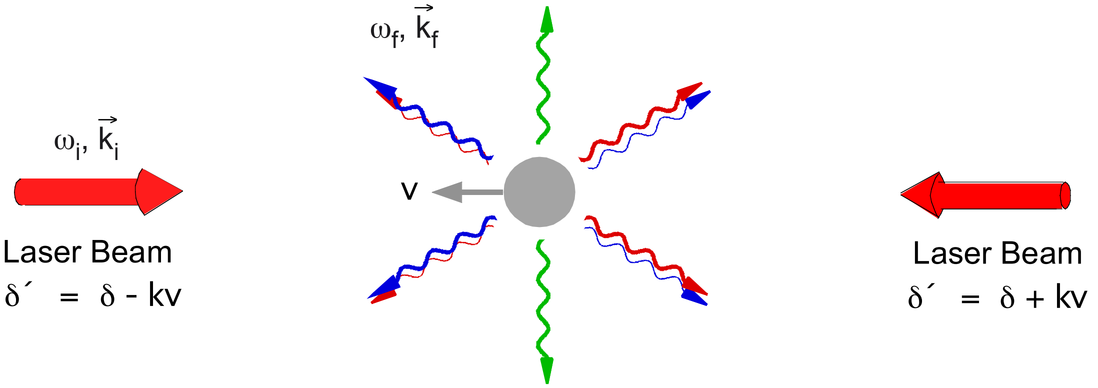




Force $=$ $\hbar k \, \Gamma \, \Pi_e$ typical acceleration: $\frac{\hbar k \, \Gamma}{m} = 10^5$ g

population of excited state: $\Pi_e = \frac{1}{2} \frac{s}{1+s} = \frac{\omega_1^2}{4\delta^2 + \Gamma^2 + 2\omega_1^2}$ s = saturation parameter

## Moving Atom:



$\delta' = \delta - kv$

$\delta' = \delta + kv$

Force $=$ $\hbar k \, \Gamma \, ( \, \Pi_e(\delta + kv) - \Pi_e(\delta - kv)) \approx \alpha \, v + O(v^2)$ $\alpha = \frac{16 \, \omega_1^2 \, \delta \, \Gamma}{(4\delta^2 + \Gamma^2 + 2\omega_1^2)^2} \hbar k^2$

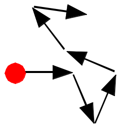
Friction coefficient $\alpha$ has a maximum for $\omega_1 = \Gamma$, $\delta = -\Gamma/2$ : $\alpha_{max} = -\frac{1}{2} \hbar k^2$

# Energy Budget for Heating and Cooling

Heating:

1. Random Direction of Spontanous Emission (1 Beam):
   Assume atom initially at rest. Random momentum kicks accelerate atom.
   This yields random walk in momentum space with an increase of kinetic energy linear in time.

$$P_1(t) = \sum_{i=1}^{N = \Pi_e \Gamma t} \hbar \mathbf{k}_i \quad \Rightarrow \quad P_1(t)^2 = \sum_{i=1,\, j=1}^{N = \Pi_e \Gamma t} \hbar^2 \, \mathbf{k}_i \, \mathbf{k}_j = \Pi_e \hbar^2 k^2 \Gamma t$$



2. Absorption Shot Noise (1 Beam): Number of absorption events is $N \pm \Delta N$, $\Delta N = \sqrt{N}$
   Different atoms experience different momentum transfer per time -> velocity distribution spreads out

$$P_2(t) = \hbar k \, \Delta N = \hbar k \sqrt{N} \quad \Rightarrow \quad P_2(t)^2 = \Pi_e \hbar^2 k^2 \, \Gamma t$$

Total change of kinetic energy after time t (n Beams):

$$\Rightarrow \left(\frac{\partial}{\partial t}\right)_{\mathrm{diff}} E_{\mathrm{kin}} = \frac{D}{m} \, , \qquad D = n \, \Pi_e \hbar^2 k^2 \, \Gamma \qquad \text{Diffusion constant}$$

Cooling: Radiation Pressure Damping:

$$\left(\frac{\partial}{\partial t}\right)_{\mathrm{fric}} E_{\mathrm{kin}} = \frac{P}{m} \frac{\partial P}{\partial t} = \frac{P}{m} \, \alpha \, \frac{P}{m} = \frac{2\alpha}{m} E_{\mathrm{kin}}$$

# Doppler Limit

in Steady State: $$\left(\frac{\partial}{\partial t}\right)_{\text{diff}} E_{\text{kin}} + \left(\frac{\partial}{\partial t}\right)_{\text{fric}} E_{\text{kin}} = 0 \quad \Rightarrow \quad \overline{E_{\text{kin}}} = \frac{D}{2\alpha}$$

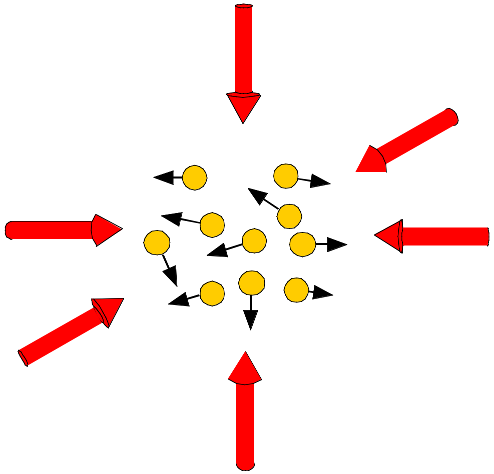
$$\Rightarrow \quad k_B\,T = \frac{n}{4d}\,\frac{\left(\delta^2 + \left(\frac{\Gamma}{2}\right)^2 + \frac{\omega_1^2}{2}\right)}{|\delta|\,\Gamma}\,\hbar\Gamma \qquad d \equiv \text{number of degrees of freedom}$$

Minimum with Respect to $\delta$ : $$|\delta| = \frac{\Gamma}{2}\sqrt{1 + 2\left(\frac{\omega_1}{\Gamma}\right)^2} \quad \Rightarrow \quad k_B\,T = \frac{n}{4d}\sqrt{1 + 2\left(\frac{\omega_1}{\Gamma}\right)^2}\;\hbar\Gamma$$

- Temperature acquires minimum value at vanishing laser intensity ($\omega_1 = 0$).

  However, if $\omega_1$ tends to zero, the time needed to reach the steady state temperature $1/\Pi_e\,\Gamma$ approaches infinity.
- Dopplerlimit (low saturation): $\frac{\hbar\Gamma}{2k_B} = 240\,\mu K$ for Sodium $\frac{\hbar\Gamma}{2k_B} = 139\,\mu K$ for Rubidium
- Model does not account for interference and polarization effects.

# 3D Optical Molasses



laser beams, Ø ≈ 1 cm

- Atoms inside the illuminated volume perform a diffusive motion under strong friction -> optical molasses
- No trapping occurs in optical molasses, however it can take seconds to drift out of the illuminated volume
- Typical Geometry for 3D Optical Molasses: 3 degrees of freedom, 6 beams -> n = 2d,

  however, other geometries are possibel, e.g., with four beams.
- Dopplerlimit (low saturation): $\frac{\hbar\Gamma}{2k_B}$ = 240 $\mu$K for Sodium
- First experimental realization with sodium 1985: S. Chu et al., Phys. Rev. Lett 55, 48 (1985).

  Experiment seemed to confirm Doppler theory. However, later experiments (P. Lett et al., Phys. Rev. Lett 61, 169 (1988))

  showed much lower temperatures, which could not be explained by Doppler cooling.

# Temperature Measurement (Time of Flight Method)

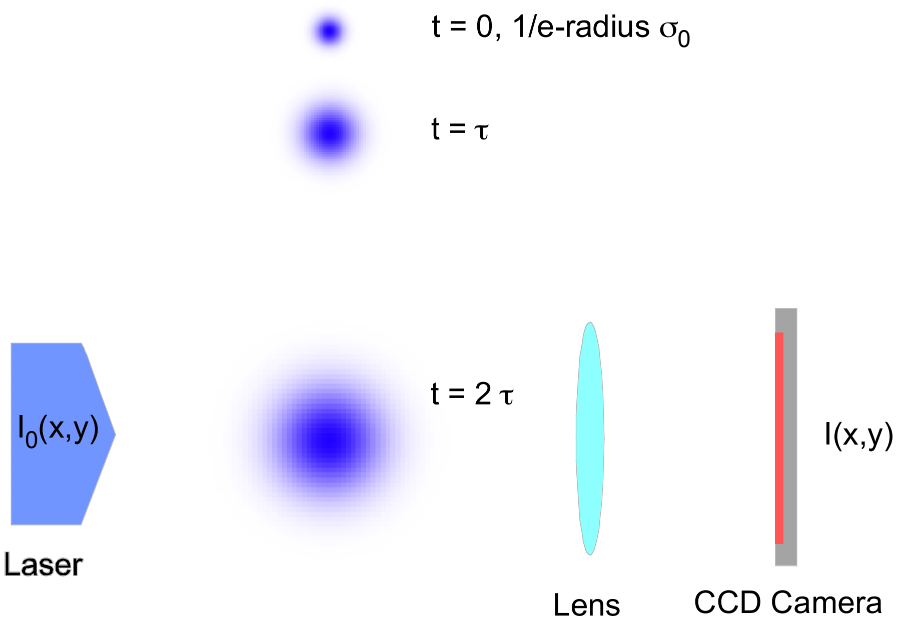
Assumption: $t=0$: $\rho(r,0) = \rho_0 \exp(- \frac{r^2}{\sigma_0{}^2})$

Gaussian density distribution at $t=0$

$$\Rightarrow \quad \rho(r) = \rho_0(t) \exp(- \frac{r^2}{\sigma(t)^2}) \;, \quad \rho_0(t) = \rho_0 \left(\frac{\sigma_0}{\sigma(t)}\right)^3$$

$$\sigma(t) = \sqrt{\sigma_0{}^2 + \bar{v}^2 t^2}$$

$$\bar{v} = \sqrt{\frac{2\,k_B T}{m}}$$



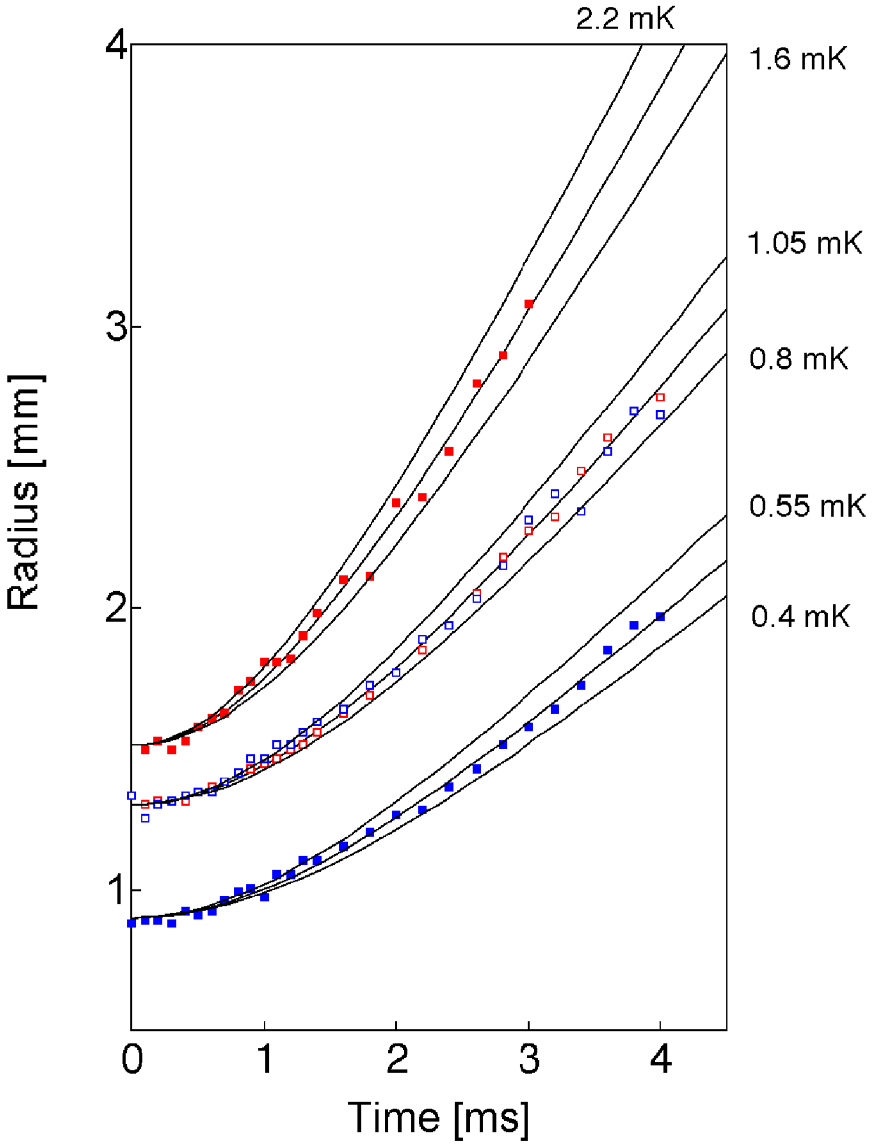
$$\frac{I(x,y)}{I_0(x,y)} = \exp\left(- \Sigma(\lambda) \int_{-\infty}^{\infty} dz\, \rho(x,y,z)\right) \;, \quad \Sigma(\lambda) = \text{absorption cross-section}$$

$$\Sigma(\lambda) = \frac{k}{\varepsilon_0}\, \beta = \frac{k}{\varepsilon_0} \frac{|\mu|^2}{\hbar} \frac{\gamma}{\delta^2 + \gamma^2} \frac{1}{1+s} = \Sigma_0(\lambda) \frac{1}{1 + 4\,(\delta/\Gamma)^2} \frac{1}{1+s}$$

resonant absorption cross-section: $\Sigma_0(\lambda) = \frac{3\lambda^2}{2\pi}$

imaginary part of atomic polarizability

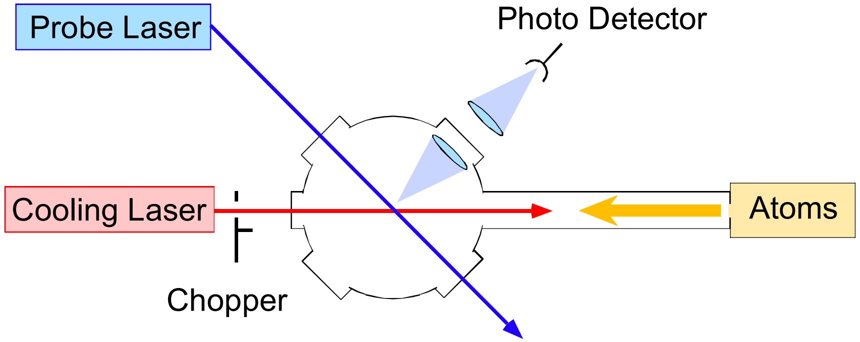
$$|\mu|^2 = \Gamma \frac{\varepsilon_0\, 3\pi\, \hbar\, c^3}{\omega^3} \;, \quad \gamma = \Gamma/2$$

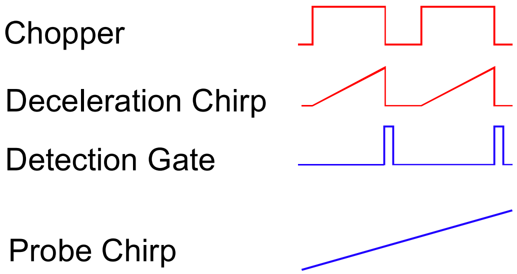

2.2 mK
1.6 mK
1.05 mK
0.8 mK
0.55 mK
0.4 mK
Radius [mm]
4
3
2
1
0
1
2
3
4
Time [ms]

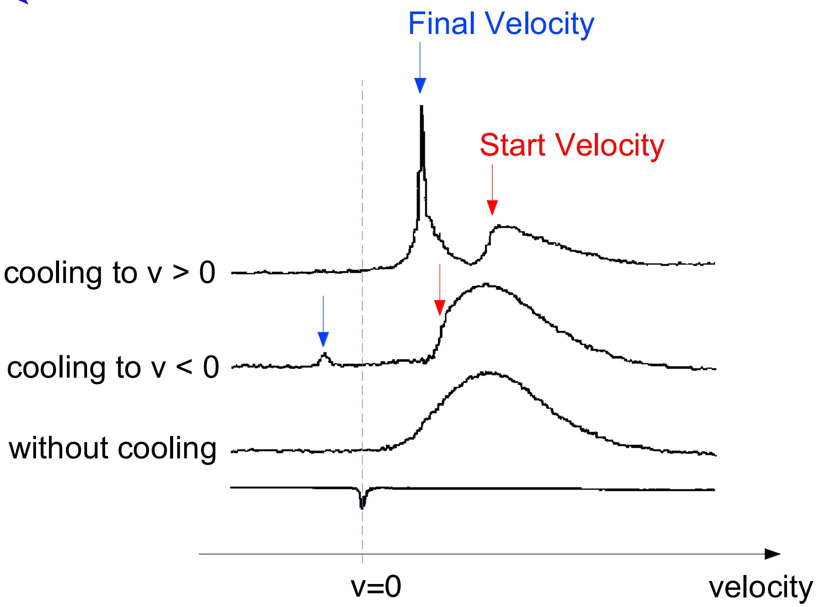
# Deceleration of fast atoms with Chirp Technique

Problem: During deceleration atoms are tuned out of resonance -> velocity capture range $\Delta v = \Gamma/k \approx 10$ m/sec

Solution: Tune frequency of deceleration laser during deceleration in order to compensate decreasing Doppler-effect.





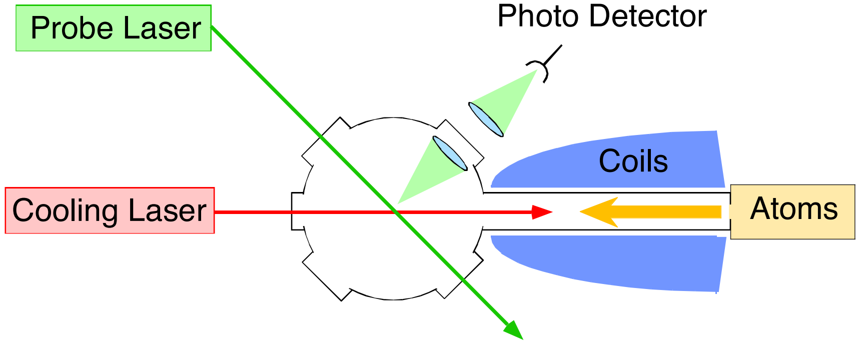


V. Balykin, et al., Sov. Phys. JETP 53, 919 (1981)
W. Ertmer, et al., Phys. Rev. Lett. 54, 996 (1985)

# Deceleration with Zeeman Technique

Problem: During deceleration atoms are tuned out of resonance -> velocity capture range $\Delta v = \Gamma/k \approx 10$ m/sec

Solution: Tune transition frequency of atom (by means of Zeeman-effect) during deceleration in order to compensate Doppler-effect.

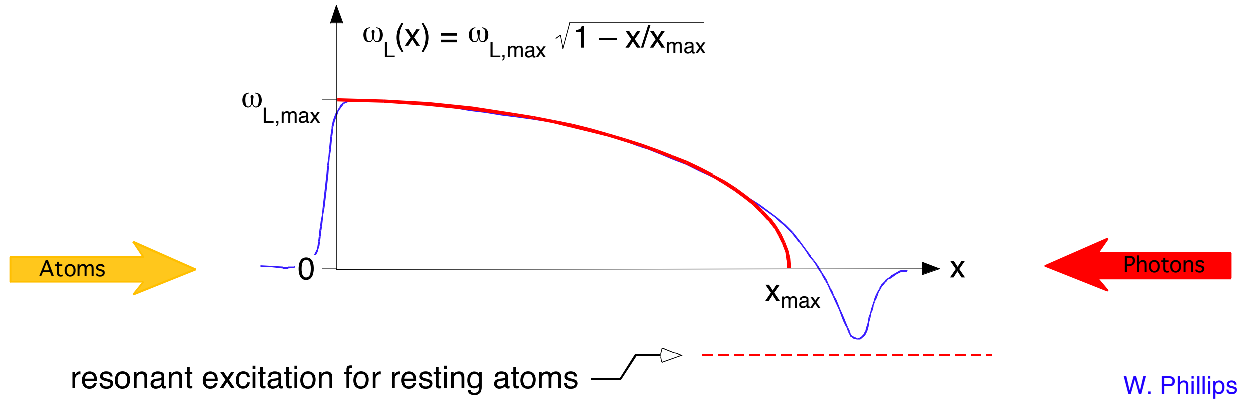


Adjusting optimal magnetic field gradient:

choose max. velocity $v_{max}$ to be decelerated: $\Rightarrow$ stopping distance: $x_{max} = \frac{v_{max}^2}{2a}$ , $a = \Pi_e \frac{\hbar k \Gamma}{m}$

$$\begin{aligned} x(t) &= v_{max}\, t - \tfrac{1}{2} a t^2 \\ v(t) &= v_{max} - a t \end{aligned} \quad \Rightarrow \quad v(x) = v_{max} \sqrt{1 - x/x_{max}} \quad \Rightarrow \quad \omega_L(x) = \omega_{L,max} \sqrt{1 - x/x_{max}}$$

max. Lamor-frequency $\omega_{L,max} = k\, v_{max}$



W. Phillips and H. Metcalf, Phys. Rev. Lett. 48, 596 (1982)

J. Prodan et al., Phys. Rev. Lett. 49, 1149 (1982)

# Can Radiation Pressure be Used for Trapping ?

radiation pressure and Poynting-vector $\mathbf{S}(r,t) = \mathbf{E}(r,t) \times \mathbf{H}(r,t)$:

$$\mathbf{S} \equiv \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}^*, \quad \mathrm{Re}(\mathbf{S}) = \langle\, \mathbf{S}(r,t)\,\rangle$$ time averaged Poynting vector

$$\nabla\times\mathbf{E} = -i\omega\,\mathbf{B}\ ,\ \nabla\times\mathbf{B} = i\,\frac{\omega}{c^2}\,\mathbf{E} \quad \Rightarrow \quad \mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}^* = \frac{-i}{\mu_0\,\omega}\,\mathbf{E}\times(\nabla\times\mathbf{E}^*)$$

Assume spatially constant polarization: $\mathbf{E}(x) = \hat{\mathbf{e}}\, f(x)$ , $\hat{\mathbf{e}}\,\hat{\mathbf{e}}^* = 1 \quad \Rightarrow \quad \mathbf{S} = \frac{-i}{\mu_0\,\omega}\, f\, \nabla\, f^*$

use $f(x) = \sqrt{I(x)/\varepsilon_0}\ e^{-i\psi(x)}$ and $\nabla\mathbf{E} = 0 \quad \Rightarrow \quad S = \frac{c^2}{\omega}\,(\,I\,\nabla\psi - \frac{i}{2}\,\nabla I\,)$

i.e., for isotropic atoms in light fields with spatially constant polarization: $\mathbf{F}_{RAP} = \beta\, I\, \nabla\psi = \beta\, \frac{\omega}{c^2}\, \mathrm{Re}(\mathbf{S})$

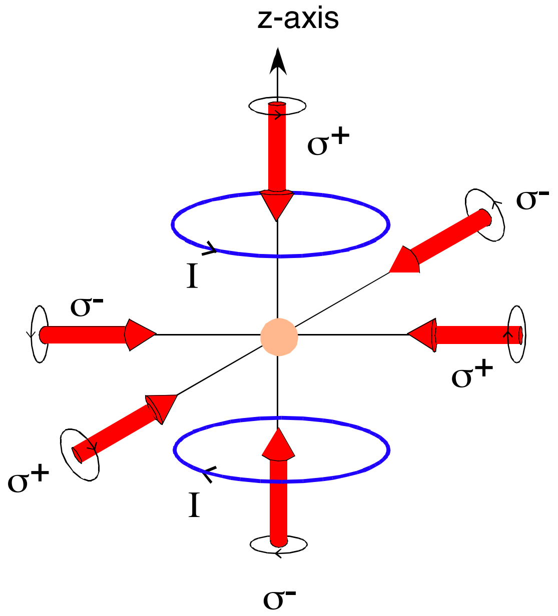
if $\beta$ is spatially constant: $\nabla\, \mathbf{F}_{RAP} = \nabla\mathbf{S} = 0 \quad \Rightarrow$ radiation pressure trapping impossible

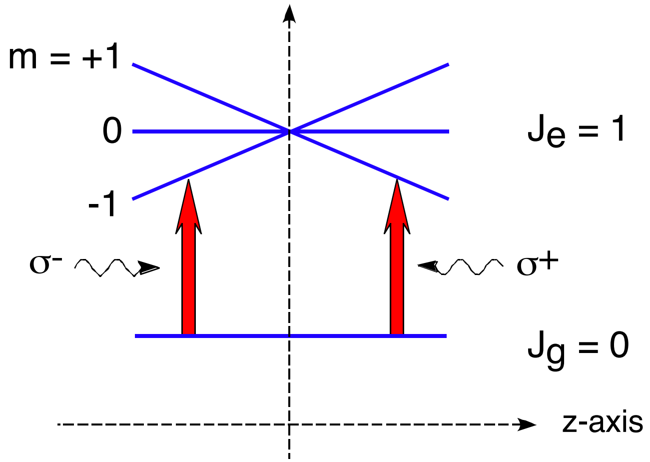
Traps based upon radiation pressure need spatially varying imaginary part of polarizability $\beta = \beta(r)$

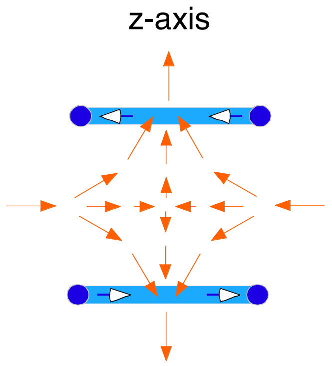
- use fields with spatially varying intensity and make use of the effect of saturation
- use static magnetic field to taylor $\beta(r)$ ——▷ Magneto-Optic Trap

# Magneto-Optic Trap:

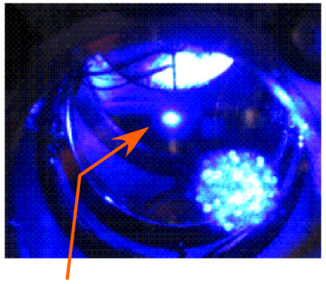
E. Raab, et al., Phys. Rev. Lett. **59**, 2631 (1987)







Typical MOT parameters:

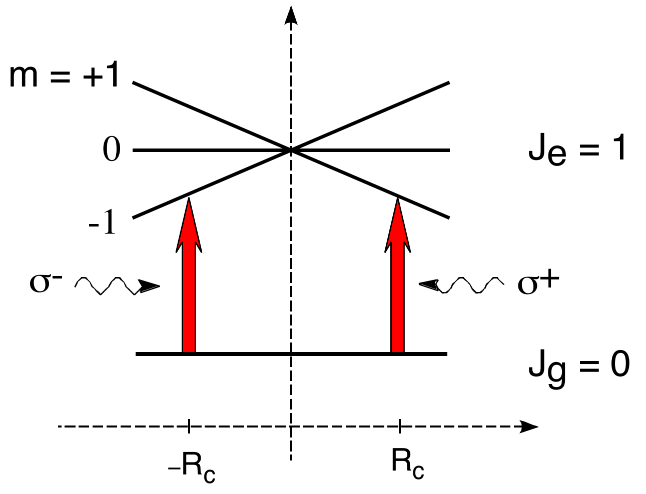


trapped atoms

| | |
|---|---|
| Diameter of Laser Beams | 1 cm |
| Power/Laser Beam | 10 mW |
| Detuning of Laser Frequency | 1 Γ |
| Magnetic Field Gradient | 10 Gauss/cm |

| | |
|---|---|
| Number of trapped Atoms | $10^9$ |
| Peak Density of trapped Atoms (Limited by Fluorescence) | $10^{11}$ atoms/cm$^{-3}$ |
| Temperature (below Doppler Limit) | 10 μK |
| Phase Space Density $\rho\Lambda^3$ | $10^{-6}$ |

# Doppler Theory of MOT



Doppler Cooling and Trapping:  (Neglecting Interference)

$$\frac{F}{m} = \left(\Pi_{+1}(\delta + kv + \beta z) - \Pi_{-1}(\delta - kv - \beta z)\right)\frac{\hbar k \Gamma}{m} = \gamma\, v + \omega_{vib}^{2}\, z + O(v^2, z^2)$$

damped harmonic oscillator !

$$\gamma = 16\, \omega_{rec}\, \frac{\omega_1^{2}}{\tilde{\Gamma}^2}\, \frac{\Gamma}{\tilde{\Gamma}}\, \frac{\tilde{\delta}}{(1 + \tilde{\delta}^2)^2}\,, \qquad \omega_{vib}^{2} = \gamma\, \frac{\beta}{k}$$

$\tilde{\Gamma} = \Gamma \sqrt{1 + 2\frac{\omega_1^{2}}{\Gamma^2}}$ = power broadened linewidth

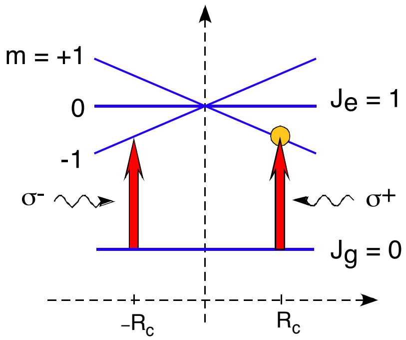
$\tilde{\delta} = \frac{\delta}{\tilde{\Gamma}/2}\,, \quad \beta = \frac{\partial \omega_B}{\partial z}$

$\omega_{rec} = \hbar k^2/2m$ = recoil frequency, $\omega_B$ = Larmor frequency

Relative Maximum of $\gamma$ : $\tilde{\delta} = -\frac{1}{\sqrt{3}}\,, \ \omega_1 = \Gamma$

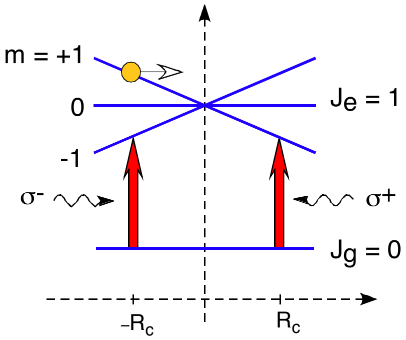
Steady State Temperature: $k_B T = -\frac{\hbar \tilde{\Gamma}}{4}\left(\tilde{\delta} + \tilde{\delta}^{-1}\right)$ Minimum: $\tilde{\delta} = -1, \ \omega_1 = 0$

# Optimizing MOT parameters: capture radius $R_c$ , magnetic gradient, capture veleocity $V_c$



## capture radius $R_c$ & magnetic gradient

$2\beta R_c = 2|\delta| = \tilde{\Gamma}|\tilde{\delta}|$ ⇒ $\beta = \frac{\tilde{\Gamma}|\tilde{\delta}|}{2\,R_c}$ ≈ few Gauss/cm



## capture velocity $v_c$ & detuning

$k\,V_c = 2|\delta| = \tilde{\Gamma}|\tilde{\delta}|$ ⇒ $V_c = \frac{\tilde{\Gamma}|\tilde{\delta}|}{k}$ choose large $\tilde{\delta}$

Stopping Length $S_c = \frac{V_c^2}{2a}$ = Path Length for Stopping Atoms with velocity $v_c$ :

$$\frac{S_c}{R_c} = \frac{\beta}{\beta_{max}} \;,\quad \beta_{max} = \frac{\omega_1^2}{\tilde{\Gamma}^2}\,\frac{\Gamma}{\tilde{\Gamma}}\,\frac{2k\,\omega_{rec}}{|\tilde{\delta}|} \approx \text{GHz/cm}$$

MOT strongly overdamped ⇒ Do not maximize $\gamma$ but rather $v_c$

# Number of Trapped Atoms:

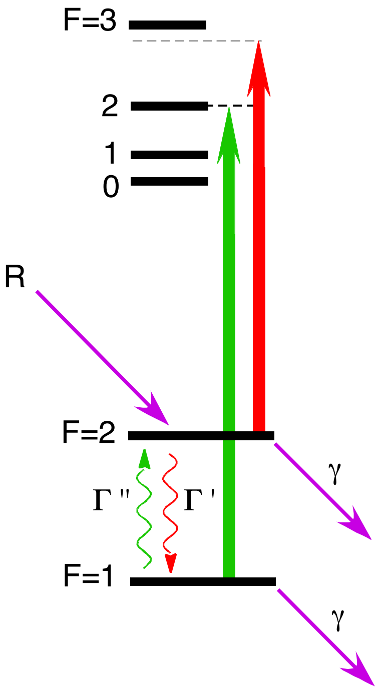
$$\dot{N}_2 = R - \gamma N_2 + \Gamma'' N_1 - \Gamma' N_2$$

$$\dot{N}_1 = -\gamma N_1 - \Gamma'' N_1 + \Gamma' N_2$$

Steady State Solution:

$$N_2 = R \, \frac{\gamma + \Gamma''}{\gamma\,(\gamma + \Gamma' + \Gamma'')}$$

$$N_1 = N_2 \, \frac{\Gamma'}{\gamma + \Gamma''}$$



R = Capture Rate

γ = Hot Background

Γ' = Optical Depumping

Γ'' = Optical Repumping

$\Gamma' \approx \Pi(F{=}2 \rightarrow F{=}2)\ \Gamma \approx \Gamma/1600$

$\Gamma'' \approx \Pi(F{=}1 \rightarrow F{=}2)\ \Gamma \approx \Gamma/2$

$$\Pi(A \rightarrow B) = \frac{1}{2} \, \frac{s_{AB}}{1 + s_{AB}}$$

$$s = \frac{1}{2} \, \frac{\omega_1^2}{(\Gamma/2)^2 + \delta^2}$$ Saturation Parameter

$\delta(F{=}2 \rightarrow F{=}2) = 20\ \Gamma$

$\omega_1 = \Gamma$

$\Rightarrow \Pi(F{=}2 \rightarrow F{=}2) \approx 1 / 1600$

$\delta(F{=}1 \rightarrow F{=}2) = 0$

$\omega_1 = \Gamma$

$\Rightarrow \Pi(F{=}1 \rightarrow F{=}2) \approx 1 / 2$

Approximation: $\gamma << \Gamma' << \Gamma'' \quad \Rightarrow \quad N_2 = 800 N_1 = \frac{R}{\gamma}$

Typical Parameter Values: $\gamma \approx 1\ s^{-1}$ , $R \approx 10^9\ s^{-1} \quad \Rightarrow \quad N_2 = 500 N_1 = 10^9$

# Binary Collision Losses:

D. Sesko et al., Phys.Rev.Lett 63,961 (1989)
J. Grünert, A. Hemmerich, Appl. Phys. B, 815 (2001)

$$\dot{N} = R - \Gamma N - \beta \int d^3r\, \rho^2(r)$$

linear loss term:
- collisions with hot background atoms
- optical pumping losses

quadratic loss term:
binary collisions between trapped atoms

assumption for density profile: $\rho(r) = \rho_{peak}\, e^{-(r/a)^2}$ $\Rightarrow$ equation has analytic solutions

decay of trap: $$\frac{N}{N_{max}} = \frac{(1-\xi)\, e^{-\Gamma t}}{1 - \xi\, e^{-\Gamma t}}$$

$$\xi = \frac{\beta\, \rho_{peak}}{\sqrt{8}\,\Gamma + \beta\, \rho_{peak}} \in [0,1]$$

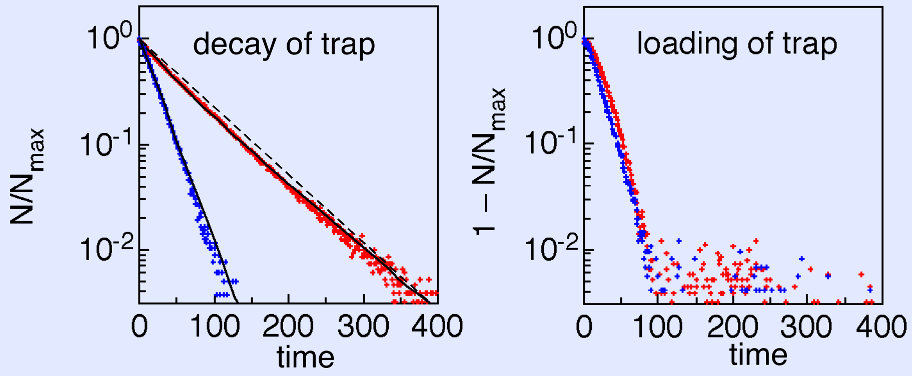
loading of trap: $$1 - \frac{N}{N_{max}} = \frac{(1+\xi)\, e^{-\gamma t}}{1 + \xi\, e^{-\gamma t}}$$

$$\gamma = \frac{1+\xi}{1-\xi}\,\Gamma$$

→ indication of binary collision losses: loading time < decay time, non-exponential dynamics

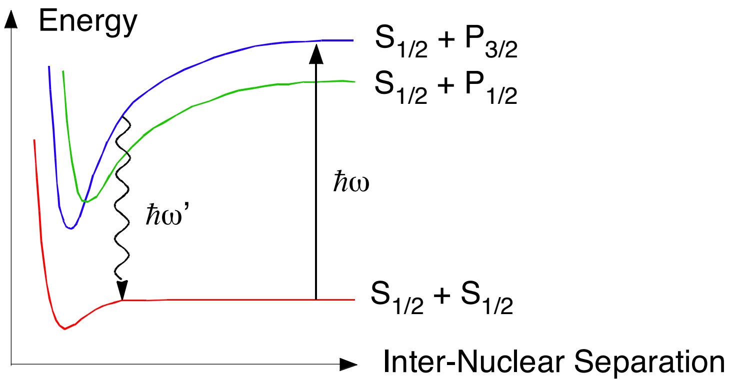
no repumping: $\Gamma \approx 44.2\ s^{-1}$ , $\xi < 10^{-4}$

repumping: decrease $\Gamma \approx 13.9\ s^{-1}$ $\Rightarrow$ increase $\xi \approx 0.32$

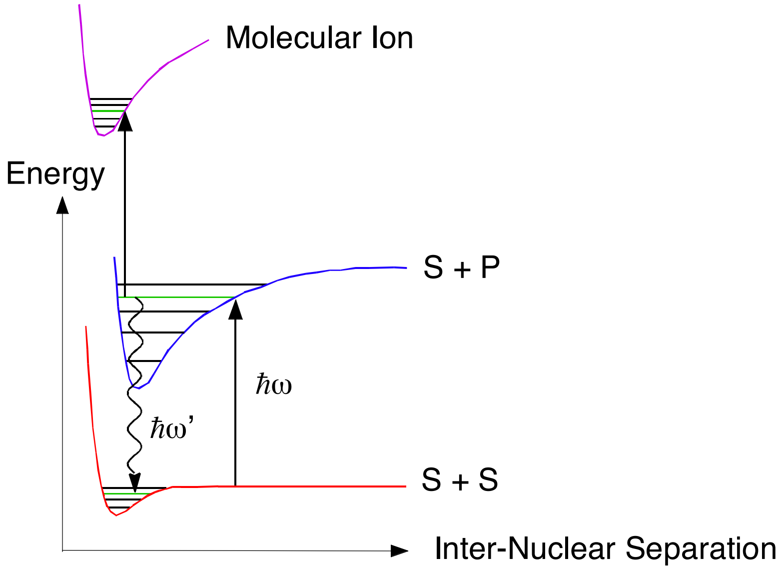


# Binary Collisional Loss Processes

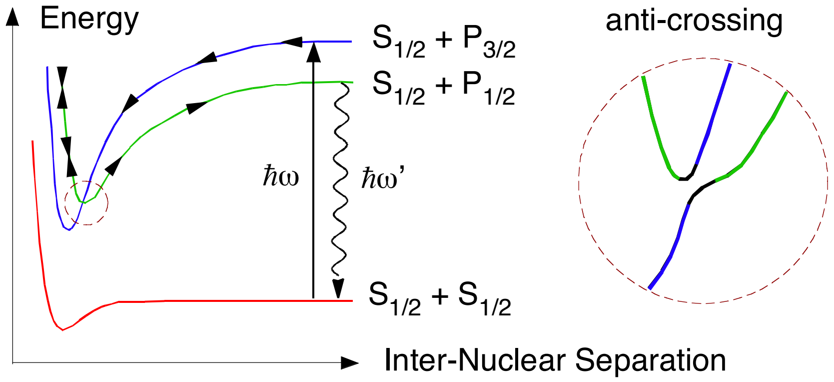
## Radiative Escape:



## Photo-Association:



## (Hyper) Fine-Structure Changing Collisions:



# Regimes of MOT-Operation, Density Limitations

Constant Volume Regime: at low density consider thermal atoms in harmonic trap potential $\rho(r) = \rho_{peak} \exp\left(- \frac{m\,\omega_{Vib}^2\, r^2}{2\, k_B T}\right)$

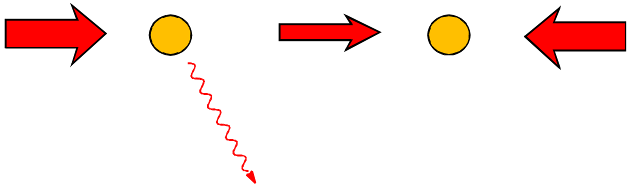
1/e Radius : $R_e = \sqrt{\frac{2\, k_B T}{m\, \omega_{Vib}^2}}$

→ sample size does not depend on particle number N

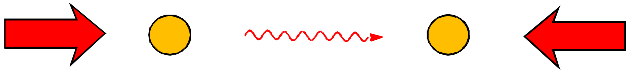
→ peak density $\rho_{peak}$ increases linearly with N

Constant Density Regime: at higher densities onset of light induced repulsive interaction among atoms

shielding of laser photons → attractive force ~ $1/\, r^2$ :



reabsorption of fluorescence photons → repulsive force ~ $1/\, r^2$ :



fluorescence photons involve near resonant contribution (Mollow Triplet) → repulsive force exceeds attractive force

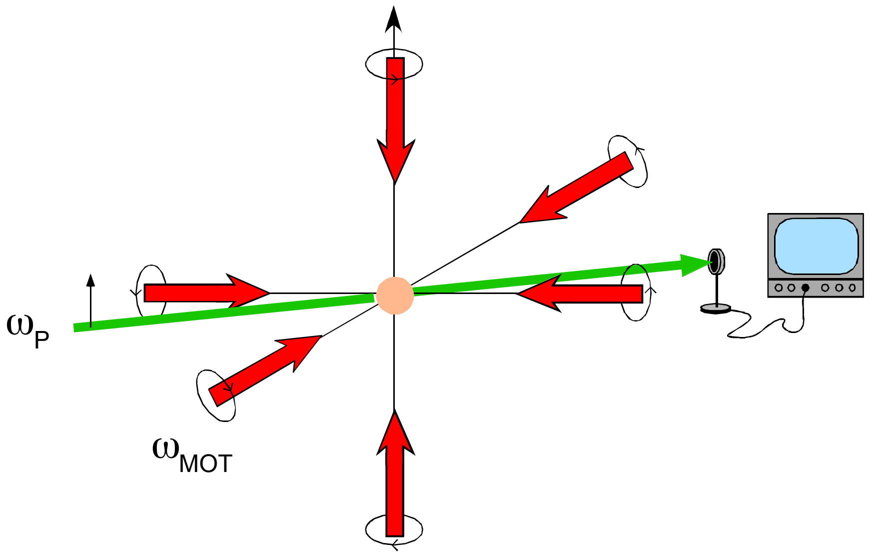
→ sample size increases linearly with N

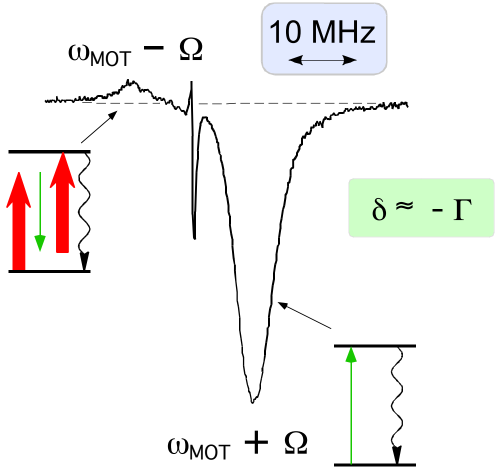
→ peak density $\rho_{peak}$ takes constant maximum value

J. Dalibard, Opt. Commun. 68,203 (1988)
T. Walker et al., Phys.Rev.Lett 64,408 (1990)
T. Townsend et al., Phys.Rev.A 52, 1423 (1995)

# Probe Transmission in a Magneto-Optic Trap

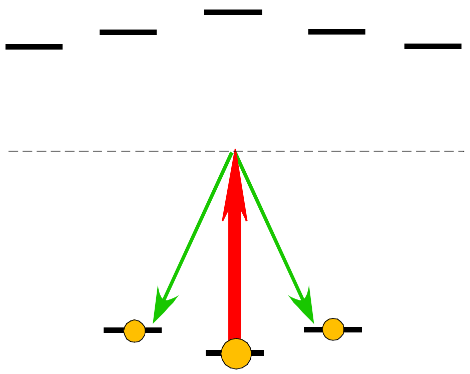
D. Grison et al., Europhys. Lett. 15, 149 (1991).
A. Hemmerich et al., Europhys. Lett. 21, 445 (1993).

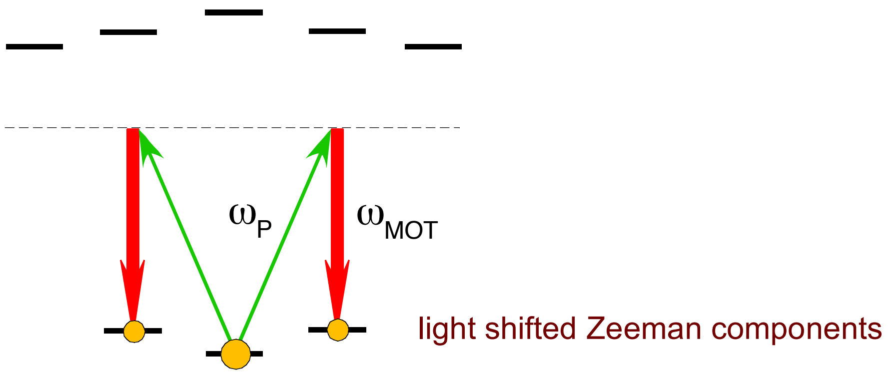




## Origin of Central Resonance:

- consider local quantization axis along linear MOT polarization
- probe polarization has both circular components
- Zeeman shifts smaller than light shifts for trapped atoms





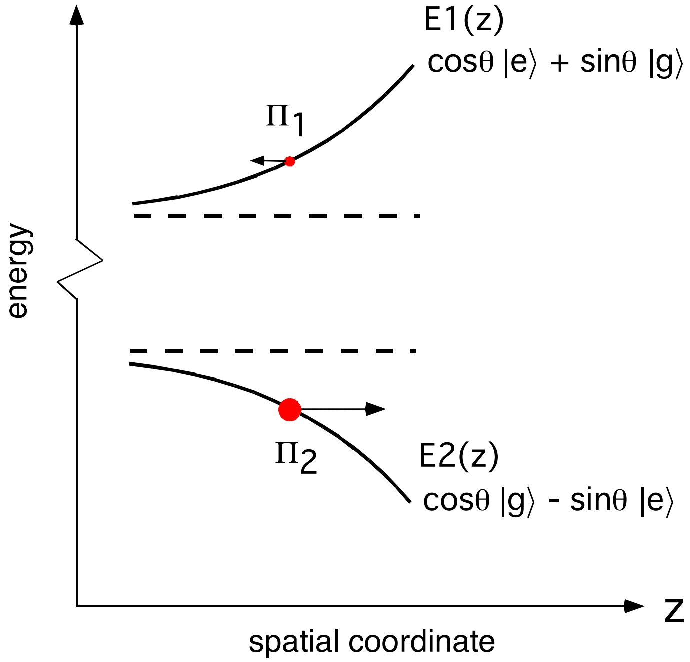
# Dipole Forces

classical viewpoint:

$\delta < 0$: atomic dipole moment oscillates in phase with driving field -> atom is dragged towards intensity maximum

$\delta > 0$: atomic dipole moment oscillates with 180° phase delay -> atom is dragged towards intensity minimum

quantum mechanical picture:



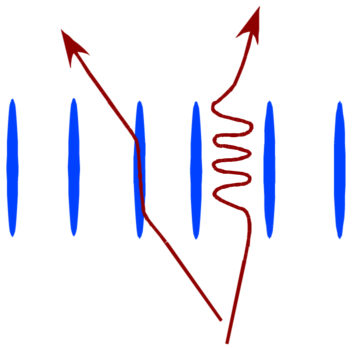
$$\text{Force} = \Pi_1\,\nabla E_1(z) + \Pi_2\,\nabla E_2(z) = -\nabla U\,, \quad U(z) = \frac{\hbar\delta}{2}\ln(1+s(z))$$

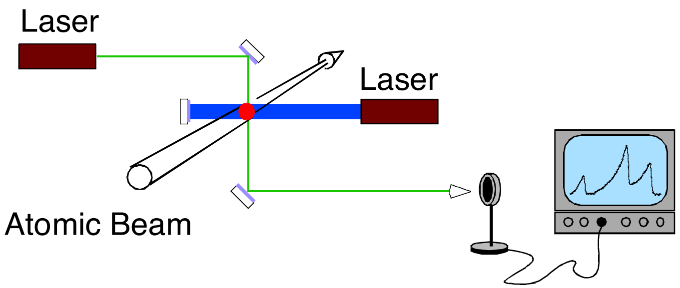
$$s = \frac{1}{2}\,\frac{\omega_1^2}{(\Gamma/2)^2+\delta^2}$$

Saturation Parameter

# Channeling Atoms in a Standing Wave

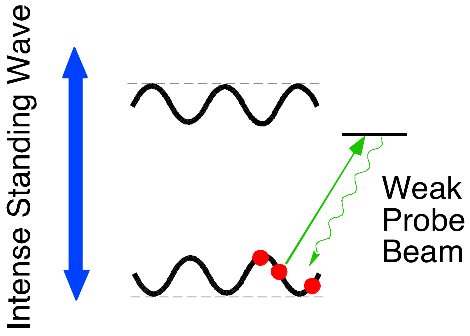
C. Salomon, et al., Phys. Rev. Lett 59, 1659 (1987)
A. Hemmerich et al., Phys. Rev. A. 44, 1910 (1991)

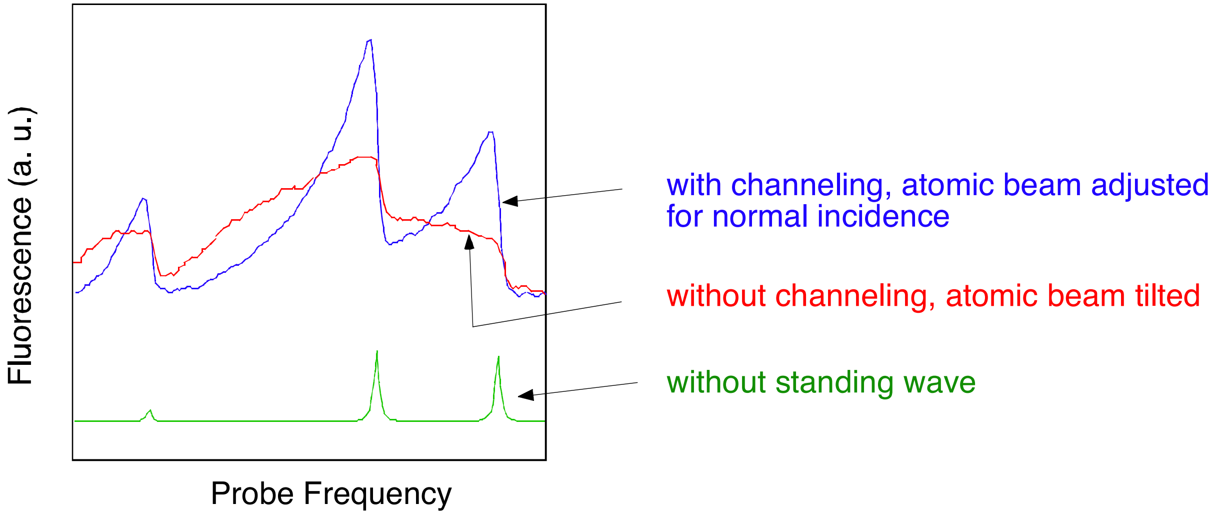




condition for transverse confinement (channeling): $\frac{1}{2}\, m\, V_{trans}^{2} < U_{max}$

## Spectroscopy of channeled atoms:





# Non-Dissipative 3D Light Shift Potentials

$F_{RAP} \propto \hbar k \Gamma \Pi_e$

$F_{DIP} \propto \hbar k \delta \Pi_e$

$\delta >> \tilde{\Gamma} \quad \Rightarrow \quad \Pi_e \propto I / \delta^2 \quad \Rightarrow$

$F_{RAP} \propto I / \delta^2$

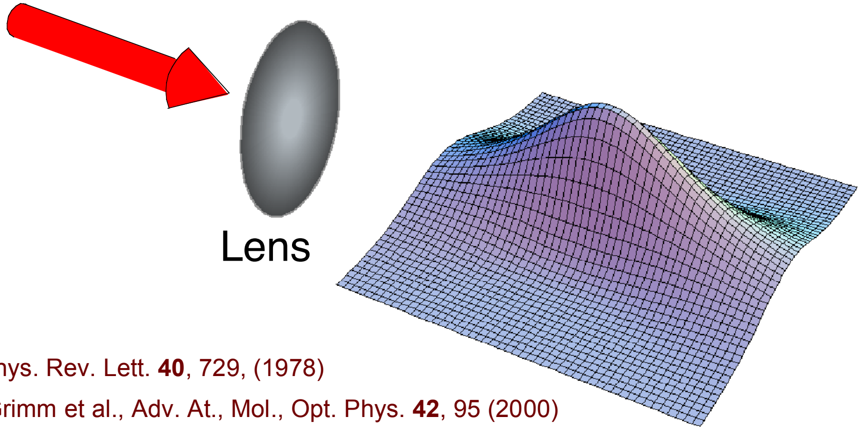
$F_{DIP} \propto I / \delta$

Large intensity **I** and detuning $\delta$ leads to significant light shift potentials, allthough atomic excitation $\Pi_e$ remains negligible

→ Trapping atoms in non-dissipative dipole traps

However: temperature reached by Doppler cooling is too high for efficiently loading atoms into such traps → polarization gradient cooling

Simple trapping geometry: strongly focused laser beam, $\delta << 0$



Typical Parameters:

| | |
|---|---|
| power: | several W |
| detuning: | few 100 nm |
| focus: | below 100 μm |
| trap depth: | few 100 μK |
| scattering rate: | 1 $s^{-1}$ |

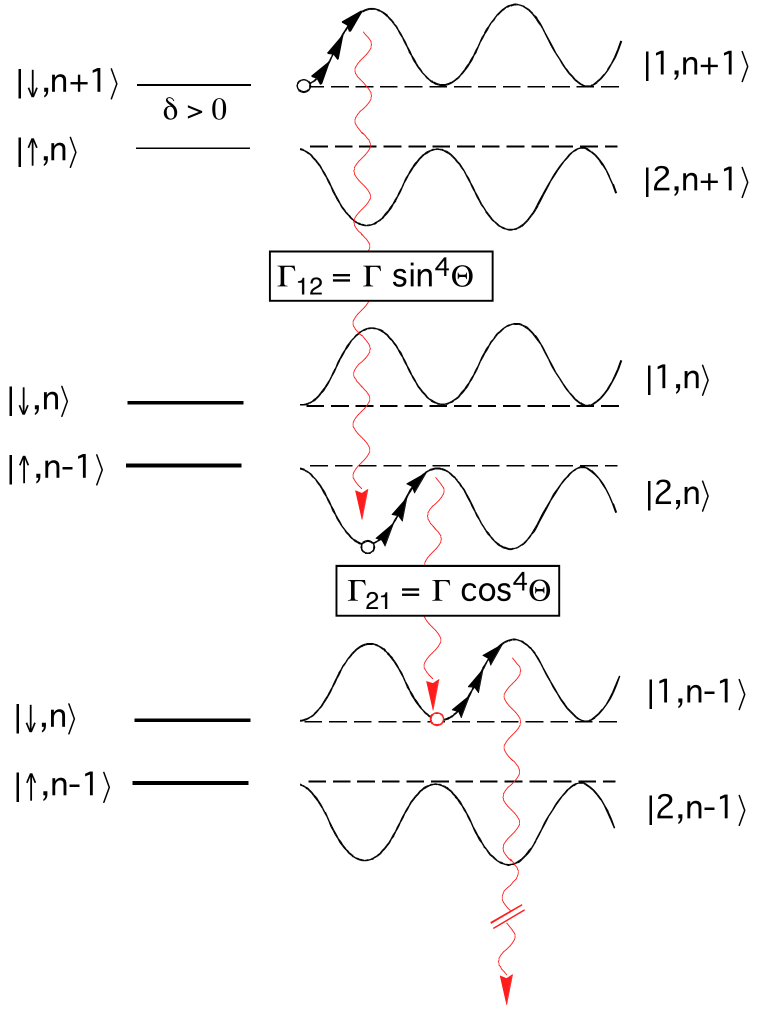
A. Ashkin, Phys. Rev. Lett. **40**, 729, (1978)

Review: R. Grimm et al., Adv. At., Mol., Opt. Phys. **42**, 95 (2000)

# Dipole Force Cooling

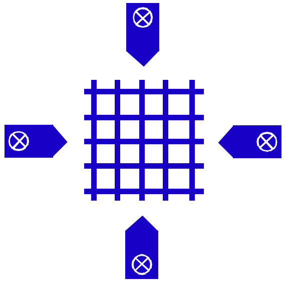
Positive detuning: $\delta > 0$

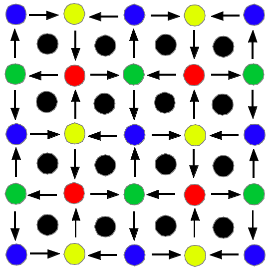
Limit of well resolved Lines: $\Omega >> 0$



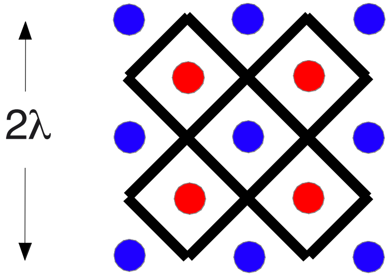
J. Dalibard and C. Cohen-Tannoudji, J. Opt. Soc. Am B 2,1707 (1985).

# Interference of two standing waves





$\phi = 90\,°$



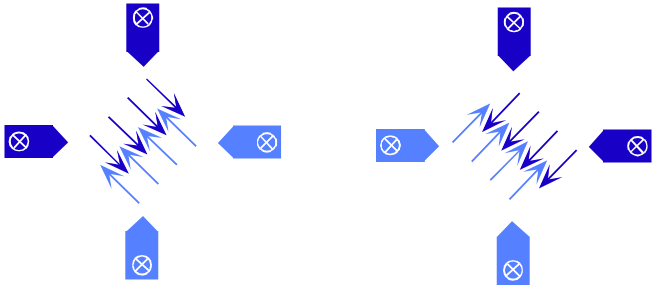
$\phi = 0°$

Interference effects and light forces: $E(x,y) \;=\; \hat{z}\,\sqrt{I_0}\;\left(\cos(kx) + \cos(ky)e^{i\phi}\right)$

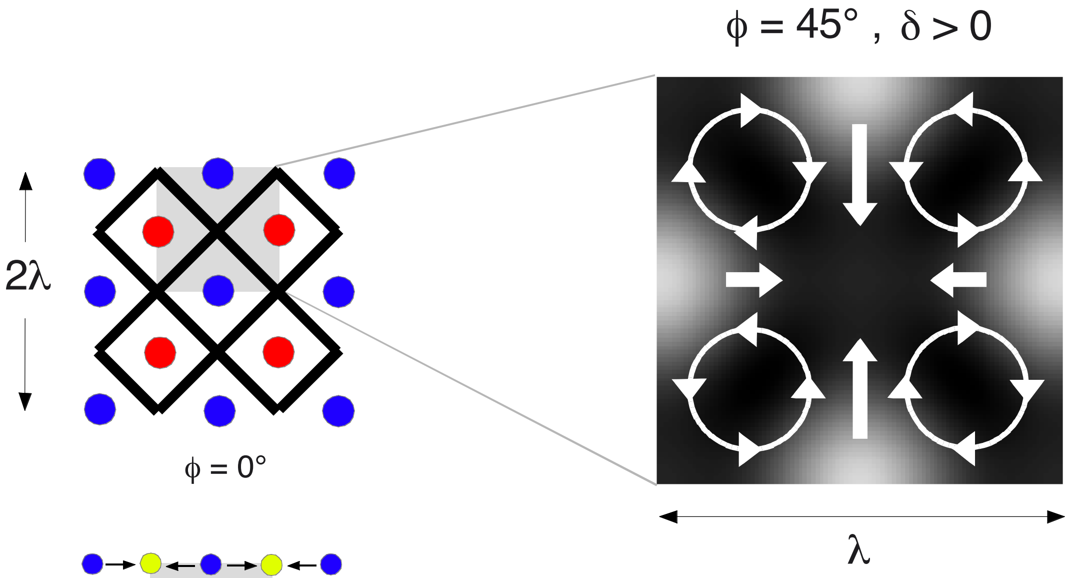
Dipole Force $\sim\; \nabla I(x,y) \;=\; \nabla\; I_0\left(\cos^2(kx) + \cos^2(ky) + 2\cos(\phi)\cos(kx)\cos(ky)\right)$

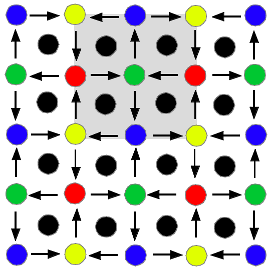
Radiation Pressure $\sim\; I(x,y)\,\nabla\,\psi(x,y) \;=\; \nabla \times\; \hat{z}\; I_0\, k\, \sin(\phi)\sin(kx)\sin(ky)$

Physical explanation of rotating poynting vector:



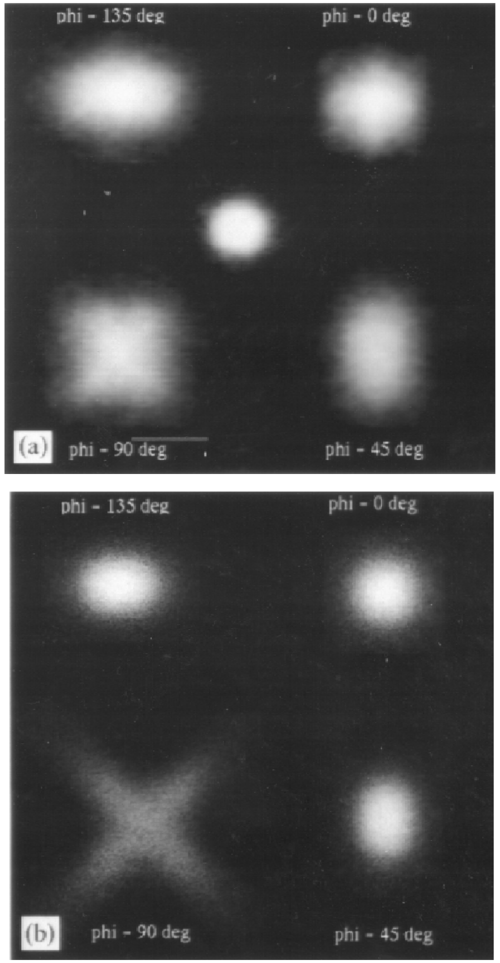
# Observation of radiation pressure vortices





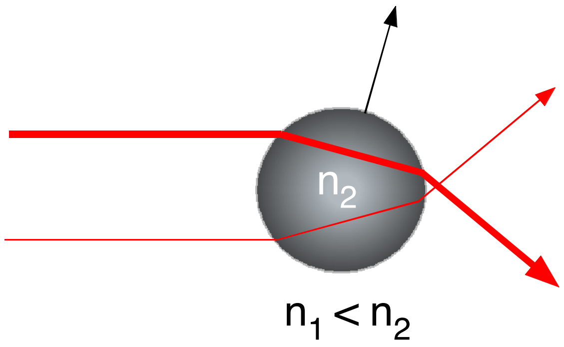
φ = 90 °

| | |
|---|---|
| φ = 0°<br>φ = 90° | ● |
| φ = 45 ° | ⬮ |
| φ = 135 ° | ⬬ |

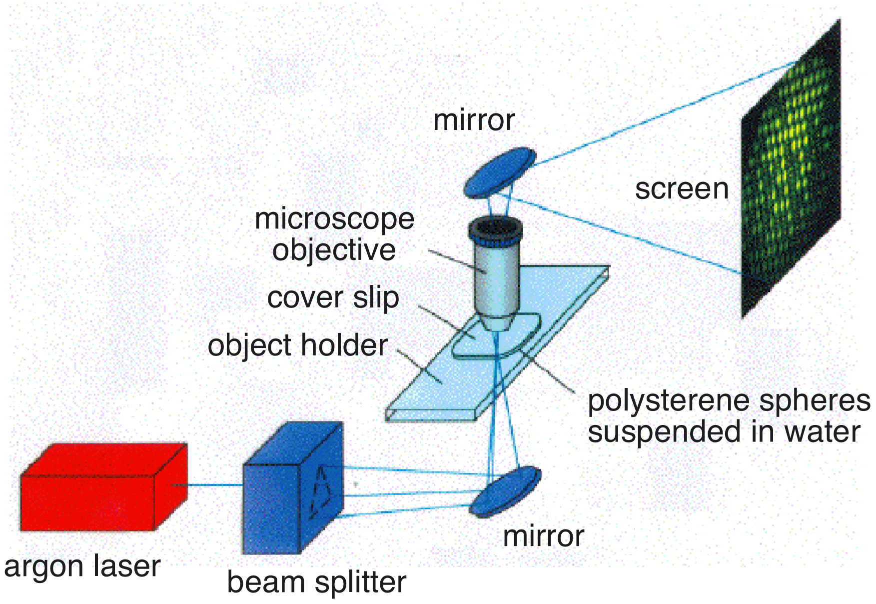


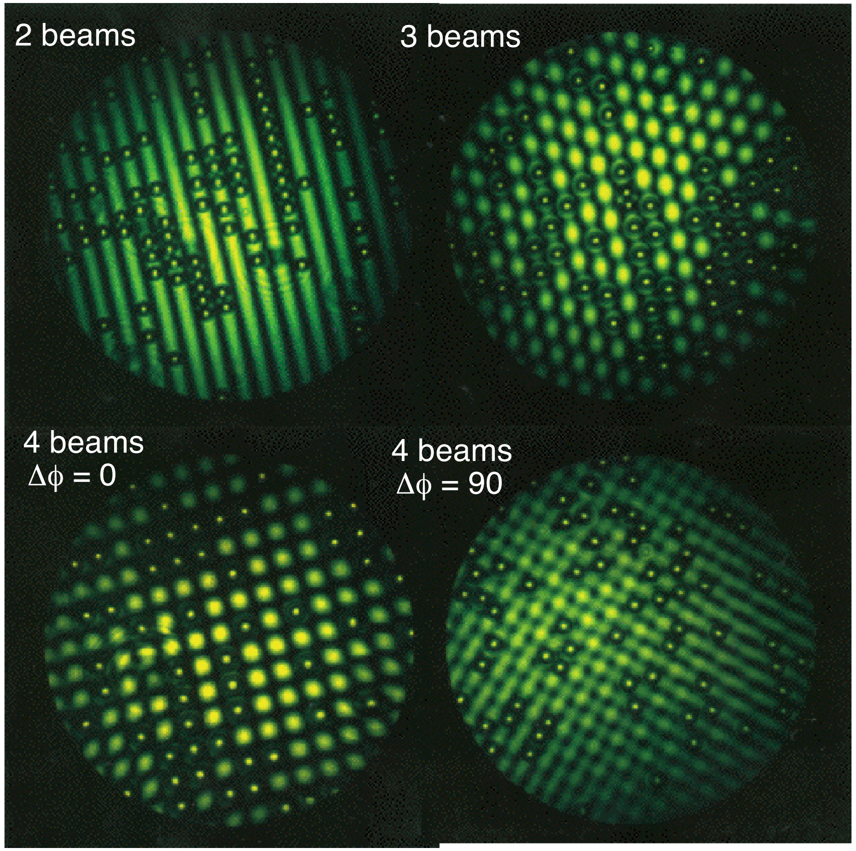
A. Hemmerich and T. W. Hänsch, Phys. Rev. Lett. 68, 1492 (1992)

# Optical Lattices: 4 μm polysterene spheres in water



M. Burns et al., Science 249, 749 (1990)

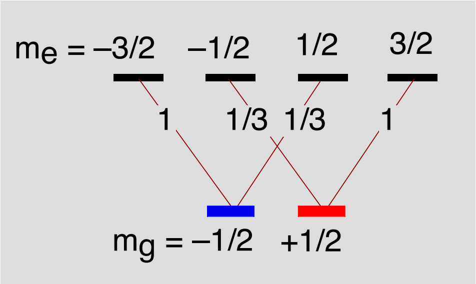




- Spheres are dragged towards high intensity: green argon laser beam is red detuned with respect to resonances in the UV.
- Cooling to room temperature by water matrix is sufficient to trap the spheres in the intensity maxima.
- Green argon laser light is visible due to Rayleigh scattering from water molecules.

# Cooling below the Doppler-Limit (Ellipticity-Gradient Cooling, Sisyphus-Cooling)

Consider atom with J → J+1 -Transition, e.g., J=1/2 :

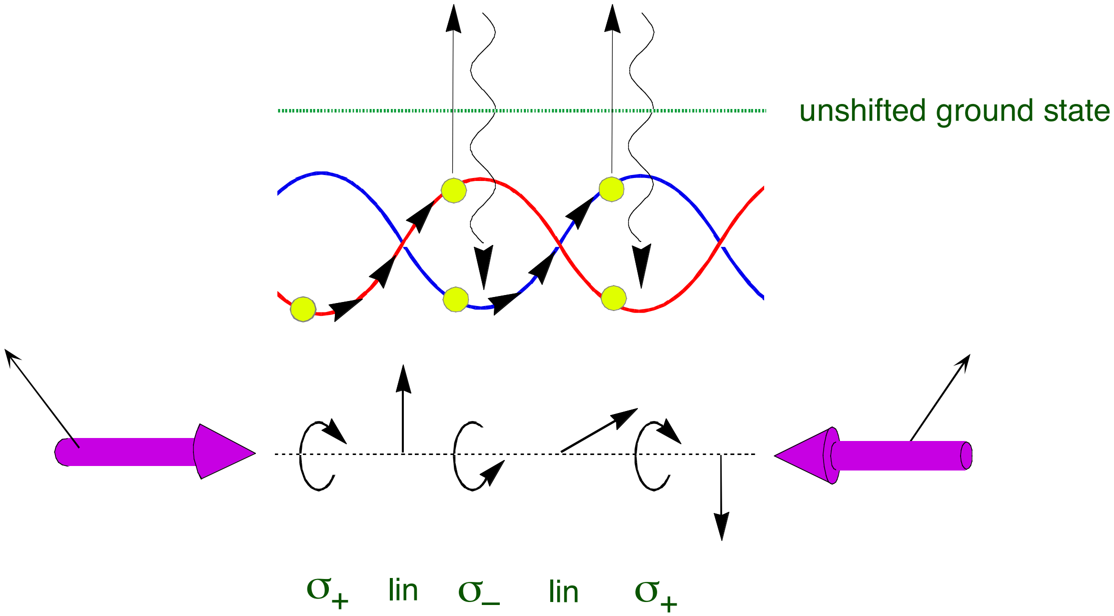


radiation selction rules → polarization-dependent coupling

→ polarization-dependent light-shifts

→ polarization-dependent optical pumping

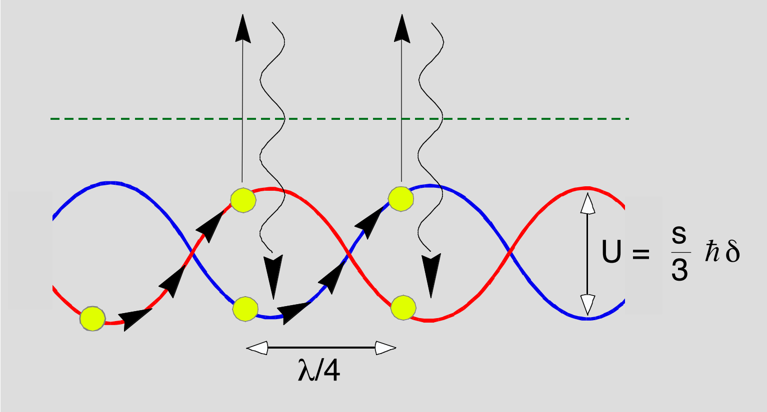
Consider light field with negative detuning $\delta$ and polarization gradient, e.g., lin⊥lin:

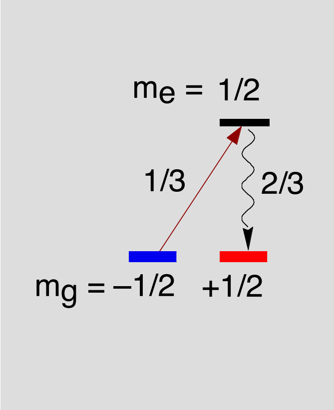


spatial correlation between light shifts and optical pumping: optical pumping populates the most light shifted Zeeman component

J. Dalibard and C. Cohen-Tannoudji, J. Opt. Soc. Am. **B 6**, 2023 (1989)

# Energy Budget in Ellipticity-Gradient Cooling:





**Capture Range:**

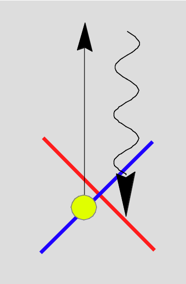
$$v_{max}\tau_p = \frac{\lambda}{4} \quad \Rightarrow \quad kv_{max} \approx \Gamma_p = \frac{2}{9}\,\Gamma\,\frac{s}{2} \qquad ( s << 1 \Rightarrow \Pi_e = \frac{1}{2}\,\frac{s}{1+s} \approx \frac{s}{2} )$$

**Friction Coefficient:**

$$v_{max}\, F = \frac{\partial W}{\partial t} = -\,U\,\Gamma_p \quad \Rightarrow \quad F = \gamma\, v_{max}, \quad \gamma = -\,U\,k^2\tau_p = 3\,\hbar\,k^2\,\frac{\delta}{\Gamma} = 6\,\gamma_{sp}\,\frac{|\delta|}{\Gamma}$$

**Diffusion Coefficient:**

$$P(t) = \sum_{i=1}^{N=\Gamma_p t} f_i\,\tau_p\,, \quad |f_i| = \hbar\,\nabla\Omega = k\,U \quad \Rightarrow \quad P(t)^2 = (\,f_i\,\tau_p)^2\,N$$



**Steady State:**

$$\left(\frac{\partial}{\partial t}\right)_{Diff} E_{kin} = \frac{D}{m}\,, \quad D = \frac{1}{2}\,\tau_p\,k^2\,U^2 = \frac{1}{4}\,\hbar^2\,k^2\,\frac{\delta^2}{\Gamma}\,s = \frac{1}{2}\,D_{sp}\,\left(\frac{\delta}{\Gamma}\right)^2$$

diffusion coefficient of Doppler-cooling

$$\bar{E}_{kin} = \frac{D}{2\gamma} = \frac{1}{4}\,U \quad \Rightarrow \quad k_B T = \frac{1}{2}\,U$$

# Limitations of semiclassical model, recoil limit:

If U is reduced, capture velocity decreases faster then RMS velocity ⇒ Model fails, if optical pumping time exceeds oscillation time

$$U \gtrless 18 \left(\frac{\delta}{\Gamma}\right)^2 E_{rec} \quad \Leftrightarrow \quad v_{max} \gtrless v_{rms} \quad \Leftrightarrow \quad 1 \gtrless \Omega_{vib}\, \tau_p \qquad \Omega_{vib} \equiv k\sqrt{\frac{2U}{m}} = \text{vibrational frequency}$$

Model can be extended to the oscillatory regime, however other limitations occur:

If U approaches $E_{rec}$ :

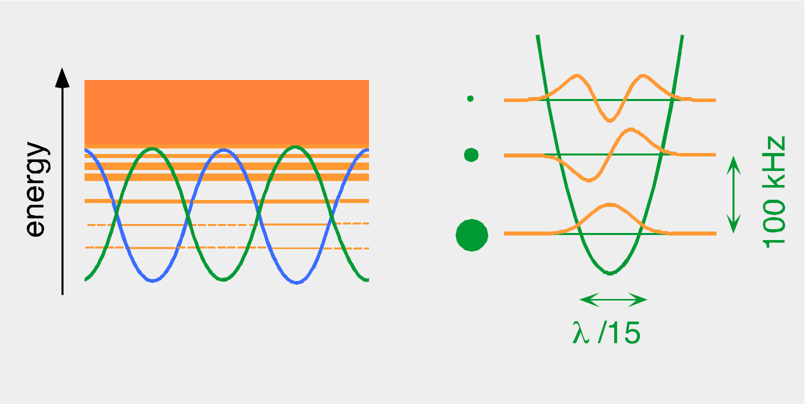
- neglection of the optical pumping recoils in the energy budget for cooling is no longer possible
- de Broglie Wavelength of atoms aproaches optical wavelength ⇒ atomic motion must be described by quantum mechanics

use band structure theory to describe cooling near recoil limit:

⇒
- for shallow potential wells, the temperature is limited to a few recoil temperatures $T_{rec}$
- for deep wells atoms are trapped in the first few lowest vibrational levels
- atoms are arranged in a periodic structure → Optical Lattice

$$k_B T_{rec} \equiv E_{rec}$$

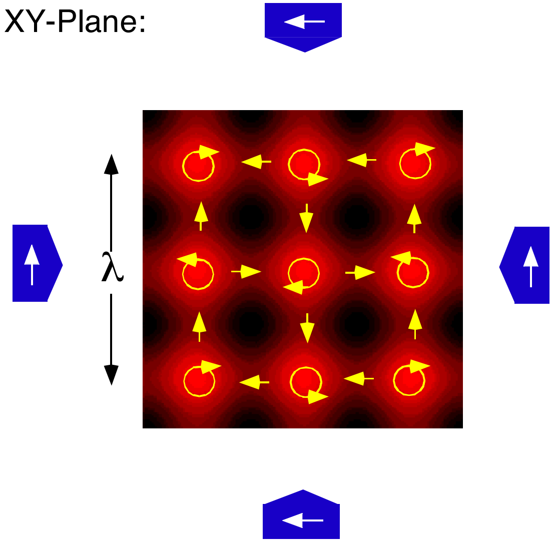
$$E_{rec} \equiv \frac{(\hbar k)^2}{2m}$$



Y. Castin and J. Dalibard, Europhys. Lett. 14, 761 (1991)
V. S. Letokhov et. al., Zh. Eksp. Teor. Fiz. 12,1328 (1977)

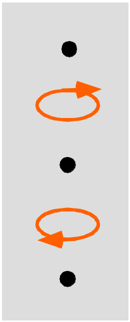
# Polarization Geometry for a 3D Optical Lattice

2D-Field in XY-Plane:



+

1D-Wave along Z-Axis:



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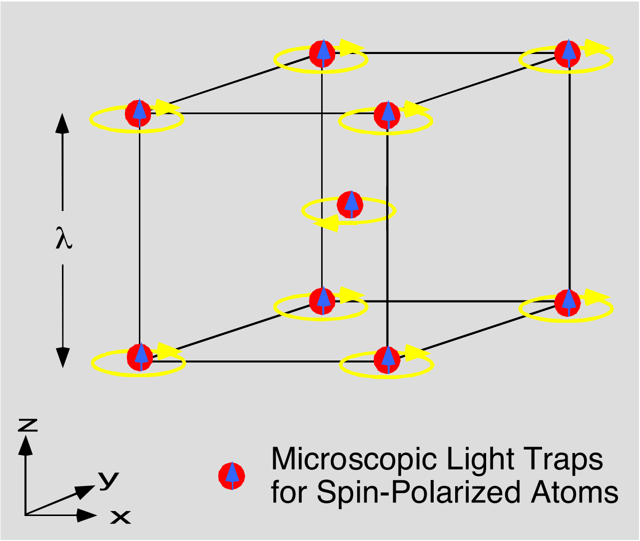
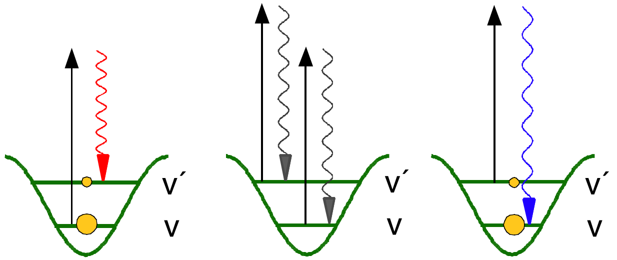
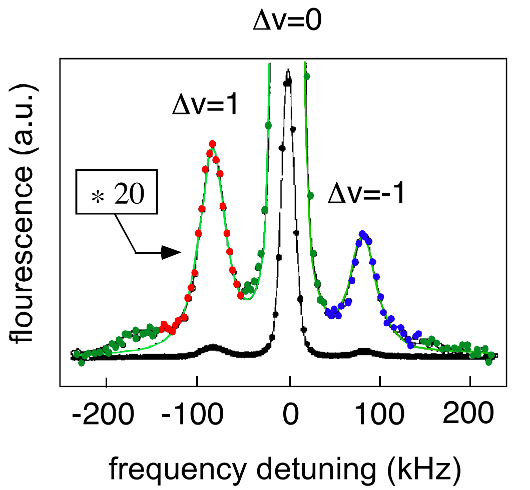


# Fluorescence in an Optical Lattice





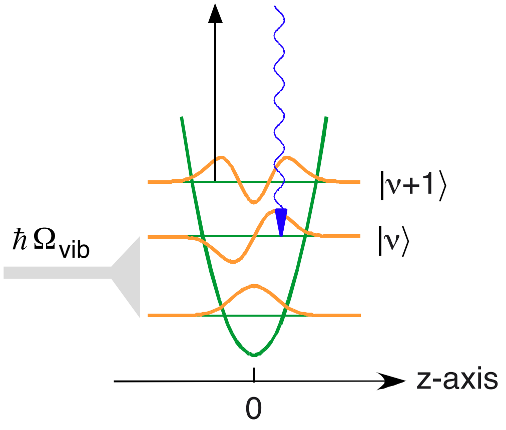
P.S. Jessen et al., Phys. Rev. Lett.69,49 (1992)

asymmetry of sidebands → high population of vibrational ground state

vibrational resonances have linewidth far smaller than optical pumping rate for free atoms

## Why can vibrational resonances be resolved?

Y. Courtois and G. Grynberg, Phys. Rev. A 46, 7060 (1992)



Franck-Condon effect for well bound vibrational modes:

$$\Gamma_{\nu,\mu} = \Gamma' \; |\langle\nu|\cos(kz)\,e^{ikz}\,|\mu\rangle|^2 , \quad \cos(kz)\,e^{ikz} = 1 + ikz + O[(z/\lambda)^2]$$

$$\Rightarrow \quad \Gamma_{\nu,\nu} = \Gamma' \, |\langle\nu|\cos(kz)\,e^{ikz}\,|\nu\rangle|^2 \approx \Gamma' \, |\langle\nu|\nu\rangle|^2 = \Gamma'$$

$$\Gamma_{\nu+1,\nu} = \Gamma' \; |\langle\nu|\cos(kz)\,e^{ikz}\,|\nu+1\rangle|^2 \approx \Gamma' \; |\langle\nu|\,kz\,|\nu+1\rangle|^2$$

for well localized states: take first non-vanishing order

$$z = \frac{z_0}{\sqrt{2}}(a^+ + a) \quad \text{with} \quad z_0 = \sqrt{\langle 0|\,z^2\,|0\rangle} = \sqrt{\frac{\hbar}{m\Omega_{vib}}} = \text{size of ground state}$$

$$= \Gamma' \; (\nu+1)\,(kz_0)^2/2 = \Gamma' \; (\nu+1)\,\frac{1}{\hbar\Omega_{vib}}\,\frac{\hbar^2 k^2}{2m} = \Gamma' \; (\nu+1)\,\frac{E_{rec}}{\hbar\Omega_{vib}} << \Gamma'$$

(for small ν)

# Franck-Condon effect for well bound vibrational modes = Lamb-Dicke effect:

R. Dicke, Phys. Rev. **89**, 472, (1953)

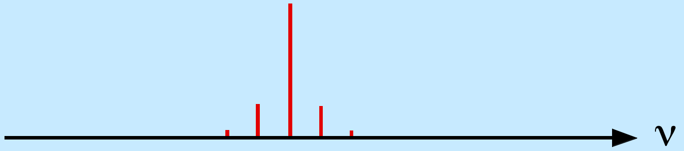
Phase Modulation:

$$A(t) = A_0 \; e^{i\phi(t)}$$

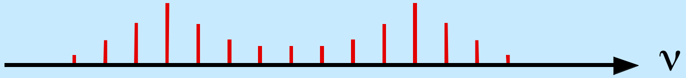
$$\phi(t) = 2\pi \,( \nu\, t + M \; \sin(\Omega t) \;)$$

M = modulaton index

$$\omega(t) = 2\pi \,( \nu + M\Omega \; \cos(\Omega t) \,)$$

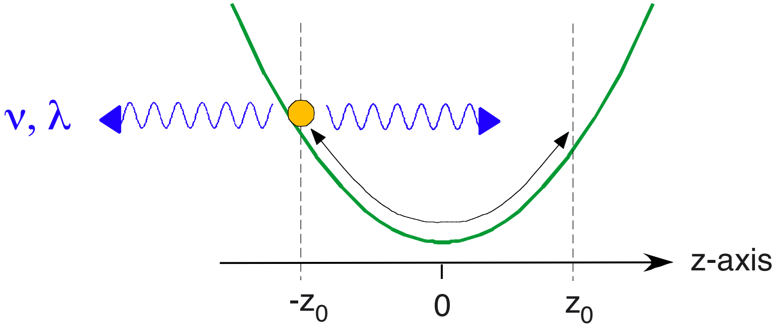


M < 1 : most power in carrier, only few sidebands



M > 1 : power distributed over many sidebands

Atom oscillating in external potential = phase modulated light source:



position: $z(t) = z_0 \sin(\Omega t)$

velocity: $v(t) = v_0 \cos(\Omega t)\;,\; v_0 = z_0\,\Omega$

$$\nu(t) = \nu + v(t) / \lambda = \nu + (z_0\Omega / \lambda) \; \cos(\Omega t) \;)$$

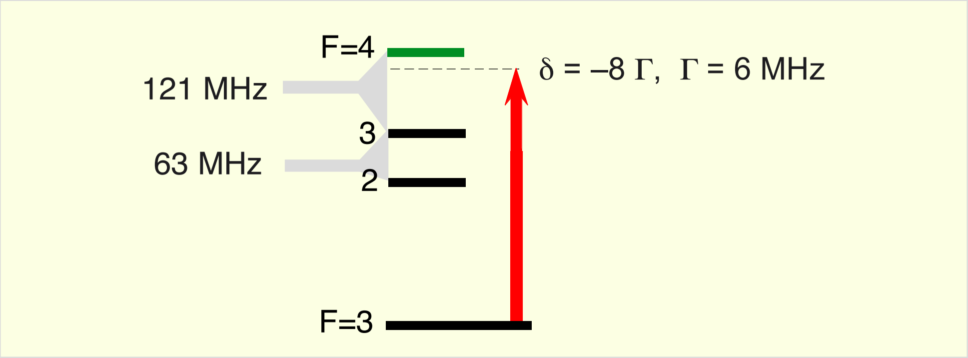
modulaton index: $M = \dfrac{z_0}{\lambda}$

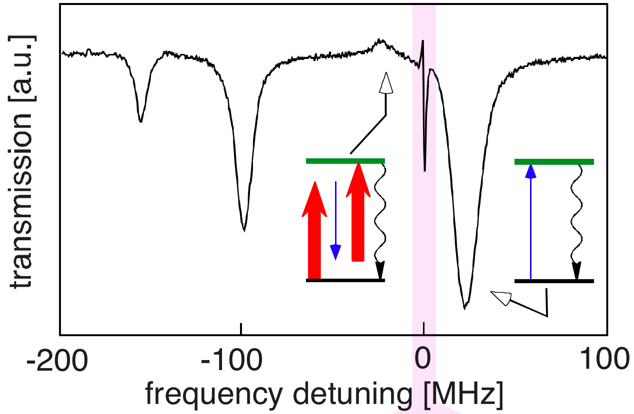
$$M \gtrless 1 \quad \Leftrightarrow \quad z_0 \gtrless \lambda$$

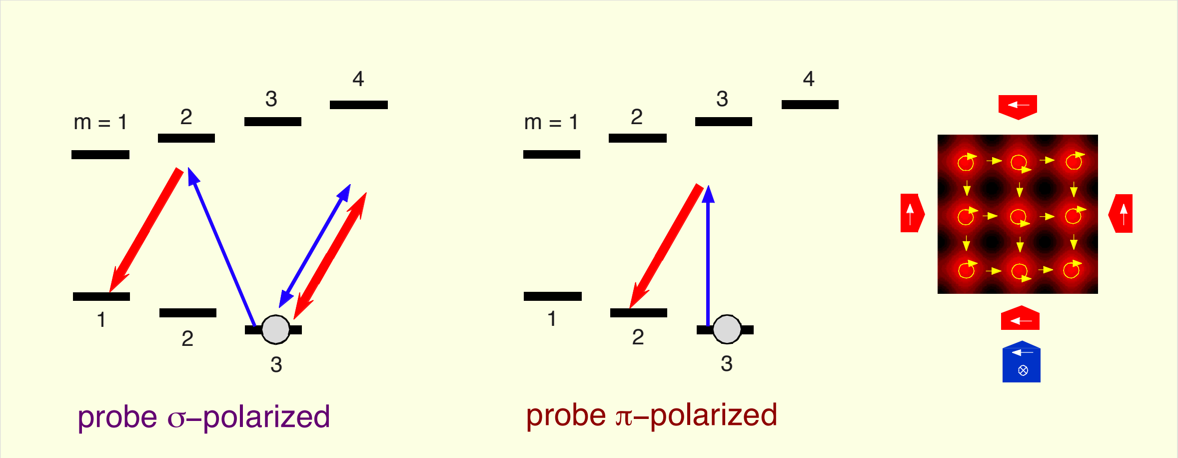
if particle is trapped in a box smaller than the optical wavelength

⇒ most of the flourescence power is emittted via the carrier which has no Doppler-shift

# Probe Transmission in an Optical Lattice with Rubidium

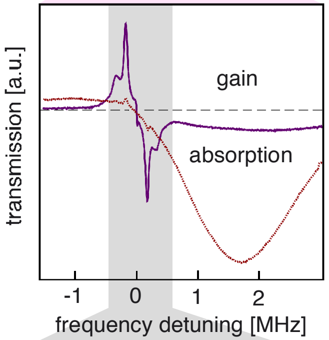


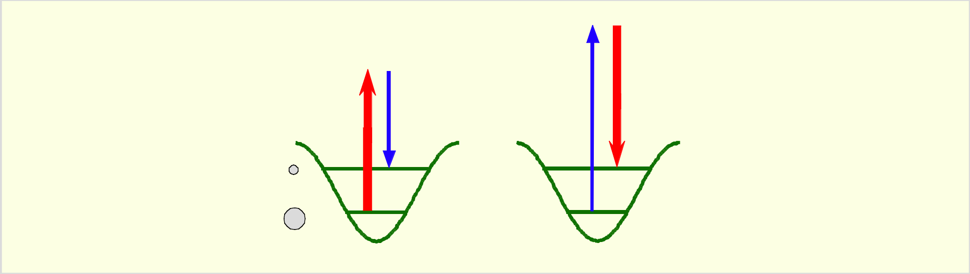


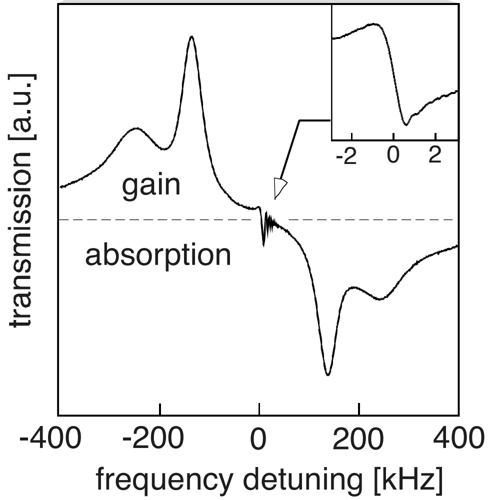


probe σ–polarized

probe π–polarized



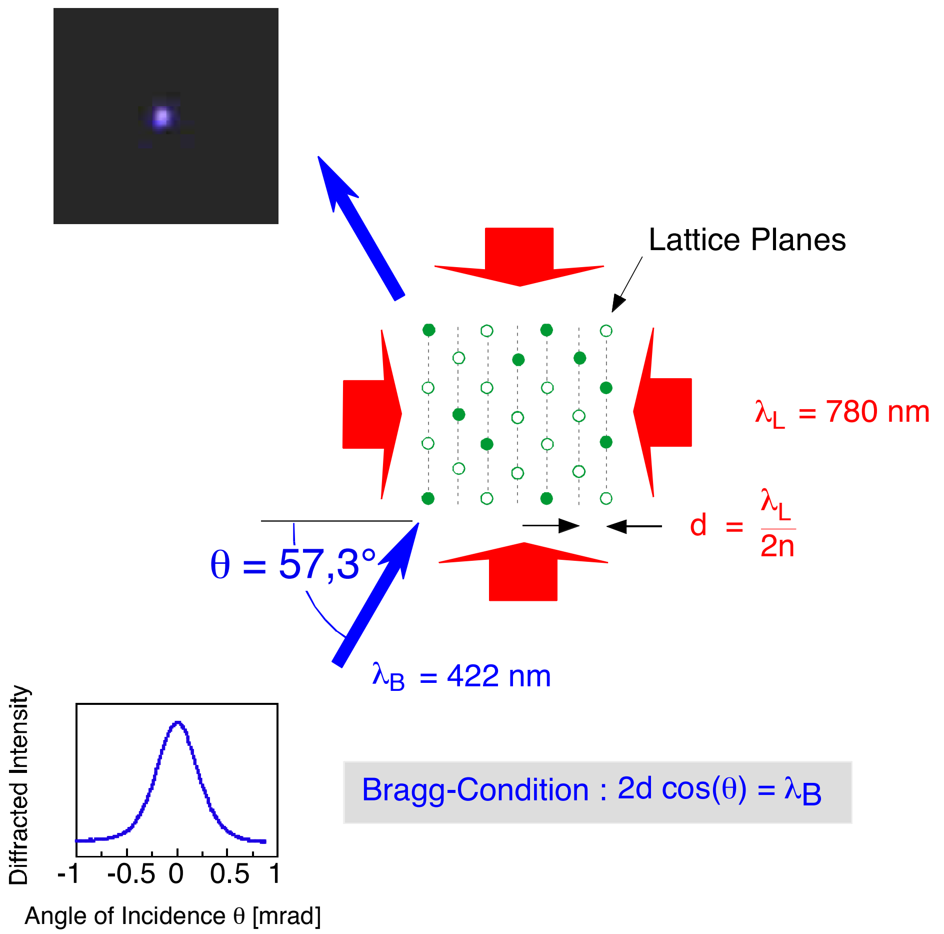




P. Verkerk, et al., Phys. Rev. Lett.68, 3861 (1992)

A. Hemmerich and T. Hänsch, Phys. Rev. Lett.70, 410 (1993)

# Bragg-Diffraction in Optical Lattices

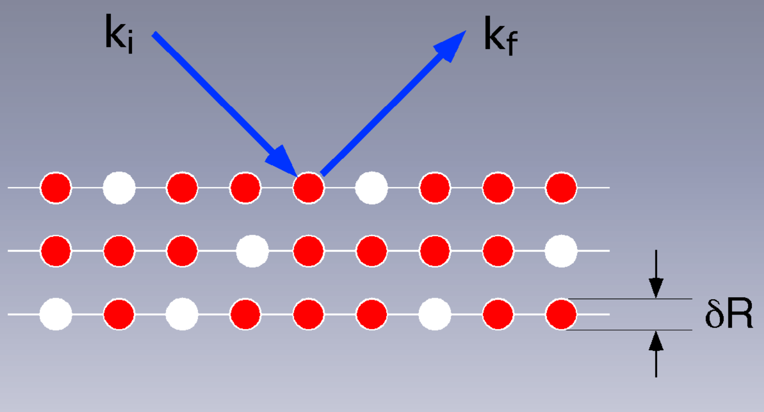


Bragg-Condition : $2d \cos(\theta) = \lambda_B$

- Scattering contrast yields information on atomic localization (Debye-Waller-factor)
- Bragg-angles yield information on separation of lattice planes

M. Weidemüller, et al., Phys. Rev. Lett **75**, 4583 (1995)
G. Birkl, et al., Phys. Rev. Lett **75**, 2823 (1995)

# Bragg diffraction

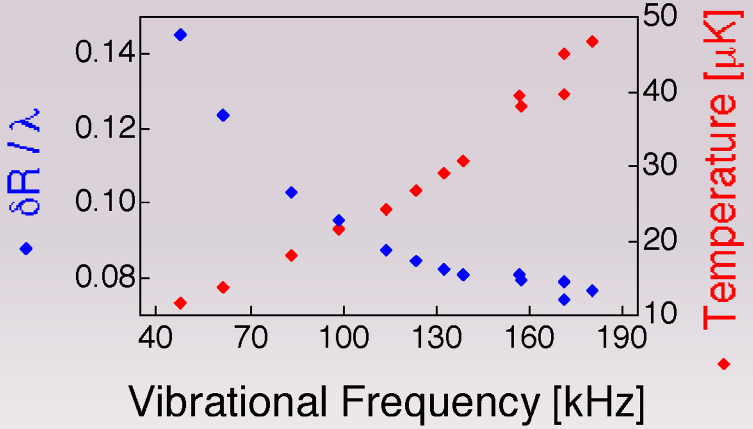


diffracted power $P = \text{const.} \cdot e^{-2W}$

Debye-Waller factor $W = 1/6 \mid k_i - k_f \mid (\delta R)^2$

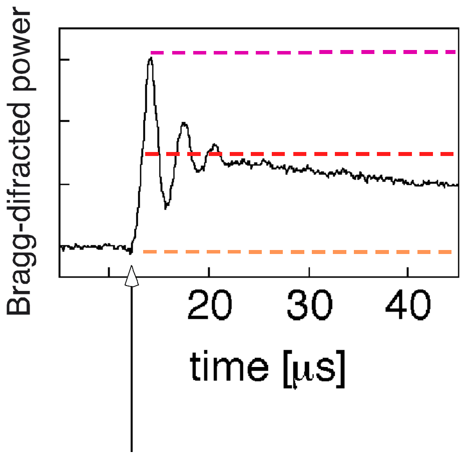
Measure mean spatial atomic extension δR by comparing power P for two different Bragg angles

$$(\delta R)^2 = \left| \frac{\ln(P_1) - \ln(P_2)}{\Delta k_1^2 - \Delta k_2^2} \right|$$

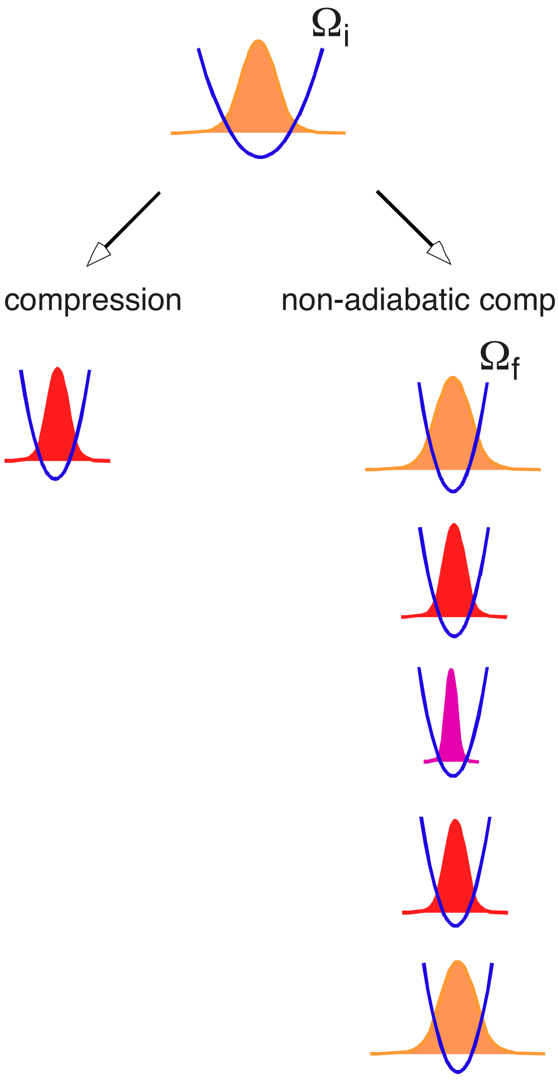


A. Görlitz, et al., Phys. Rev. Lett **78**, 2096 (1997)
G. Raithel, et al., Phys. Rev. Lett **78**, 2928 (1997)

# Observing position spread oscillations:



non-adiabatic compression: $\Omega_i \rightarrow \Omega_f$



$\Omega_i$

adiabatic compression

non-adiabatic compression: $\dot{\Omega}/\Omega >> \Omega$

$\Omega_f$

oscillation of position spread with $2\Omega_f$

A. Görlitz, et al., Phys. Rev. Lett **78**, 2096 (1997)
G. Raithel, et al., Phys. Rev. Lett **78**, 2928 (1997)

# Backaction of Atoms upon the Lattice

Increase of Density → Increase of Refractive Index

$$n = 1 + \frac{1}{4}\left(1 + \exp(-W_0) + 4\exp(-W_{90})\right)\chi(\sigma^+)$$

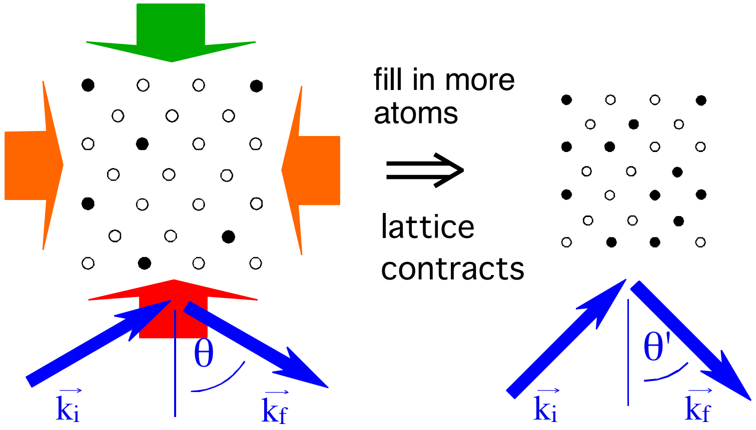
Forward–Scattering

Backward–Scattering

$$W_0 = \frac{2}{3}k^2\,\delta R^2$$

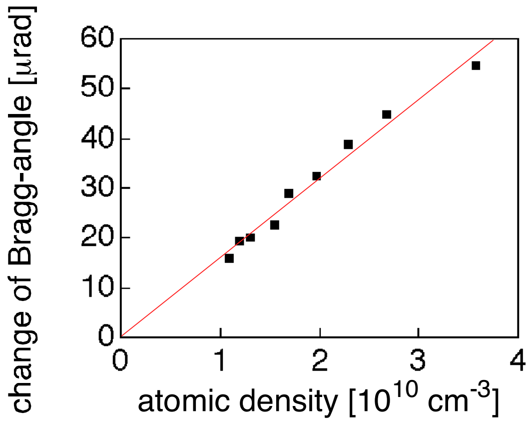
90°–Scattering

$$W_{90} = \frac{1}{3}k^2\,\delta R^2$$



Recall: Debeye-Waller factor for Bragg-scattering:

$$W_{\delta k} = \frac{1}{3}\,\delta k^2\,\delta R^2$$

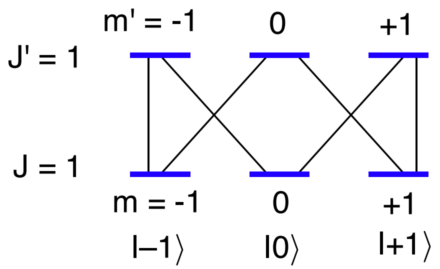


→ Decrease of Lattice Constant: $d = \frac{\lambda_L}{n}$

→ Increase of Scattering Angle: $\cos(\theta) = \frac{\lambda_B}{2d}$

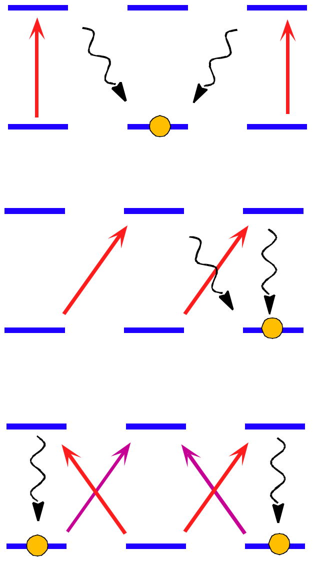
M. Weidemüller et al., Phys. Rev. A **58**, 4647 (1998)

# Dark States:



Consider J = 1 → J = 1 level scheme

- for every polarization a dark state exists
- optical pumping populates this dark state



$\pi$-light : dark state = $|0\rangle$

$\sigma_+$ -light : dark state = $|+1\rangle$

$\sigma$ -light : dark state is a superposition of $|-1\rangle$ and $|1\rangle$ such that transition matrix elements destructively interfere

$$|\psi_{NC}> \quad = \quad \alpha_-|-1\rangle \ + \ \alpha_+|1\rangle$$

A. Arimondo, Prog. Opt. **25**, 257 (1996)

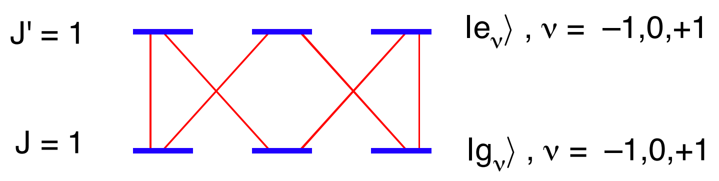
# Determination of Dark State:

Hamiltonian:

$$H_0\,|e_\nu\rangle \;=\; \hbar\omega\,|e_\nu\rangle$$

$$H_0\,|g_\nu\rangle \;=\; 0$$

$$H \;=\; H_0 \;+\; W \;+\; P^2/2m$$



$J' = 1$ $\quad$ $|e_\nu\rangle$ , $\nu = -1,0,+1$

$J = 1$ $\quad$ $|g_\nu\rangle$ , $\nu = -1,0,+1$

angular momentum:

$$J_z\,|e_\nu\rangle \;=\; \nu\,\hbar\,|e_\nu\rangle$$

$$J_z\,|g_\nu\rangle \;=\; \nu\,\hbar\,|g_\nu\rangle$$

dipole operator:

$$d \;=\; d^- \;+\; d^+ \;, \quad d^+ \;=\; P_e\,d\,P_g \;,\quad d^- \;=\; P_g\,d\,P_e \;,\quad P_e \;=\; \sum_\nu |e_\nu\rangle\langle e_\nu| \quad , \; P_g \;=\; \sum_\nu |g_\nu\rangle\langle g_\nu|$$

interaction (RWA):

$$E(r,t) \;=\; \tfrac{1}{\surd 2}\,(E(r)\,e^{-i\omega t} \;+\; E^*(r)\,e^{i\omega t}\,)$$

$$W \;=\; -\,d^+\,E(r) \;-\; d^-\,E^*(r) \quad = \; -\sum_{\nu,\mu} \; |e_\nu\rangle\langle e_\nu|\;d^+\,E(r)\;|g_\mu\rangle\langle g_\mu| \quad + \quad \text{c.c}$$

$$= \quad -\;D\;\sum_{\nu,\mu,\kappa} |e_\nu\rangle \quad E_\kappa(r)\;C^\kappa{}_{\nu\mu}\;\;\langle g_\mu| \quad + \quad \text{c.c}$$

change to cartesian basis:

$$|g_x\rangle \;=\; \tfrac{1}{\surd 2}(|g_{-1}\rangle \;-\; |g_{+1}\rangle\;)$$

$$|g_y\rangle \;=\; \tfrac{i}{\surd 2}(|g_{-1}\rangle \;+\; |g_{+1}\rangle\;)$$

$$|g_z\rangle \;=\; |g_0\rangle$$

$$\Rightarrow \quad W \;=\; -\,\tfrac{i}{\surd 2}\,D\,\sum_{n,m,k} \varepsilon_{nmk}\,|e_n\rangle \quad E_m(r)\quad\langle g_k|$$

$\varepsilon_{nmk}$ = Levi-Civita symbol define dark state: $|\psi_{NC}\rangle = \sum_{n} G_n(r)\ |g_n\rangle \quad \Rightarrow \quad \langle e_n|\, W\, |\psi_{NC}\rangle = \sum_{m,k} \varepsilon_{nmk}\, E_m(r)\, G_k(r) = [\vec{E} \times \vec{G}]_n$

$|\psi_{NC}\rangle$ stationary, i.e., Eigen-state of the total Hamiltonian $H = H_0 + W + P^2/2m \quad \Leftrightarrow$

$$\begin{array}{ll} \text{A)} & \vec{E} \times \vec{G} = 0 \\ \text{B)} & P^2\, |\psi_{NC}\rangle = p^2\ |\psi_{NC}\rangle \end{array} \quad \Leftrightarrow \quad \boxed{\begin{array}{l} \vec{G}(r) = f(r)\, \vec{E}(r) \\ [\, \Delta + (p/\hbar)^2\, ]\ \vec{G} = 0 \end{array}}$$

solution 1 : E-field has no polarization gradient: choose $\vec{E}(r) = \vec{E}_0\ \alpha(r)$ with $f(r) = e^{\,i\,(p/\hbar)\, r} / \alpha(r)$

$\Rightarrow \vec{G}(r) = \alpha(r)\, f(r)\, \vec{E}_0$ and $[\, \Delta + (p/\hbar)^2\, ]\ \vec{G} = 0 \Rightarrow$ A) and B)

$|\psi_{NC}\rangle$ is dark ( $\langle e_n|\, W\, |\psi_{NC}\rangle = 0$ ) and stationary with respect to $P^2/2m$ independent of the value of p.

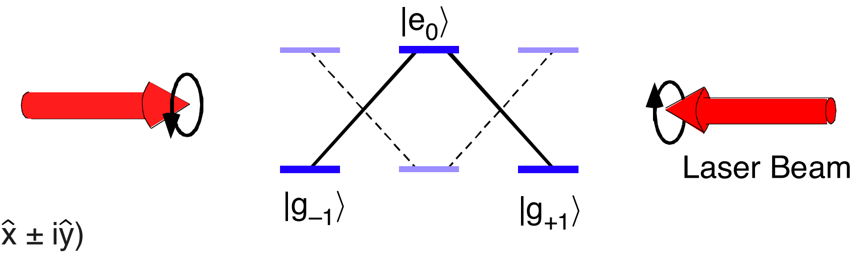
solution 2 : chose $f(r) = \text{constant} \Rightarrow$ because $[\, \Delta + k^2\, ]\ \vec{E} = 0$ we get $[\, \Delta + k^2\, ]\ \vec{G} = 0 \Rightarrow$ B) holds if $|p| = |\hbar k|$

$|\psi_{NC}\rangle$ only remains dark if $|p| = |\hbar k|$ , otherwise $|\psi_{NC}\rangle$ is not stationary with respect to $P^2/2m$
Atoms with $\pm\hbar k$ momentum are decoupled from the light field, while faster atoms may interact.

## → Velocity selective coherent population trapping VSCPT :

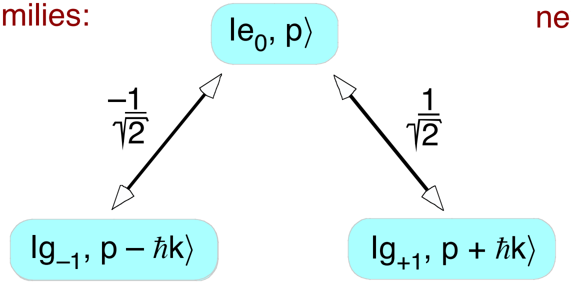
Atoms undergo random walk in momentum space until they incidentally have $\pm\hbar k$ momentum and become trapped in the dark state.

# VSCPT in the $\sigma^+\sigma^-$ configuration:



Electric Field: $E(r) \;=\; E\;(\hat{\varepsilon}_+ e^{ikz} + \hat{\varepsilon}_- e^{-ikz}) \qquad \varepsilon_\pm \;=\; \frac{1}{\sqrt{2}}(\hat{x} \pm i\hat{y})$

Closed excitation families:



$|e_0, p\rangle$

$\frac{-1}{\sqrt{2}}$ $\quad$ $\frac{1}{\sqrt{2}}$

$|g_{-1}, p - \hbar k\rangle$ $\quad$ $|g_{+1}, p + \hbar k\rangle$

new basis:

$$|e_0, p\rangle$$

$$|\psi_C(p)\rangle \;=\; \frac{1}{\sqrt{2}}\Big[\,|g_{+1}, p + \hbar k\rangle \;-\; |g_{-1}, p - \hbar k\rangle\,\Big]$$

$$|\psi_{NC}(p)\rangle \;=\; \frac{1}{\sqrt{2}}\Big[\,|g_{+1}, p + \hbar k\rangle \;+\; |g_{-1}, p - \hbar k\rangle\,\Big]$$

Interaction:

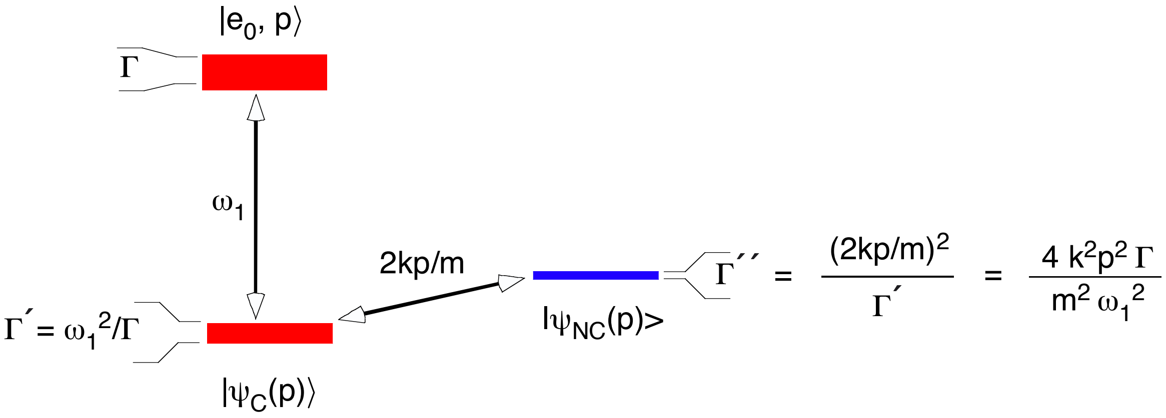
$$W \;=\; \frac{\hbar\omega_1}{2}\sum_p \frac{1}{\sqrt{2}}\Big[\,|e_0, p\rangle\langle g_{+1}, p + \hbar k| \;-\; |e_0, p\rangle\langle g_{-1}, p - \hbar k| \;+\; \text{c.c}\,\Big] \;=\; \frac{\hbar\omega_1}{2}\sum_p \Big[\,|e_0, p\rangle\langle\psi_C(p)| \;+\; \text{c.c}\,\Big]$$

$$\langle e_0, p|\, W\, |\psi_C(p)\rangle \;=\; \frac{\hbar\omega_1}{2}$$

$$\langle e_0, p|\, W\, |\psi_{NC}(p)\rangle \;=\; 0$$

$$\langle\psi_{NC}(p)|\, P^2/2m\, |\psi_C(p)\rangle \;=\; \frac{\hbar k\, p}{m}$$

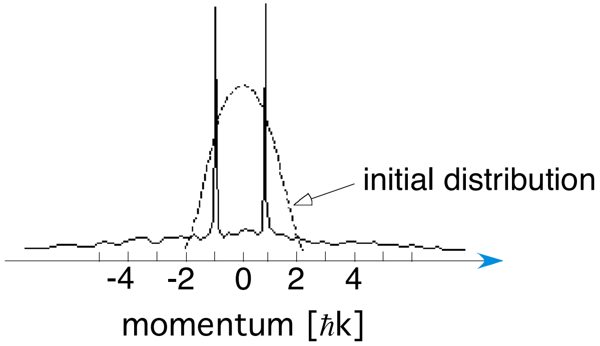
# VSCPT dynamics

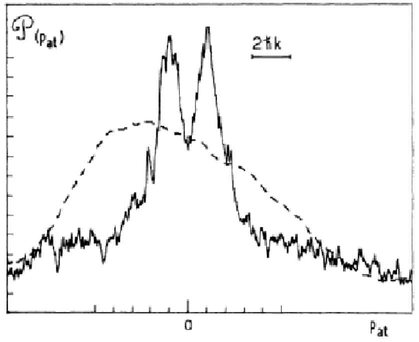


$$\Gamma'' = \frac{(2kp/m)^2}{\Gamma'} = \frac{4\,k^2p^2\,\Gamma}{m^2\,\omega_1^2}$$

- Bright state $|\psi_C(p)\rangle$ has spatially constant light shift.
- If $p \neq 0$, the kinetic energy operator induces a Rabi-oscillation with frequency 2kp/m between $|\psi_{NC}(p)\rangle$ and $|\psi_C(p)\rangle$.
- If $p = 0$, $|\psi_{NC}(p)\rangle$ is stationary and perfectly dark.
- The state $|\psi_{NC}(p)\rangle$ is populated via spontaneous emission in a mometum diffusion process
- for moderate interaction times, atoms pile up at momenta $\pm\hbar k$ . For large interaction times the atoms tend to distribute over the entire momentum space → no steady state exists.

Monte-Carlo Simulation



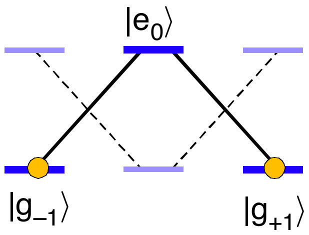


A. Aspect et al., Phys. Rev. Lett. **61**, 826 (1988)

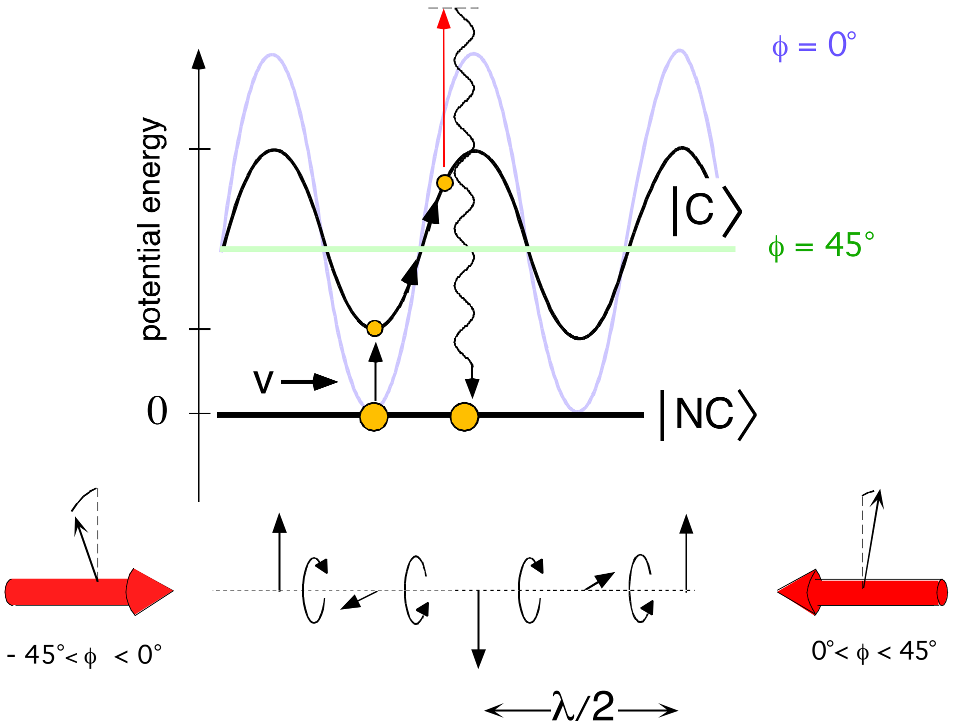
# Combining VSCPT with Sisyphus-Cooling

Problem: VSCPT has no steady state, unefficient loading of $|NC\rangle$
Solution: keep atoms within finite fraction of momentum space by sub-Doppler mechanism

Level scheme:

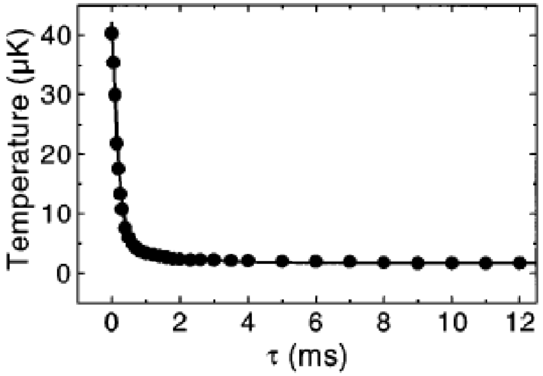


optical potentials: bright state has spatially varying light shift



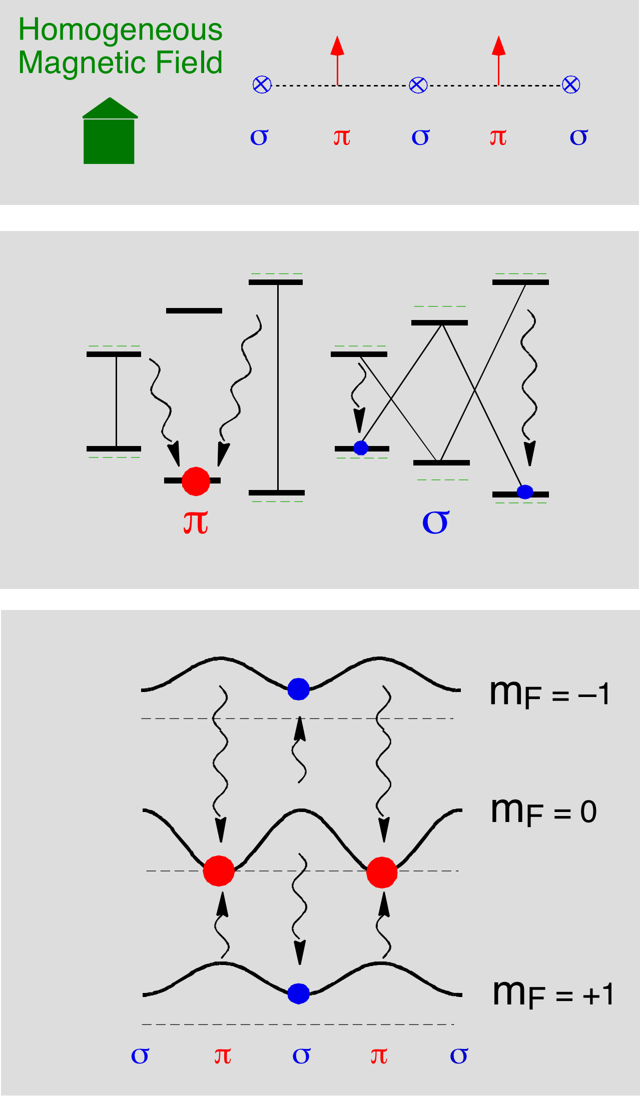
M.S. Sharhiar, et al., Phys. Rev. A 48, R4035 (1993)
M. Weidemüller, et al., Europhys.Lett. 27, 109 (1994)

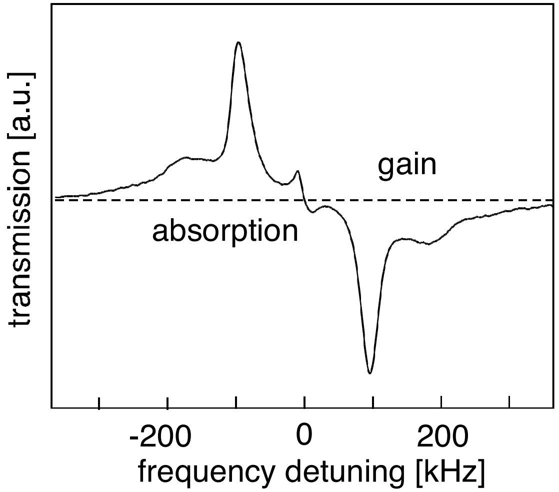
Temperature in dark optical molasses lower than in conventional optical molasses:

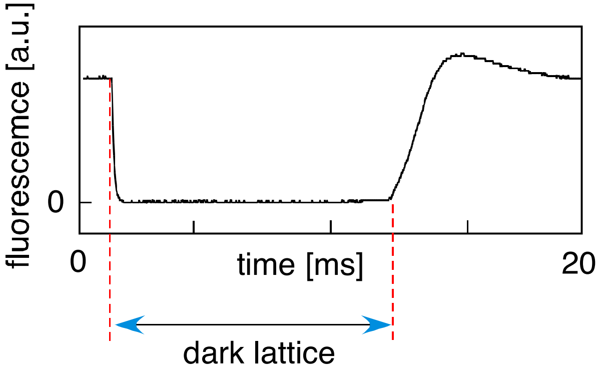


D. Boiron, et al., Phys. Rev. A .53, R3734 (1996)

# Dark Optical Lattice





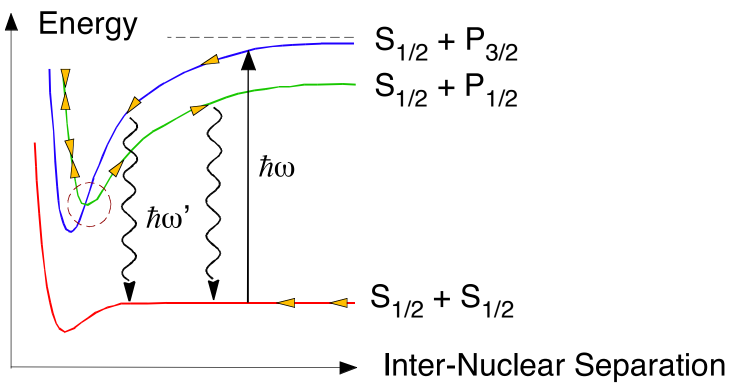


G. Grynberg and Y. Courtois, Europhys. Lett. 27, 41 (1994)

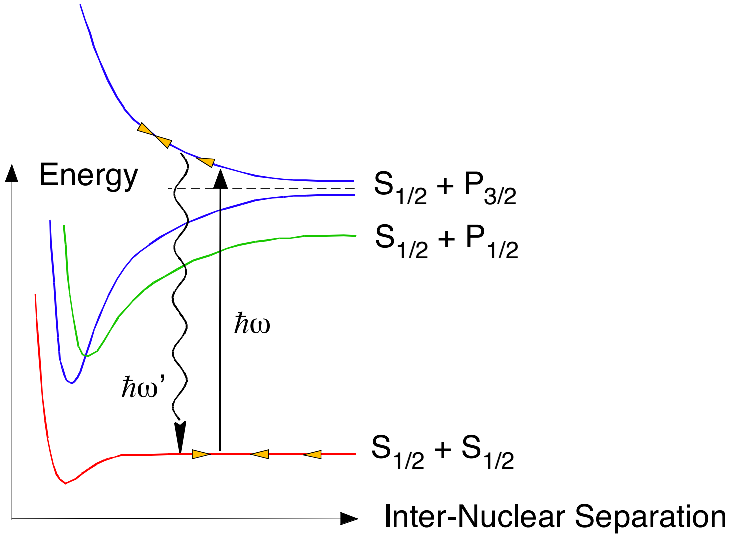
A. Hemmerich et al., Phys. Rev. Lett. 75, 37 (1995)

# Binary Collisional Loss Processes in Optical Lattices

## Conventional Optical Lattices



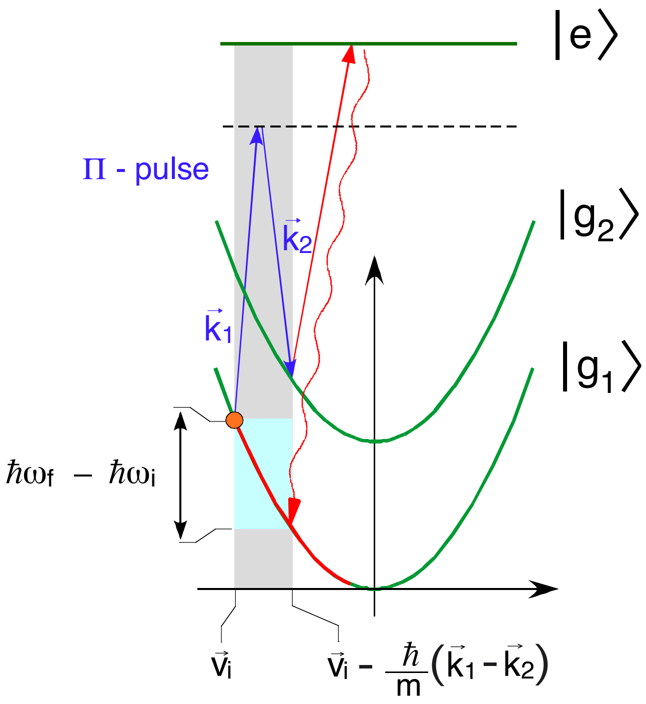
## Dark Optical Lattices

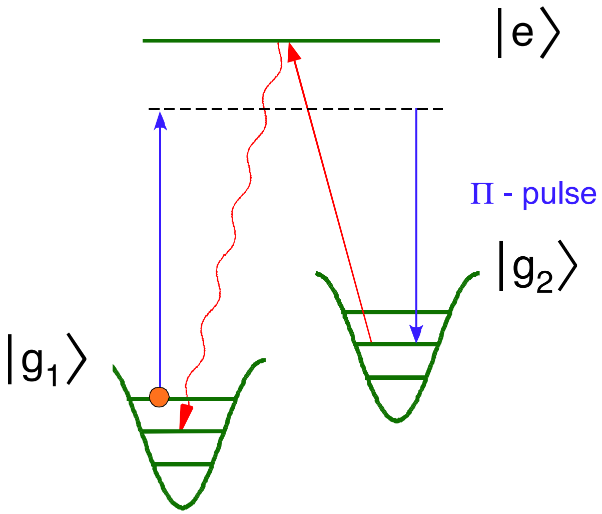


Blue detuning: radiative escape and hyperfine changing collisions surpressed -> optical shielding

J. Piilo and K.-A. Suominen, Phys. Rev. A **66**, 013401 (2002)

# Raman-Cooling





Cooling procedure: successivly apply $\Pi$ Raman pulses and optical pumping pulses

Temperature limit: state selectivity of Raman pulse -> no principle limitation

Preconditions: two stable electronic states (e.g., hyperfine levels or Zeeman levels of ground state)

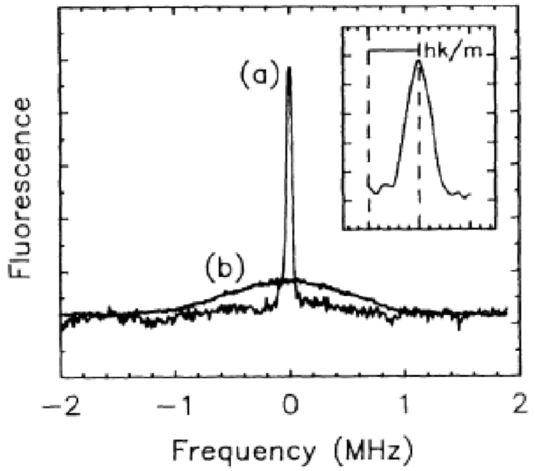
-> state selective optical pumping (e.g., selectivity via frequency or polarization)

-> nearly equidistant motional states (free or trapped)

M. Kasevich and S. Chu, Phys. Rev. Lett. **69**, 1741 (1992)
S. E. Hamann et al., Phys. Rev. Lett. **80**, 4149 (1998)

# Raman-Cooling

Free atoms, 1D:

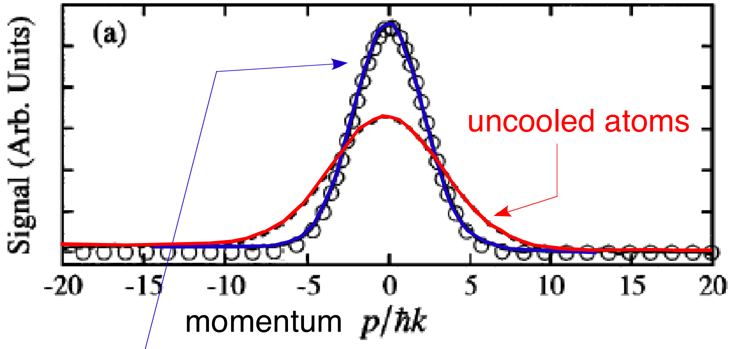


M. Kasevich and S. Chu, Phys. Rev. Lett. **69**, 1741 (1992)

FIG. 4. (a) The velocity distribution after application of the stimulated Raman cooling pulses. The inset, showing a high resolution scan of the central velocity spike, compares the velocity distribution to the velocity change $\Delta v = 3$ cm/sec from the recoil of a single photon. (b) The initial velocity distribution of sodium atoms due to polarization-gradient cooling. A uniform background signal ~3 times the size of the peak signal for curve *b* has been subtracted from curve *a*. The background was due to incomplete optical pumping from $F=2 \rightarrow F=1$ during the Raman cooling sequence, and is responsible for the increased noise on curve *a*.

Far-detuned optical lattice, 3D:

S. E. Hamann et al., Phys. Rev. Lett. **80**, 4149 (1998)



solid line:
cooled atoms

open circles:
calculation for vibrational
ground state