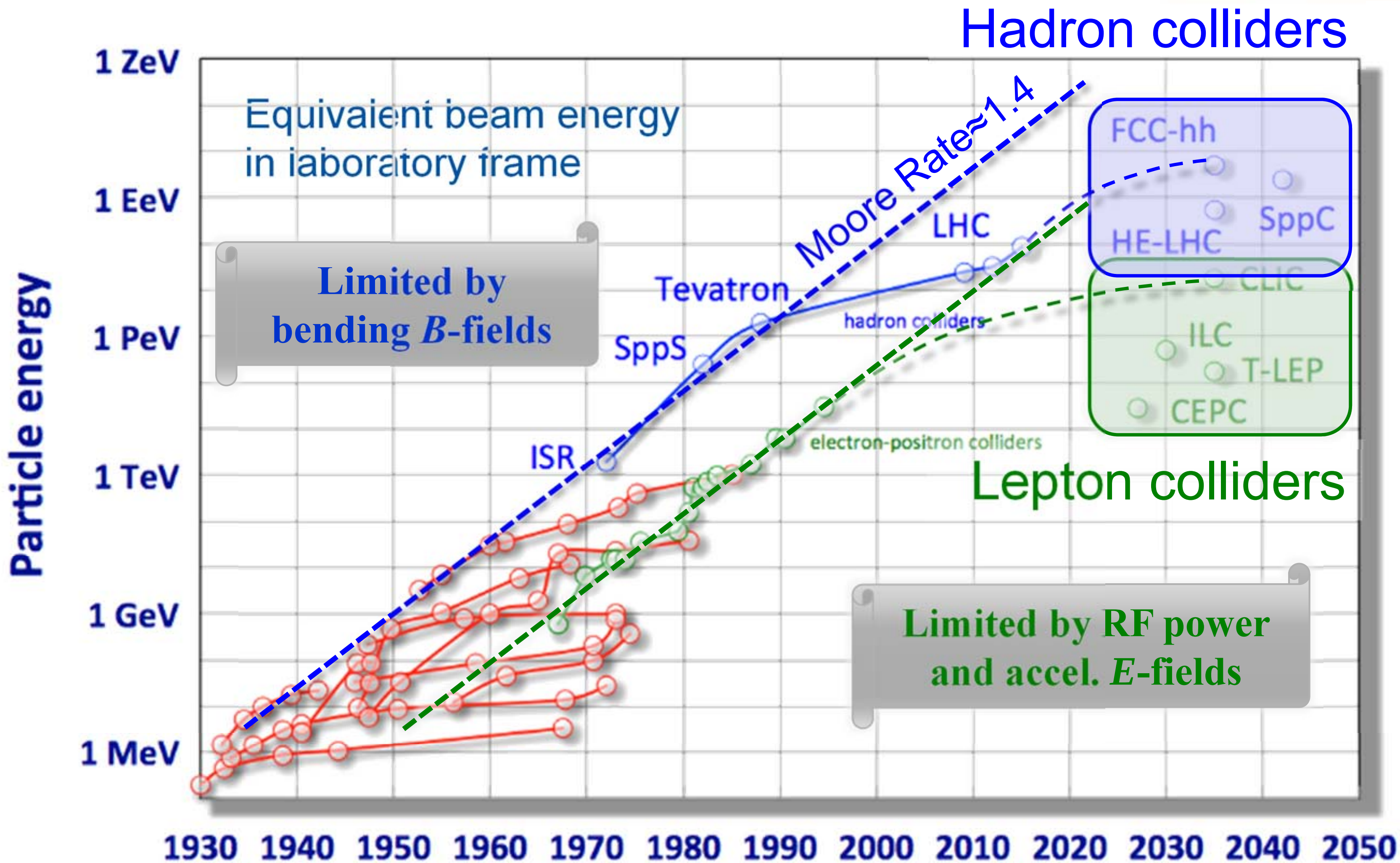
Livingston plot



Luminosity

… the unknown divinity …

The (only?) important acc. parameter for particle physicists?

- **Luminosity** $\dot{N} = \sigma \cdot \mathcal{L}$
- **Integrated Luminosity:** $\dot{N} = \sigma \cdot \int_{\text{t-meas.}} \mathcal{L} \cdot dt = \sigma \cdot \mathcal{J}$

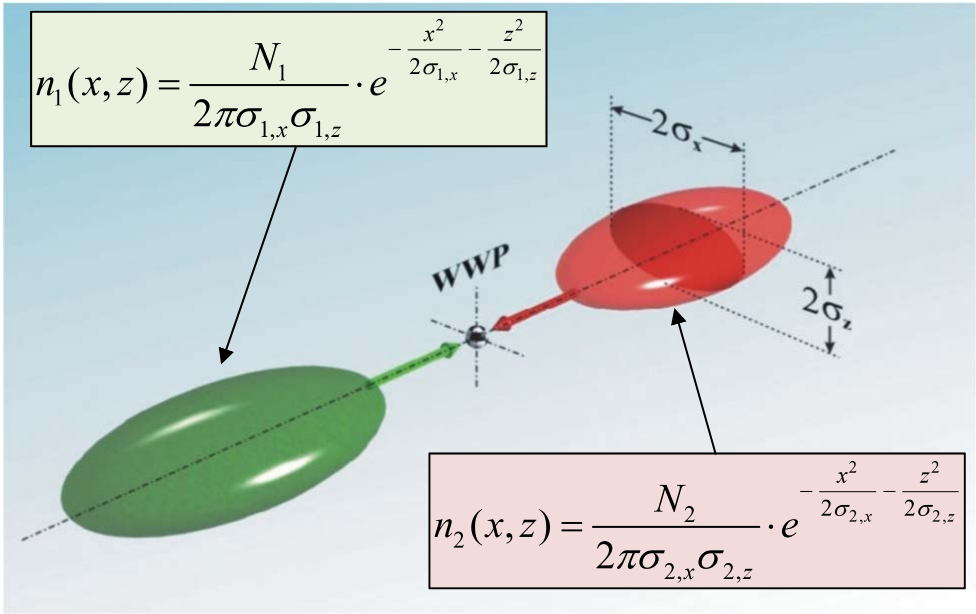
$$N_{b,b} = \sigma \cdot \iint n_1(x,z) \cdot n_2(x,z) \cdot dx\,dz$$

⇓

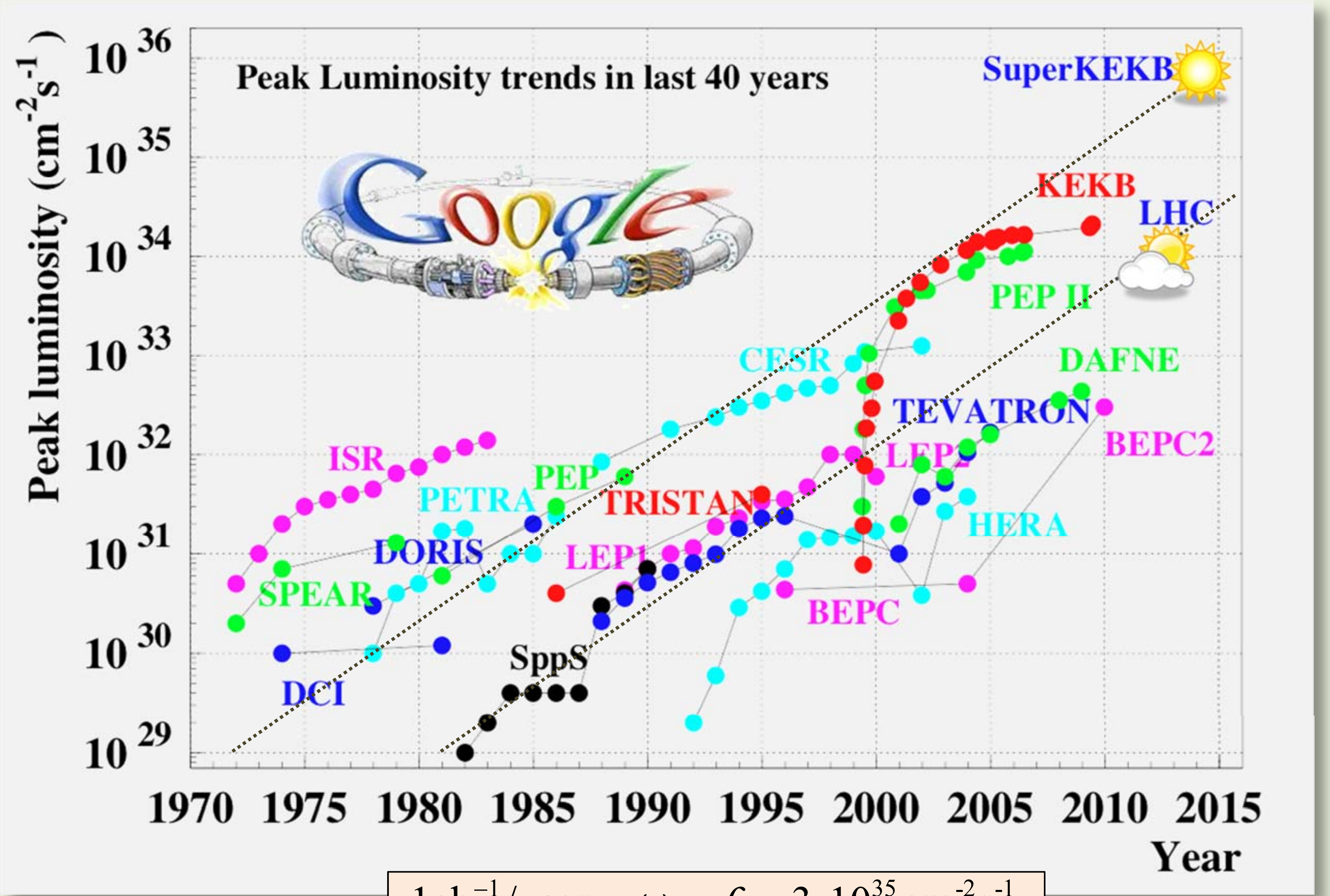
e⁺-e⁻, p-p Collider:

$$\sigma_1 = \sigma_2 = \sigma$$

$$\mathcal{L} = \frac{n_b \cdot f_{rev}}{4\pi} \cdot \frac{N_1 \cdot N_2}{\sigma_x \cdot \sigma_z}$$

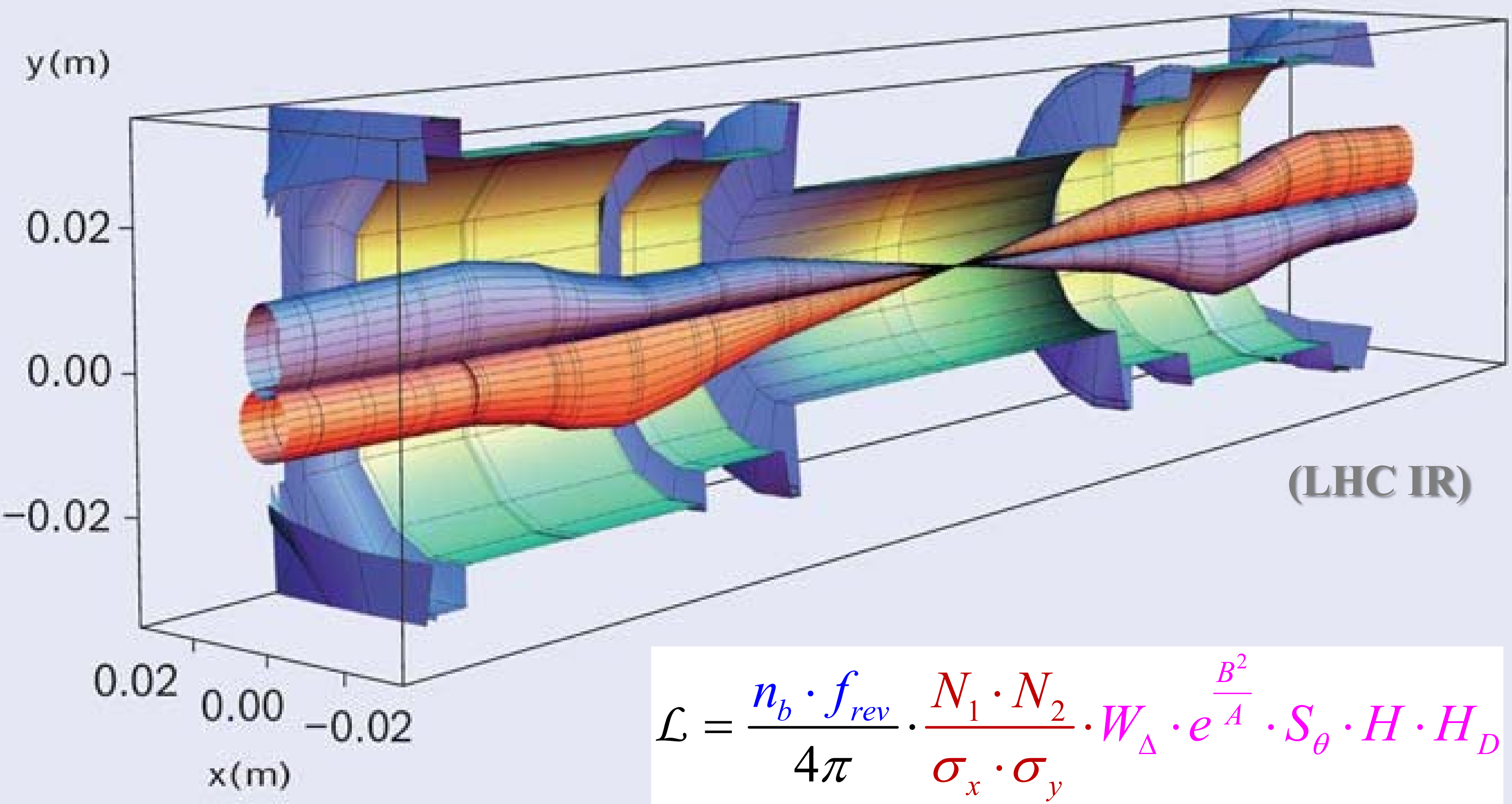


Luminosity



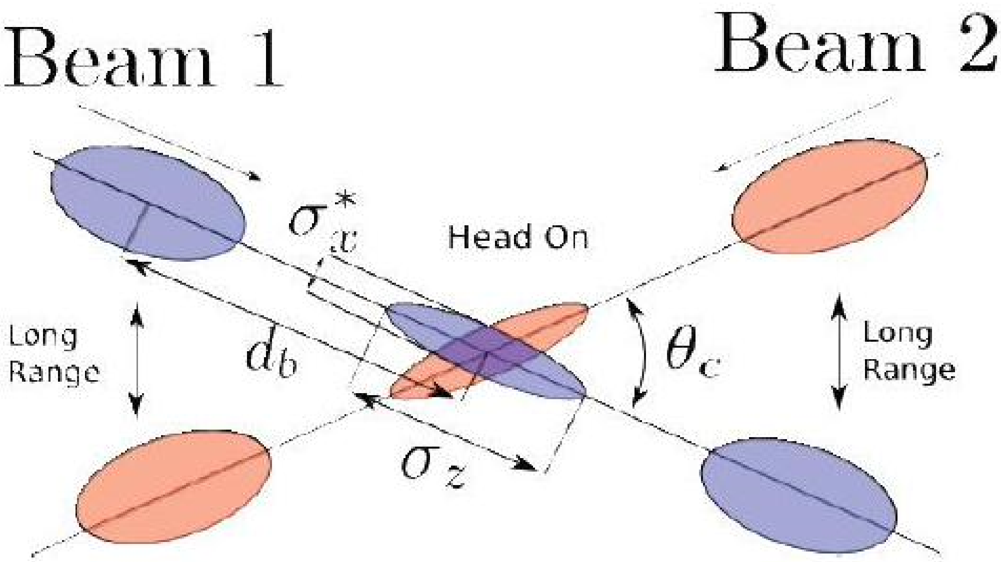
$$1\,\text{ab}^{-1}/\text{year} \quad \leftrightarrow \quad \mathcal{L} = 3 \cdot 10^{35}\,\text{cm}^{-2}\text{s}^{-1}$$

Luminosity Optimization



$$\mathcal{L} = \frac{n_b \cdot f_{rev}}{4\pi} \cdot \frac{N_1 \cdot N_2}{\sigma_x \cdot \sigma_y} \cdot W_\Delta \cdot e^{\frac{B^2}{A}} \cdot S_\theta \cdot H \cdot H_D$$

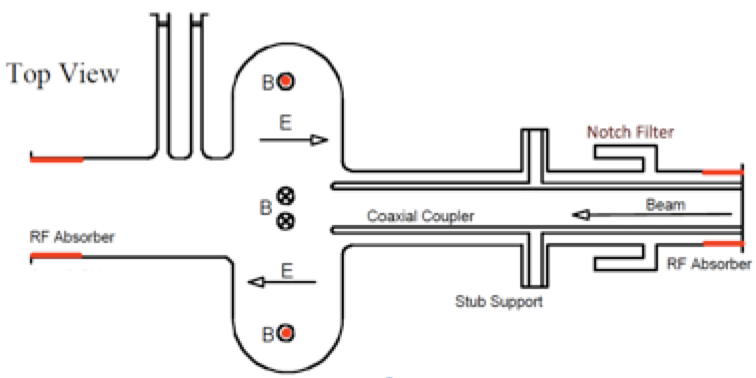
Beam Crossing

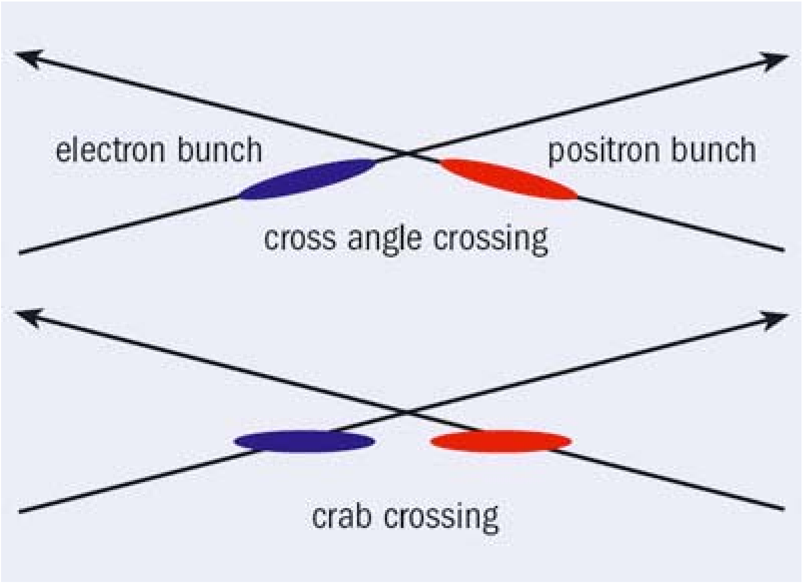


- Sufficient beam separation
- Small crossing angle
- Long bunch separation
- …

$$\mathcal{L} = \frac{n_b \cdot f_{rev}}{4\pi} \cdot \frac{N_1 \cdot N_2}{\sigma_x \cdot \sigma_y}$$

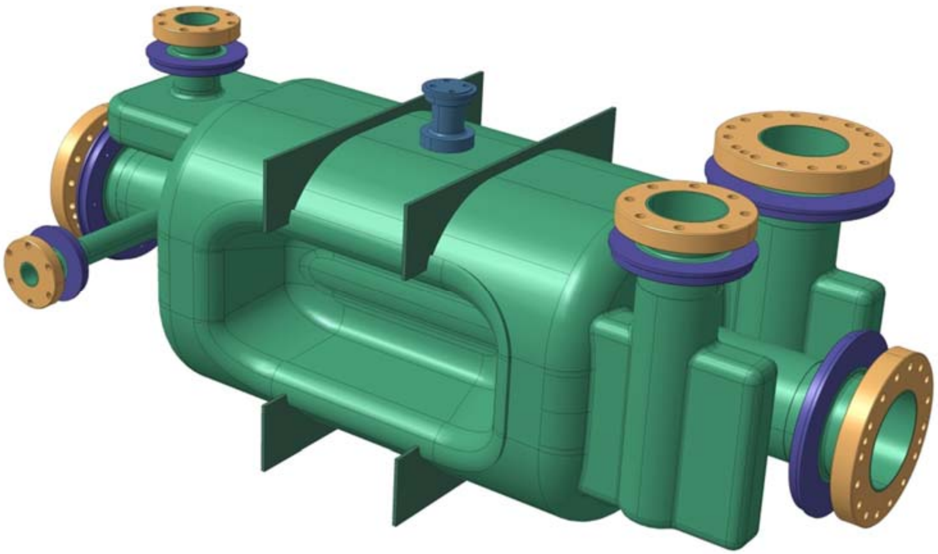
<u>Crab-Cavities:</u>





Cavities

Baseline : adopt both cavity types and exploit their natural RF topology

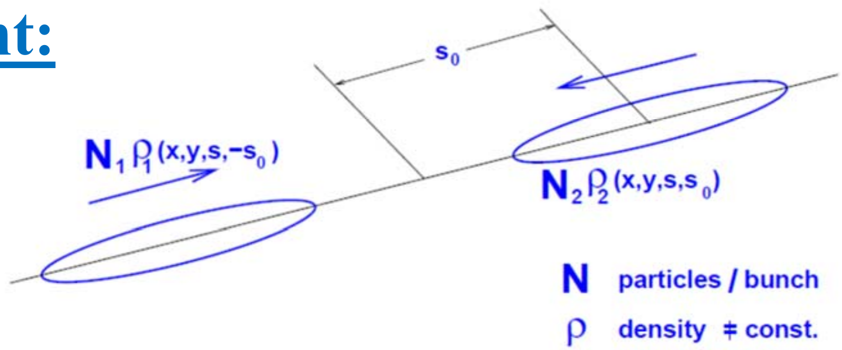


Double Quarter Wave (DQW) cavity –
Vertical – to be used in Point 1 (ATLAS)

RF Dipole (RFD) cavity –
Horizontal – to be used in Point 5 (CMS)

Beam Crossing

Full 3D treatment:



$$\mathcal{L} \propto K N_1 N_2 \int\int\int\int_{-\infty}^{+\infty} \rho_1(x,y,s,-s_0)\rho_2(x,y,s,s_0)\,dx\,dy\,ds\,ds_0$$

s_0 is "time"-variable: $s_0 = c\cdot t$

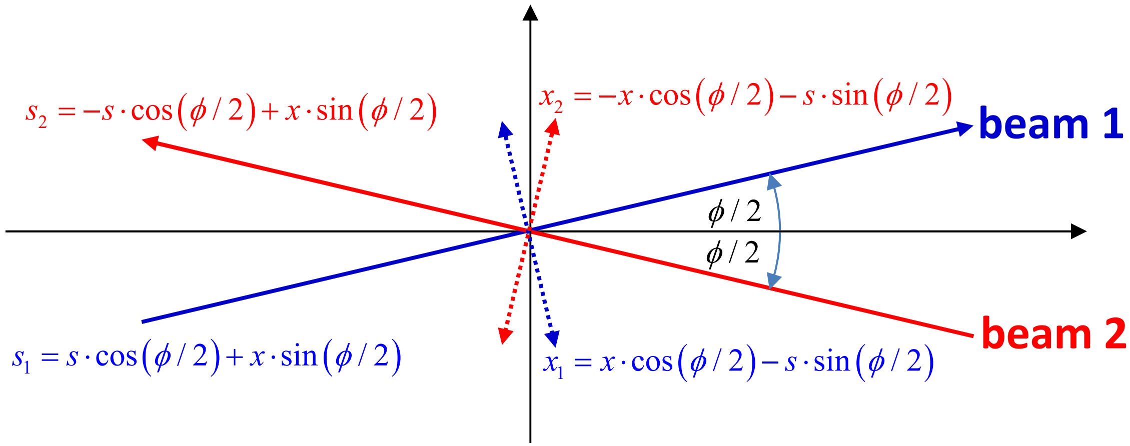
Kinematic factor: $K = \sqrt{(\vec{v_1}-\vec{v_2})^2 - (\vec{v_1}\times\vec{v_2})^2/c^2}$

Gaussian
distribution functions:

$$\rho_i(x,y,s,\pm s_0) = \rho_{i,x}(x)\cdot\rho_{i,y}(y)\cdot\rho_{i,s}(s\pm s_0)$$

$$\rho_s(s\pm s_0) = \frac{1}{\sigma_s\sqrt{2\pi}}\exp\left(-\frac{(s\pm s_0)^2}{2\sigma_s^2}\right) \qquad \rho_z(u) = \frac{1}{\sigma_u\sqrt{2\pi}}\exp\left(-\frac{u^2}{2\sigma_u^2}\right) \qquad u = x,y$$

Piwinski Angle



Assume crossing in horizontal (x, s)- plane.
Transform to new coordinates:

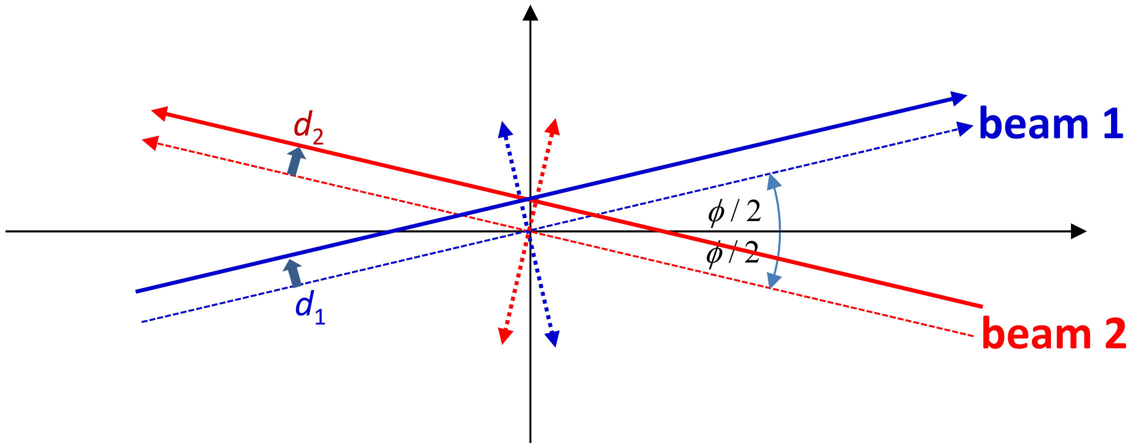
$$\begin{cases} x_1 = x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, & s_1 = s\cos\frac{\phi}{2} + x\sin\frac{\phi}{2}, \\ x_2 = x\cos\frac{\phi}{2} + s\sin\frac{\phi}{2}, & s_2 = s\cos\frac{\phi}{2} - x\sin\frac{\phi}{2} \end{cases}$$

$$\mathcal{L} \approx \frac{f_{rev} n_b}{4\pi}\frac{N_1 N_2}{\sigma_x \sigma_y}\underbrace{\frac{1}{\sqrt{1+\left(\frac{\sigma_s}{\sigma_x}\tan\frac{\phi}{2}\right)^2}}}_{=S_\theta} \approx \frac{f_{rev} n_b}{4\pi}\frac{N_1 N_2}{\sigma_x \sigma_y}\frac{1}{\sqrt{1+\left(\frac{\sigma_s}{\sigma_x}\frac{\phi}{2}\right)^2}}$$

"Piwinski Angle"

Example LHC @ 7TeV: ϕ=285µrad, σ_x ≈17µm, σ_s ≈7.5cm → S_θ=0.84

Offset and Crossing Angle



Transformations with offsets in crossing plane:

$$\begin{cases} x_1 = d_1 + x\cos\frac{\phi}{2} - s\sin\frac{\phi}{2}, & s_1 = s\cos\frac{\phi}{2} + x\sin\frac{\phi}{2}, \\ x_2 = d_2 + x\cos\frac{\phi}{2} + s\sin\frac{\phi}{2}, & s_2 = s\cos\frac{\phi}{2} - x\sin\frac{\phi}{2} \end{cases}$$

$$\mathcal{L} \approx \frac{f_{rev} n_b}{4\pi} \frac{N_1 N_2}{\sigma_x \sigma_y} \cdot W_\Delta \cdot e^{-\frac{B^2}{A}} \cdot S_\theta$$

$\sigma_x \to \sigma_y$ for vert. displacement

$$A = \frac{\sin^2\frac{\phi}{2}}{\sigma_x^2} + \frac{\cos^2\frac{\phi}{2}}{\sigma_s^2}, \qquad B = \frac{(d_2 - d_1)\sin\frac{\phi}{2}}{2\sigma_x^2},$$

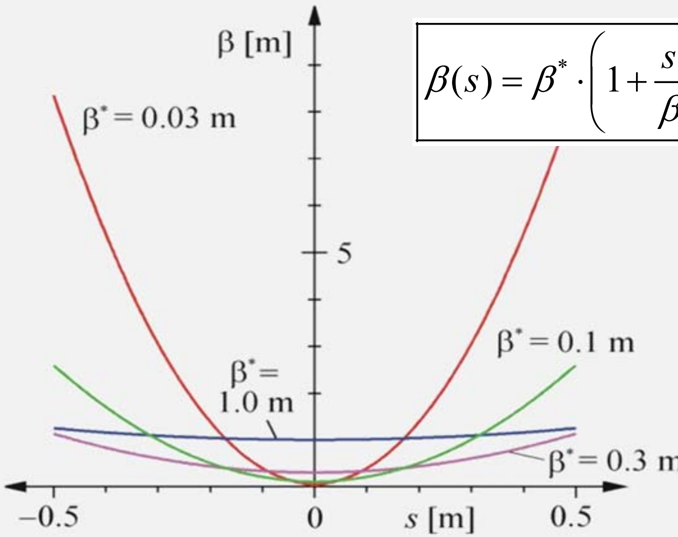
$$W_\Delta = e^{-\frac{(d_2 - d_1)^2}{4\sigma_x^2}}$$

“Beta Squeeze”

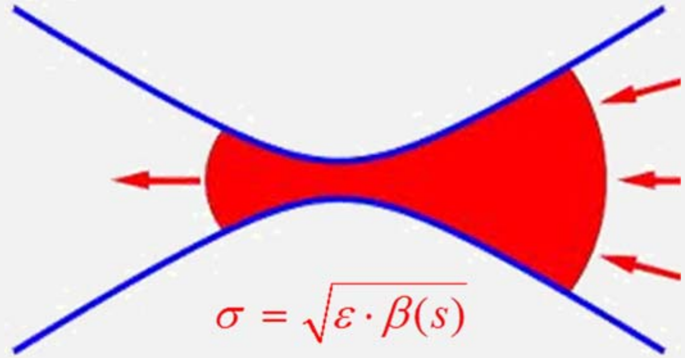
$$\sigma_{x,y} = \sqrt{\varepsilon_{x,y} \cdot \beta_{x,y}}$$

Beam Broadening:

$$\beta(s) = \beta^* \cdot \left(1 + \frac{s^2}{\beta^*}\right)$$



Hourglass Effect:

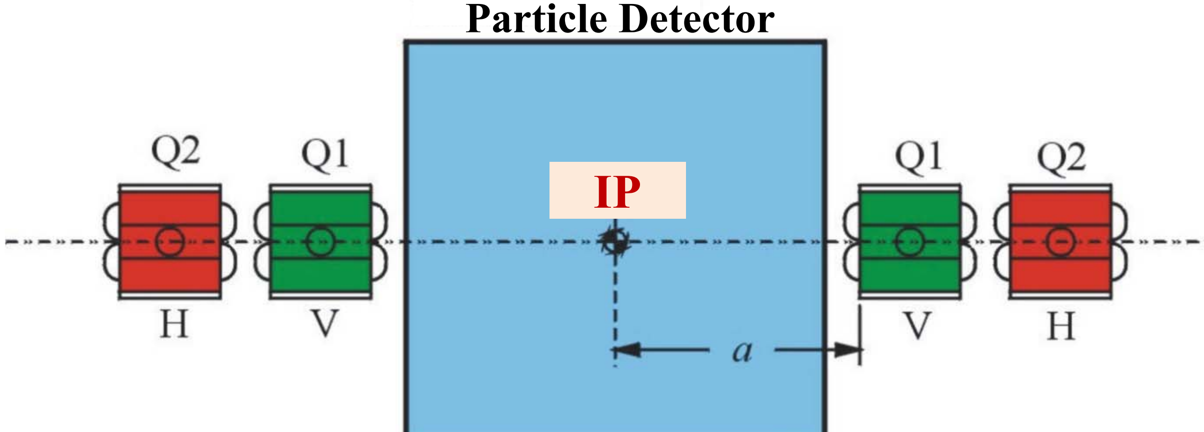


$$\sigma = \sqrt{\varepsilon \cdot \beta(s)}$$

Short Bunches!

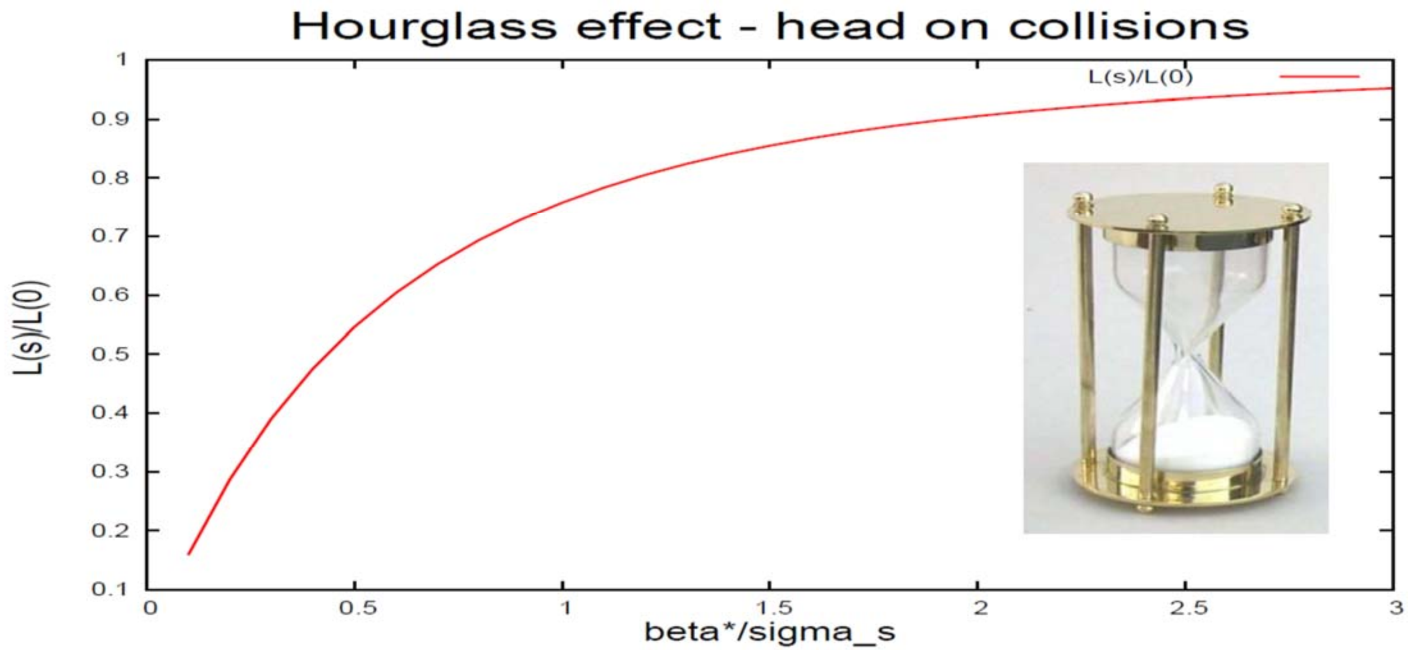
$$\sigma_s \approx \beta^*$$

$$\mathcal{L} = \frac{f_{rev} n_b}{4\pi} \cdot \frac{N_1 \cdot N_2}{\sigma_x \cdot \sigma_y} \cdot H$$



Hourglass Effect

$$\frac{\mathcal{L}(\sigma_s)}{\mathcal{L}(0)} = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\pi}} \frac{e^{-u^2}}{[1+(\frac{u}{u_x})^2]} du = \sqrt{\pi} \cdot u_x \cdot e^{u_x^2} \cdot \mathrm{erfc}(u_x)$$

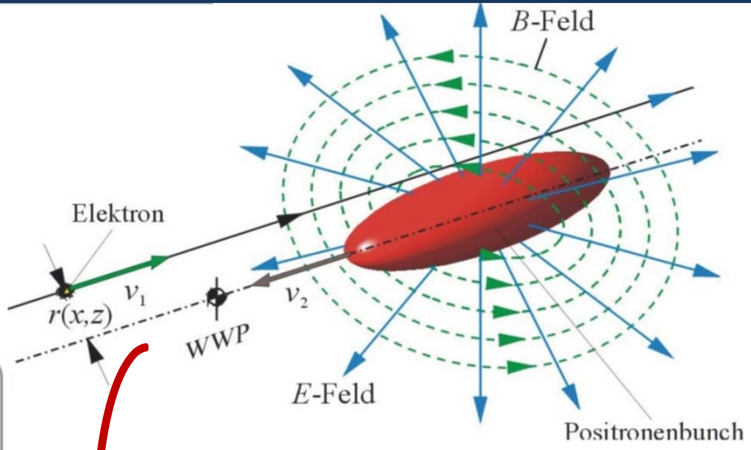


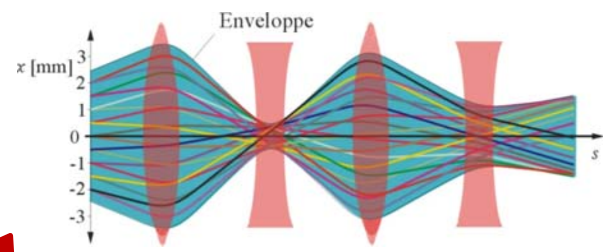
$$\mathcal{L}(\sigma_s) = \mathcal{L}(0) \cdot H \quad \text{with :} \quad H = \sqrt{\pi} \cdot u_x \cdot e^{u_x^2} \cdot \mathrm{erfc}(u_x)$$

$$u_x = \beta^* / \sigma_s$$

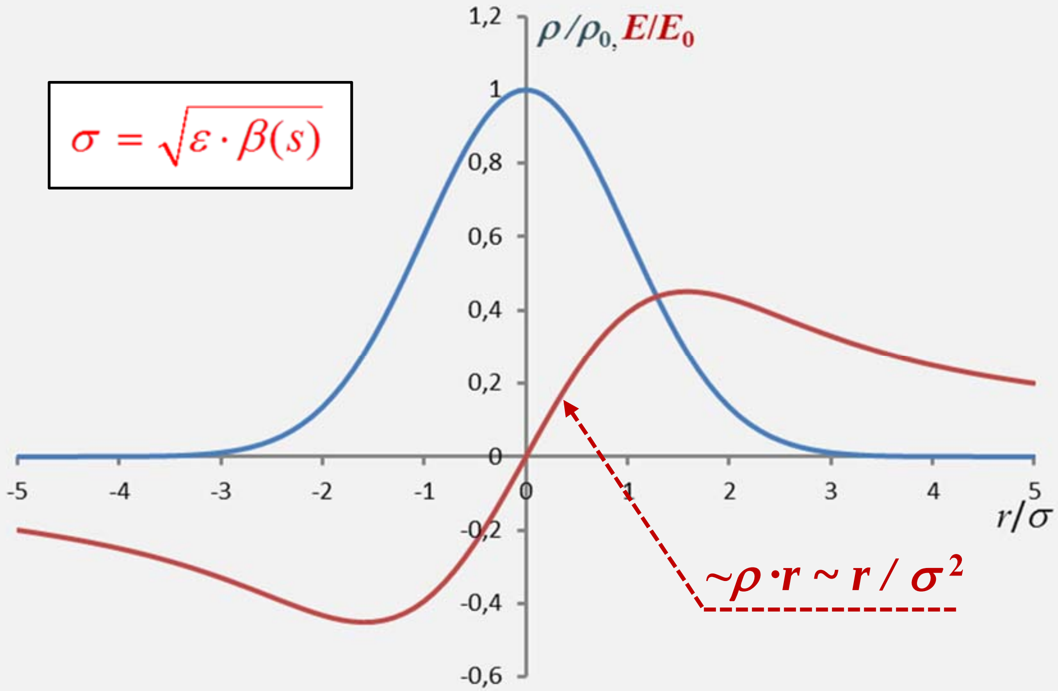
Beam-Beam Effects ↔ $\mathcal{L} = \frac{f_{rev} n_b}{4\pi} \cdot \frac{N_1 \cdot N_2}{\sigma_x \cdot \sigma_z} \cdots$

Additional Focusing or Defocusing in the Interaction Region





$$\sigma = \sqrt{\varepsilon \cdot \beta(s)}$$



Sets limit on emittance in head-on collisions!

Beam-Beam Parameters:

$$\xi_{x,y} = \frac{r_e N}{2\pi\gamma_r} \frac{\beta^*_{x,y}}{\sigma_{x,y}\left(\sigma_x + \sigma_y\right)}$$

$$\sigma_{x,y} = \sqrt{\varepsilon_{x,y}\beta_{x,y}}$$

Storage Rings

Important Relations:

a) **Luminosity**

$$\mathcal{L} = \frac{n_b \cdot f_{rev}}{4\pi} \cdot \frac{N_1 \cdot N_2}{\sigma_x \cdot \sigma_y} \cdot S_\theta \cdot H$$

b) **Beam-Beam Parameters**

$$\xi_{x,y} = \frac{r_e N}{2\pi \gamma_r} \frac{\beta^*_{x,y}}{\sigma_{x,y}\left(\sigma_x + \sigma_y\right)}$$

→ Rewrite Luminosity Formula ($\xi_y > \xi_x$)

$$\mathcal{L} = \frac{\gamma_r}{2 e r_e} \cdot \left(1 + \frac{\sigma_y^*}{\sigma_x^*}\right) \cdot \frac{I_{beam} \cdot \xi_y}{\beta_y^*} \cdot S_\theta \cdot H$$

Lorentz factor

Beam current

Beam-beam parameter

Aspect ratio at IP

Vertical beta function at IP

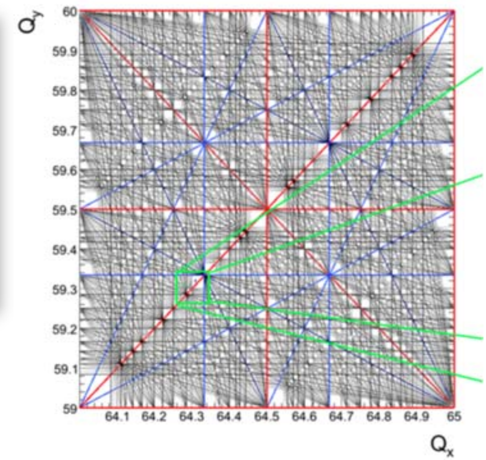
Hourglass effect: $\beta_y^* \geq \sigma_s$

Beam-Beam Parameters

$$\xi_{x,y} = \frac{r_e N}{2\pi\gamma_r} \frac{\beta^*_{x,y}}{\sigma_{x,y}\left(\sigma_x + \sigma_y\right)}, \qquad \sigma_{x,y} = \sqrt{\varepsilon_{x,y}\beta_{x,y}}$$

Circular Colliders: $\xi_{x,y} < 0.05$ typ.

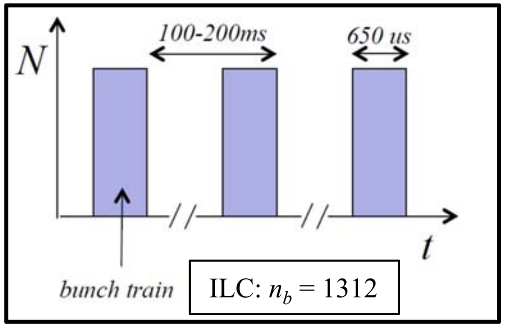
Linear Colliders: $\xi_x = 0.54,\ \xi_y = 1.44$ (ILC)



$$\mathcal{L} = \frac{\gamma_r}{2er_e} \cdot \left(1 + \frac{\sigma^*_y}{\sigma^*_x}\right) \cdot \frac{I_{beam} \cdot \xi_y}{\beta^*_y} \cdot S_\theta \cdot H$$

But:

Time structure of linear / circular colliders are different:



Comparison FCC-ee (@Higgs) ↔ ILC:

- SR: $I_{beam} = f_{rev} n_b q N = 3000 \cdot 393 \cdot q \cdot 1.5 \cdot 10^{11} = 29\,\text{mA}$
- LC: $I_{beam} = f_{rep} n_b q N = 5 \cdot 1312 \cdot q \cdot 1 \cdot 10^{10} = 11\,\mu\text{A}$

Example (e^+-e^- Storage Ring)
SUPER-KEKB

Lumi-Goal: 5(8)x10^{35} s^{-1}·cm^{-2}

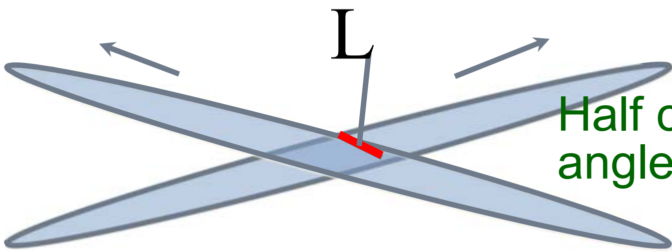
SUPER-KEKB with head-on collision:

- Circumference: 3016 m
- Number of bunches: 2500
- Collision frequency: 10 kHz
- **Beam currents: 9.4 / 4.1 A**

$$L \approx 1.24 \cdot 10^{28} \frac{I_1 \cdot I_2}{\sigma_x \cdot \sigma_y} \approx \frac{5 \cdot 10^{29}}{\sigma_x \cdot \sigma_y}$$

→ $\sigma_x \cdot \sigma_y \approx 10^{-6}$ cm^2, and with real emittance values **$\beta_y \approx 3$ mm**

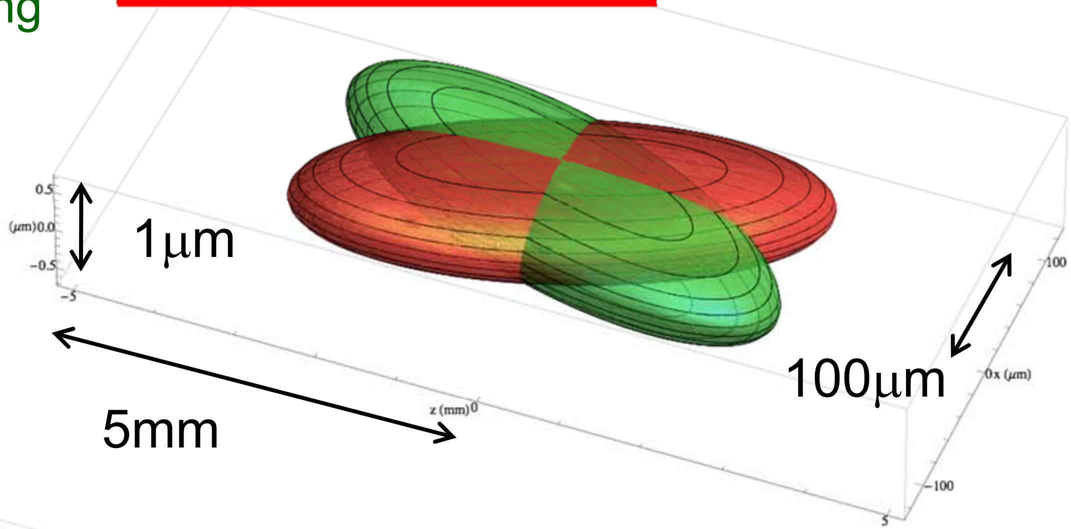
Nano-Beam Scheme



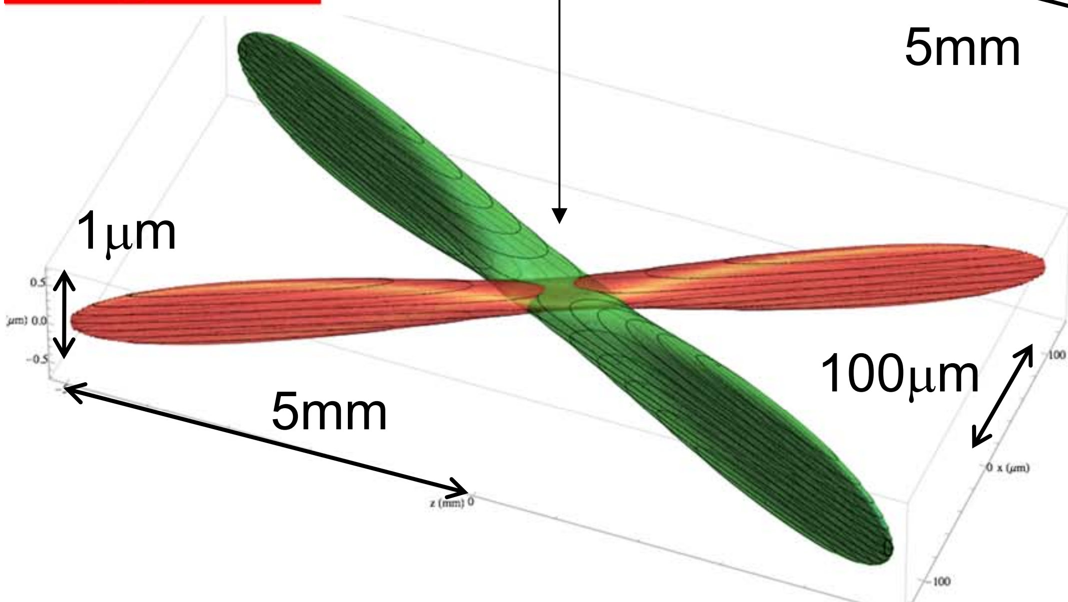
Hourglass condition:

$$\beta_y^* \geq L = \sigma_x/\phi$$

present KEKB (w/o crab)



SuperB



$$L = \frac{\gamma_\pm}{2er_e}\left(1+\frac{\sigma_y^*}{\sigma_x^*}\right)\frac{I_\pm \xi_{\pm y}}{\beta_y^*}\left(\frac{R_L}{R_y}\right)$$

⇕

$$L = \frac{N_+ N_- f}{4\pi\sigma_x^*\sigma_y^*} R_L$$

SuperKEKB Parameters as of Feb.15, 2010

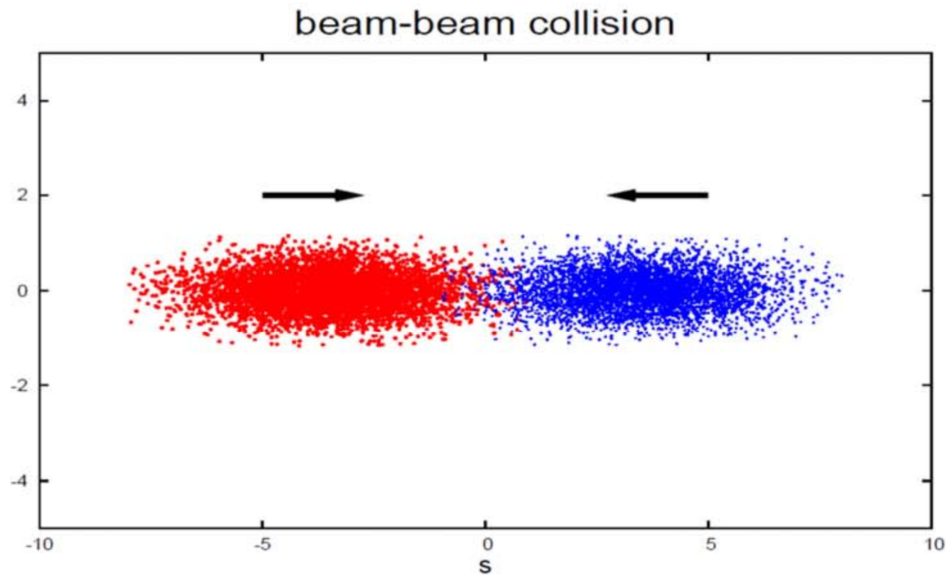
	KEKB Design	KEKB Achieved : with crab	SuperKEKB
Energy (GeV) (LER/HER)	3.5/8.0	3.5/8.0	4.0/7.0
Crossing angle (mrad)	22	0 (crab)	**83**
β_y^* (mm)	10/10	5.9/5.9	**0.27/0.41**
ε_x (nm)	18/18	18/24	**3.2/2.4**
σ_y(μm)	1.9	0.94	0.059
ξ_y	0.052	0.129/0.090	**0.09/0.09**
σ_z (mm)	4	~ 6	6/5
I_{beam} (A)	2.6/1.1	1.64/1.19	3.6/2.62
Number of bunches	5000	1584	2503
Luminosity (10^{34} cm^{-2} s^{-1})	1	2.11	80

Improves with increase of beam energies - if you can store enough beam current!!!

Linear Colliders

$\xi_y > 1$:

Pinch effect - disruption

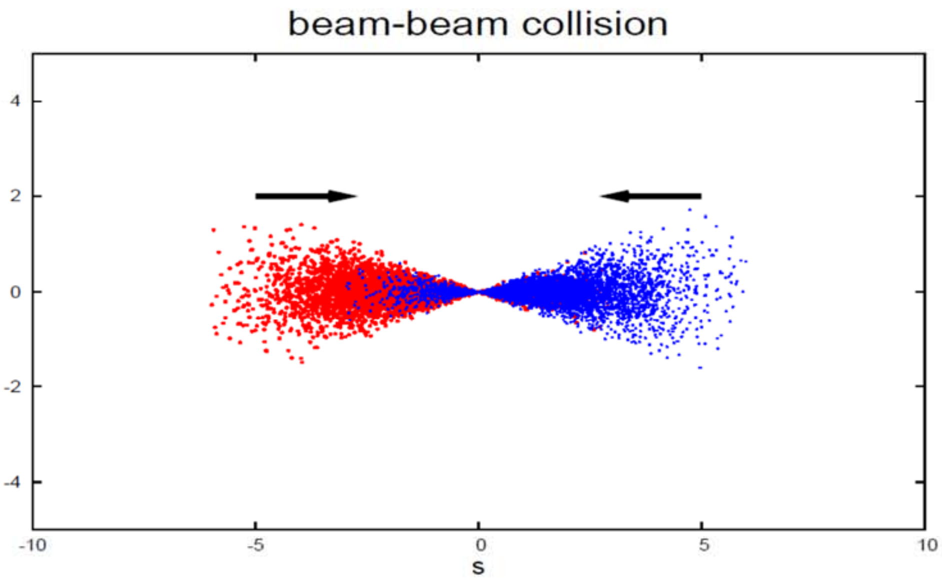


- Additional focusing by opposing beams

Linear Colliders

$\xi_y > 1$:

Pinch effect - disruption



- Additional focusing by opposing beams

Enhancement and Disruption

- Using the enhancement factor H_D:

$$\mathcal{L} = \frac{N^2 \; f_{rep} \; n_b}{4\pi\overline{\sigma_x} \; \overline{\sigma_y}} \;\rightarrow\; \mathcal{L} = \frac{H_D \cdot N^2 \; f_{rep} \; n_b}{4\pi\sigma_x \; \sigma_y}$$

- Enhancement factor H_D takes into account reduction of nominal beam size by the disruptive field (pinch effect)

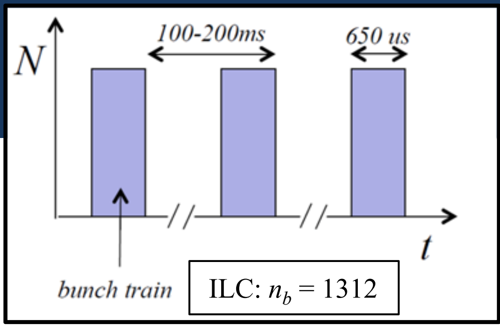
- Related to disruption parameter $\mathcal{D}$:

$$D_{x,y} = \frac{2r_e N\sigma_z}{\gamma\sigma_{x,y}\left(\sigma_x + \sigma_y\right)} \approx \frac{\sigma_z}{f_{beam}} \qquad \begin{aligned} \sigma_z &= \text{bunch length,} \\ f_{beam} &= \textit{focal length} \text{ of beam-lens} \end{aligned}$$

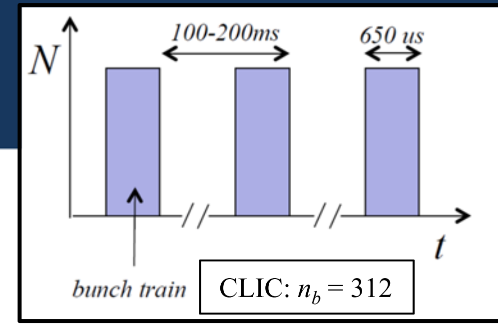
Enhancement factor (typically $H_D \sim 2$): (ILC: $D_y \approx 20 \longrightarrow f_{beam} << \sigma_z$)

$$H_{Dx,y} = 1 + D_{x,y}^{1/4}\left(\frac{D_{x,y}^3}{1 + D_{x,y}^3}\right)\left[\ln\left(\sqrt{D_{x,y}} + 1\right) + 2\ln\left(\frac{0.8\beta_{x,y}}{\sigma_z}\right)\right]$$

'hour glass' effect



Linear Collider



Important Relations:

a) **Luminosity**

$$\mathcal{L} = \frac{n_b \cdot f_{rep}}{4\pi} \cdot \frac{N \cdot N}{\sigma_x \cdot \sigma_y} \cdot H_D$$

b) **RF to beam power efficiency**

$$P_{beams} = f_{rep} n_b N \cdot E_{cm} = \eta_{RF} \cdot P_{RF}$$

→ Rewrite Luminosity Formula

$$\mathcal{L} = \frac{1}{4\pi E_{cm}} \cdot \left(\eta_{RF} P_{RF}\right) \cdot \left(\frac{N}{\sigma_x \sigma_y} \cdot H_D\right)$$

Beam-beam effects:

- beamstrahlung
- disruption

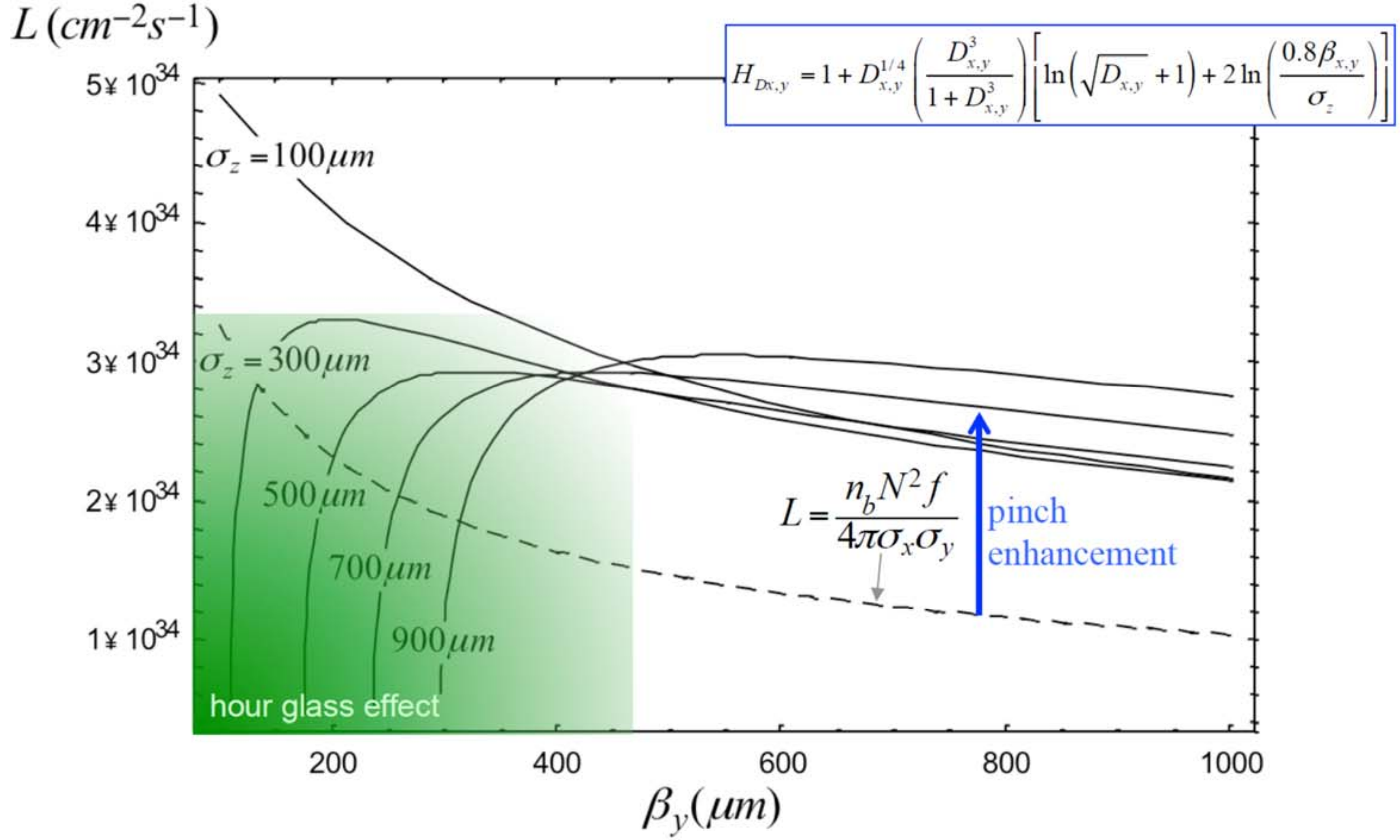
Choice of linac technology:

- efficiency
- available power

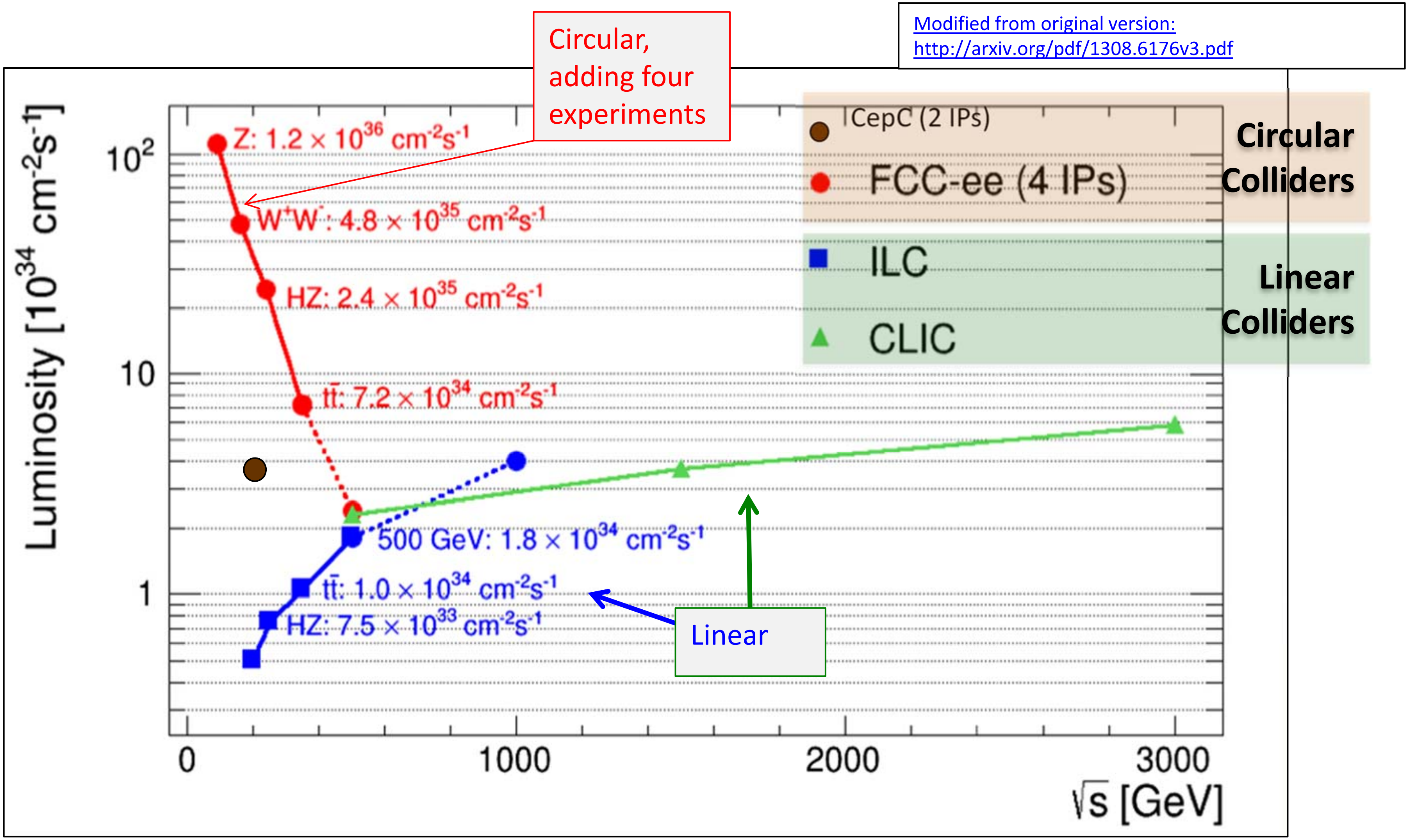
Strong final focus:

- Optical abberations
- Stability issues and tolerances

Luminosity as a function of β_y



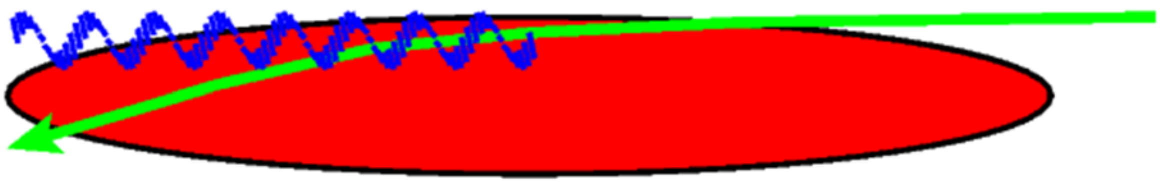
Circular vs. Linear Collider



F. Gianotti

Beamstrahlung

Particles are deflected in magnetic field of colliding bunch:



Peak field: $B_{\max} = \frac{2E_{\perp,\max}}{c} = \frac{eN}{2\pi\varepsilon_0 c\sigma_x\sigma_s}$ = up to 1000 Tesla!

Classical treatment of synchrotron radiation: $\Delta E \sim \frac{\gamma^4}{R^2} \sim \gamma^2 B^2$

- **particles with high energy loss will be lost**
- **short beam life time**

Beamstrahlung

Most important parameter: $\Upsilon = \frac{2}{3}\frac{\hbar\omega_c}{E}$

Some Numbers:

- **average and maximum value:** $\Upsilon_{av} = \frac{5}{6} \cdot \frac{N r_e \lambda_c \gamma_r}{\sigma_s \left(\sigma_x + \sigma_y\right)}, \quad \Upsilon_{max} \approx 2.4 \cdot \Upsilon_{av}$
- **# photons per electron:** $n_\gamma \approx 2.54 \left(\frac{\alpha \sigma_s}{\lambda_c \gamma_r}\right) \frac{\Upsilon_{av}}{\sqrt{1 + \Upsilon_{av}^{2/3}}}$
- **average energy loss:** $\delta_{BS} = \left\langle -\frac{\Delta E}{E} \right\rangle \approx 1.24 \cdot \left(\frac{\alpha \sigma_s}{\lambda_c \gamma_r}\right) \cdot \frac{\Upsilon_{av}^2}{\left[1 + \left(\frac{3}{2}\Upsilon_{av}\right)^{2/3}\right]^2}$

Leads to pair production: $\begin{cases} \Upsilon < 0.6 & \text{incoherent } e^+e^- \text{ pairs} \\ \Upsilon > 0.6 & \text{coherent } e^+e^- \text{ pairs } (E_{cm} > 1 \text{ TeV}) \end{cases}$

Beamstrahlung $\rightarrow \mathcal{L}$

RMS energy loss for weak beamstrahlung:

$$\delta_{BS} \approx 0.86 \frac{e r_e^3}{2 m_0 c^2} \cdot \frac{E_{cm}}{\sigma_s} \cdot \frac{N^2}{(\sigma_x + \sigma_y)^2} \propto \frac{E_{cm}}{\sigma_s} \cdot \frac{N^2}{\sigma_x^2}$$

- **use flat beams ($\sigma_x >> \sigma_y$) but keep $\sigma_x + \sigma_y$ large to reduce δ_{BS}**

a) **Luminosity**

$$\mathcal{L} = \frac{1}{4\pi E_{cm}} \cdot (\eta_{RF} P_{RF}) \cdot \left(\frac{N}{\sigma_x \sigma_y} \cdot H_D \right)$$

b) **Vertical rms beam size**

$$\sigma_y = \sqrt{\frac{\varepsilon_{n,y} \beta_y}{\gamma_r}}$$

→ Again Rewrite Luminosity Formula ($\delta_{BS} \approx$ few %)

$$\mathcal{L} \propto \frac{\eta_{RF} P_{RF}}{4\pi E_{cm}} \cdot \sqrt{\frac{\delta_{BS}}{\varepsilon_{n,y}}} \cdot \underbrace{\sqrt{\frac{\sigma_s}{\beta_y}} \cdot H}_{\text{hourglass: } \beta_y \approx \sigma_s} \cdot H_D \propto \frac{\eta_{RF} P_{RF}}{4\pi E_{cm}} \cdot \sqrt{\frac{\delta_{BS}}{\varepsilon_{n,y}}} \cdot H_D$$

damping rings!

Luminosity: Beamstrahlung Limit

$$\mathcal{L} \propto \frac{\rho P_{SR}}{E^{13/3}} \left(\frac{\xi_y \eta^2}{\varepsilon_{g,y}} \right)^{1/3}$$ **Circular ↔ Linear** $$\mathcal{L} \propto \frac{\eta_{RF} P_{RF}}{4\pi E_{cm}} \sqrt{\frac{\delta_{BS}}{\varepsilon_{n,y}}} H_D$$

P_{SR} : syn.rad.power

ρ : bending radius

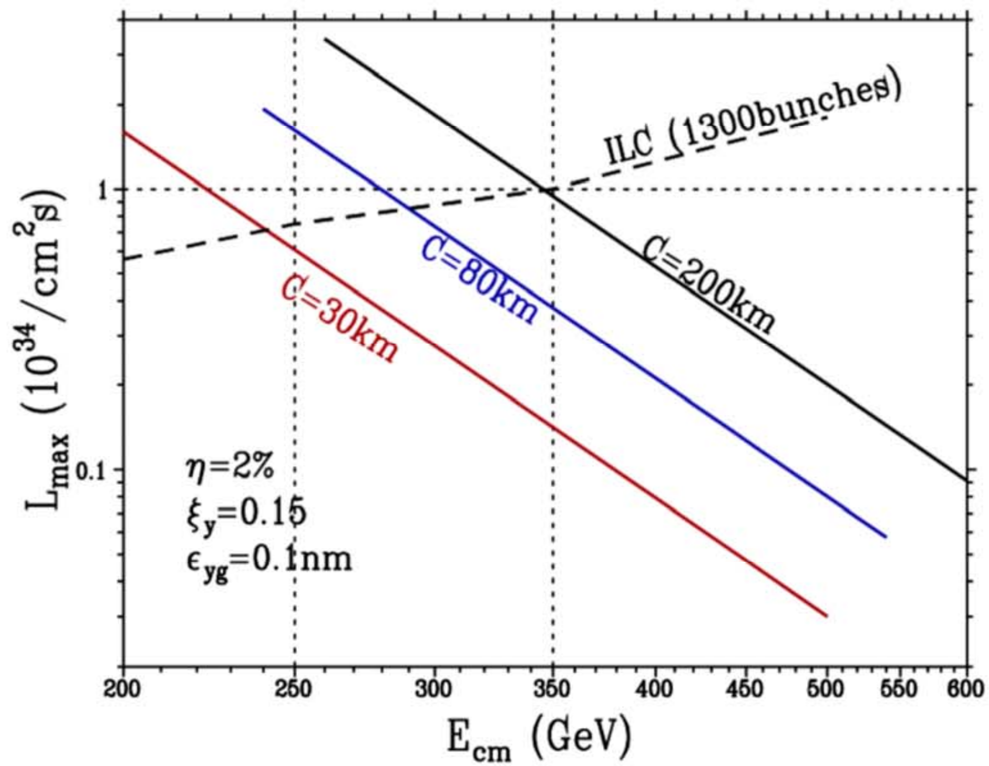
ξ_y : tune-shift

$\varepsilon_{g,y}$: geometric emit.

?????

example with

- η=2%
- ξ_y=0.15
- ε_{gy}=0.1nm



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