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Generalized Calabi-Yau compactifications with D-branes and fluxes

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We review some aspects of generalized Calabi-Yau compactifications with D-branes and Fluxes

1 Introduction

The standard way to connect string theory with particle physics is to choose a string vacuum which contains an $N = 1$ supersymmetric version of the Standard Model or some generalization thereof in four space-time dimensions ($d = 4$). Then one manufactures a mechanism which spontaneously breaks this $N = 1$ supersymmetry. Traditionally one considered the heterotic string propagating in a space-time background

$$\mathcal{M}_4 \times Y_6 \,, \tag{1}$$

where $\mathcal{M}_4$ is the $d = 4$ Minkowski-space and Y_6 is a compact Calabi-Yau manifold which determines the amount of supersymmetry left intact [1–3]. One argues that the supersymmetry is spontaneously broken by (space-time) non-perturbative effects which are, however, not yet under satisfactory theoretical control. Instead one uses field theoretic considerations to determine the structure of possible non-perturbative effects (such as gaugino condensation) and the possibility to break supersymmetry spontaneously [1].

In recent years there has been slight variation on this setup which goes under the name of 'Brane World Scenarios' where the Standard Model or its generalization lives on a stack of space-time filling D-branes in a type II bulk [3–5]. This in turn requires to replace the product space-time (1) by a warped product and the Calabi-Yau manifold by a Calabi-Yau orientifold [6–17]. The $N = 1$ supersymmetry can then be spontaneously broken by additionally turning on background fluxes in the orientifold bulk [9,14–16,18–45].

All perturbative string vacua come with a (large) number of gauge neutral scalar fields whose vacuum expectation value appear as free parameters. This unacceptably large vacuum degeneracy prohibits to extract definite predictions from a given string vacuum since gauge and Yukawa couplings depend on the scalar fields (or rather their vacuum expectation values). The fluxes do fix some of the scalar field but in general additional non-perturbative effects have to be employed in order to fix all of them and construct a (meta-stable) ground state [46–50]. This aspect is particular important if one attempts to construct de-Sitter vacua with a small cosmological constant.

In these scenarios the $N = 1$ supersymmetry is generically broken by F-terms of the moduli scalars and therefore they serve as the 'messenger sector' in that they communicate the breaking of supersymmetry to

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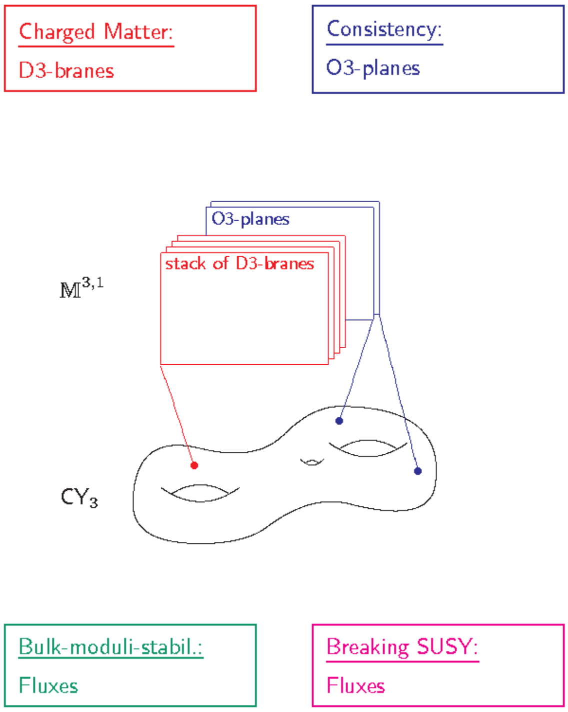

Fig. 1 (online colour at: www.fp-journal.org) Calabi-Yau orientifold with space-time filling D3-branes.

the observable (Standard Model) sector. This spontaneous breaking manifests itself in the observable sector in that a set of soft supersymmetry breaking terms are generated [51–61].

A realistic particle phenomenology needs on the one hand a realistic spectrum of light modes and on the other hand a set of viable soft supersymmetry breaking terms. The first aspects is often called 'model building' and its current status will not be covered in this review.[1] Instead we focus on the computation of the effective action or more specifically on the couplings of the matter fields to the bulk moduli since they communicate the supersymmetry breaking to the observable sector. This is of importance since the resulting soft breaking terms determine masses and couplings as could be measured by LHC in the near future. The effective action also is of crucial importance for cosmological aspects of the Brane World Scenarios – an aspects which is covered in the lecture by R. Kallosh in this meeting and reviewed in [62].

The four-dimensional effective action S_{eff} can be determined in basically two different, complementary ways. On the one hand one can compute string scattering amplitudes and from it reconstruct the effective action [63, 64]. With this methods one can obtain information about S_{eff} in the 'stringy' regime or in other words in the regime where Y_6 is of the order of the string length. However, in order to compute the scattering amplitudes one can only perturb around a string vacuum where the underlying conformal field theory correlation functions are known. The second methods which can be used (and will be discussed in these lectures) employ a Kaluza-Klein reduction of the combined ten-dimensional actions consisting of the type II bulk supergravity action plus the Dirac-Born-Infeld and Chern-Simons action governing the dynamics of the D-branes. This approach is only valid in the large volume limit (also called the supergravity limit) or in other words in a regime where Y_6 is much bigger than the string length.

In this review we discuss the ingredients necessary for the computation of the effective action separately. More specifically the outline is as follows. In Sect. 2 we discuss Calabi-Yau compactification of type II string theories. In Sect. 3 we turn on background fluxes which break supersymmetry spontaneously. In Sect. 4 we make a slight detour and discuss geometrical compactifications which replace Calabi-Yau manifolds by more general manifolds called 'manifolds with $SU(3)$ structure'. In Sect. 5 we discuss $N = 1$ Calabi-Yau

[1] This topic is reviewed for example in refs. [5].

orientifolds. In Sect. 6 we add D3 and D7-branes to orientifold compactifications of type IIB string theory. Finally in Sect. 7 we compute the resulting soft supersymmetry breaking terms. We conclude in Sect. 8.

2 Calabi-Yau compactification of type II string theory

In order to set the stage let us start by recalling Calabi-Yau compactifications of type II string theories. They preserve $N = 2$ supersymmetry in $d = 4$ and thus the low energy effective action is an $N = 2$ supergravity coupled to vector-, hyper- and tensor multiplets. Before we give the effective action in Sect. 2.4 let us first briefly summarize a few properties of Calabi-Yau manifolds (Sects. 2.1 and 2.2) and mirror symmetry (Sect. 2.3) needed in the following.

2.1 Calabi-Yau threefolds

When one considers strings propagating in the space-time background (1) the ten-dimensional Lorentz group $Spin(1,9)$ decomposes into $Spin(1,9) \to Spin(1,3) \times Spin(6)$. There is an associated decomposition of the spinor representation $\mathbf{16} \in Spin(1,9)$ according to $\mathbf{16} \to (\mathbf{2},\mathbf{4}) \oplus (\bar{\mathbf{2}},\bar{\mathbf{4}})$. In order to achieve the minimal amount of supersymmetry one chooses Y_6 to have a reduced structure group $SU(3) \subset Spin(6)$. This implies a further decomposition of the $\mathbf{4} \in Spin(6)$ under the $SU(3)$ as $\mathbf{4} \to \mathbf{3} \oplus \mathbf{1}$. Thus manifolds with a reduced structure group $SU(3)$ admit an invariant spinor η (the singlet $\mathbf{1}$) which is nowhere vanishing and globally well defined. Such manifolds are termed ‘manifolds with $SU(3)$ structure’ in the mathematical literature [65–70]. In Sect. 4 we will learn more about these manifolds but for now we impose the additional constraint that this spinor η is also covariantly constant with respect to the Levi-Civita connection. Geometrically this says that the holonomy of Y_6 is $SU(3)$.

From η one can build a globally defined two-form J and a complex three-form Ω via

$$\eta^\dagger_\pm \gamma^{mn} \eta_\pm = \pm \frac{i}{2}\, J^{mn} \;, \qquad \eta^\dagger_- \gamma^{mnp} \eta_+ = \frac{i}{2}\, \Omega^{mnp} \;, \qquad \eta^\dagger_+ \gamma^{mnp} \eta_- = \frac{i}{2}\, \overline{\Omega}^{mnp} \;, \tag{2}$$

where $\eta_\pm$ denotes the two chiralities of the spinor. They are normalized as ($\eta^\dagger_\pm \eta_\pm = \frac{1}{2}$) and $\gamma^{m_1 \cdots m_p} = \gamma^{[m_1} \gamma^{m_2} \ldots \gamma^{m_p]}$ are anti-symmetrized products of six-dimensional γ-matrices. Using appropriate Fierz identities one shows that with this normalization for the spinors J and Ω are not independent but satisfy

$$J \wedge J \wedge J = \tfrac{3i}{4}\, \Omega \wedge \bar{\Omega} \;, \qquad J \wedge \Omega = 0 \;. \tag{3}$$

Furthermore, lowering one index of the two-form J with the metric results in a complex structure $\mathcal{J}$ in that it satisfies $\mathcal{J}^2 = -\mathbf{1}$ and the associated Nijenhuis-tensor vanishes. For a fixed metric and fixed complex structure J is a closed $(1,1)$-form while Ω is a closed $(3,0)$-form. This says that Y_6 is a Ricci-flat Kähler manifold with holonomy $SU(3)$ or in other words it is a Calabi-Yau manifold which we denote by Y henceforth.

Since type II string theories have two supersymmetries (or 32 supercharges) in $d = 10$ Calabi-Yau compactifications thereof lead to two supersymmetries (or 8 supercharges) in $d = 4$. Compactification in a space-time background (1) where the compact manifold Y_6 is chosen to be a Calabi-Yau manifold Y leads to a simplification for the ten-dimensional equations of motion. For example the ten-dimensional Laplace equation for a scalar field ϕ splits in the background (1) according to

$$\Delta_{10}\, \phi = (\Delta_4 + \Delta_6)\, \phi = (\Delta_4 + m^2)\, \phi \;=\; 0 \;, \tag{4}$$

where the second equation assumed that ϕ is an eigenfunction of the internal, six-dimensional Laplace operator of the Calabi-Yau manifold $\Delta_6 \phi = m^2 \phi$. Eq. (4) implies that the massless modes of the $d = 4$ theory correspond to zero modes of Δ_6. These zero modes are in one-to-one correspondence with the harmonic forms on Y which in turn are in one-to-one correspondence with elements of the Dolbeault cohomology groups $H^{(p,q)}(Y)$. Here (p,q) denotes the number of holomorphic and anti-holomorphic differentials of the harmonic forms. The dimensions of $H^{(p,q)}(Y)$ are called Hodge numbers and denoted as $h^{p,q} = \dim H^{p,q}(Y)$. They are conventionally arranged in a Hodge diamond which on a Calabi-Yau manifold simplifies as follows

$$
\begin{array}{ccccccc}
 & & & h^{(0,0)} & & & \\
 & & h^{(1,0)} & & h^{(0,1)} & & \\
 & h^{(2,0)} & & h^{(1,1)} & & h^{(0,2)} & \\
h^{(3,0)} & & h^{(2,1)} & & h^{(1,2)} & & h^{(0,3)} \\
 & h^{(3,1)} & & h^{(2,2)} & & h^{(1,3)} & \\
 & & h^{(3,2)} & & h^{(2,3)} & & \\
 & & & h^{(3,3)} & & &
\end{array}
\quad = \quad
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 0 & & 0 & & \\
 & 0 & & h^{(1,1)} & & 0 & \\
1 & & h^{(1,2)} & & h^{(1,2)} & & 1 \\
 & 0 & & h^{(1,1)} & & 0 & \\
 & & 0 & & 0 & & \\
 & & & 1 & & &
\end{array}
\; . \tag{5}
$$

Or in other words the $h^{(p,q)}$ satisfy

$$
\begin{aligned}
& h^{(1,0)} = h^{(0,1)} = h^{(2,0)} = h^{(0,2)} = h^{(3,1)} = h^{(1,3)} = h^{(3,2)} = h^{(2,3)} = 0 \,, \\
& h^{(0,0)} = h^{(3,0)} = h^{(0,3)} = h^{(3,3)} = 1 \,, \qquad h^{(2,1)} = h^{(1,2)} \,, \qquad h^{(1,1)} = h^{(2,2)} \,.
\end{aligned} \tag{6}
$$

We see that $h^{(1,1)}$ and $h^{(1,2)}$ are the only non-trivial, i.e. arbitrary Hodge numbers on a Calabi-Yau threefold.

Apart from the zero modes of the scalar fields also all the other zero modes arising from the other massless ten-dimensional fields correspond to harmonic forms on Y [2]. In particular the deformations of the Calabi-Yau metric which do not disturb the Calabi-Yau condition correspond to moduli scalars in the low energy effective action. These scalar fields can be viewed as the coordinates of what is called the geometrical moduli space of the Calabi-Yau manifolds [71, 72]. Let us briefly review some properties of this moduli space.

2.2 The moduli space of Calabi-Yau threefolds

The deformations of the Calabi-Yau metric $g_{i\bar{\jmath}}$, $i, \bar{\jmath} = 1, \ldots, 3$ can be naturally split into deformations of the complex structure δg_{ij} and deformations of the Kähler form $\delta g_{i\bar{\jmath}}$. The latter are in one to one correspondence with the harmonic $(1,1)$-forms and thus can be expanded as

$$
\delta g_{i\bar{\jmath}} = i v^a \, (\omega_a)_{i\bar{\jmath}} \,, \qquad a = 1, \ldots, h^{(1,1)} \,, \tag{7}
$$

where ω_a are harmonic $(1,1)$-forms on Y which form a basis of $H^{(1,1)}(Y)$. The v^a denote $h^{(1,1)}$ moduli which in the effective action appear as scalar fields. Similarly the deformations of the complex structure are parameterized by complex moduli z^k which are in one-to-one correspondence with harmonic $(1,2)$-forms via

$$
\delta g_{ij} = \frac{i}{||\Omega||^2} \, \bar{z}^k (\bar{\chi}_k)_{i\bar{\imath}\bar{\jmath}} \, \Omega^{\bar{\imath}\bar{\jmath}}{}_j \,, \quad k = 1, \ldots, h^{(1,2)} \,, \tag{8}
$$

where Ω is the holomorphic (3,0)-form, $\bar{\chi}_k$ denotes a basis of $H^{(1,2)}$ and we abbreviate $||\Omega||^2 \equiv \frac{1}{3!} \Omega_{ijk} \bar{\Omega}^{ijk}$.

The moduli v^a and z^k can be viewed as the coordinates of a moduli space $\mathcal{M}$ which locally is a direct product

$$
\mathcal{M} = \mathcal{M}_{\rm cs}^{h^{(1,2)}} \times \mathcal{M}_{\rm k}^{h^{(1,1)}} \,. \tag{9}
$$

$\mathcal{M}_{\rm cs}^{h^{(1,2)}}$ is the complex $h^{(1,2)}$-dimensional component spanned by the complex structure deformations z^k while $\mathcal{M}_{\rm k}^{h^{(1,1)}}$ is the real $h^{(1,1)}$-dimensional component spanned by the Kähler deformations v^a. The metric on $\mathcal{M}_{\rm cs}^{h^{(1,2)}}$ is a special Kähler metric with a Kähler potential given by [72]

$$g_{k\bar{l}} = \partial_{z^k}\partial_{\bar{z}^{\bar{l}}}\, K_{\rm cs}\ , \qquad K_{\rm cs} = -\ln\Big[-i\int_Y \Omega\wedge\bar{\Omega}\Big] = -\ln i\Big[\bar{Z}^K\mathcal{F}_K - Z^K\bar{\mathcal{F}}_K\Big]\ . \tag{10}$$

The second form of $K_{\rm cs}$ is obtained from the expansion of Ω

$$\Omega(z) = Z^K(z)\,\alpha_K - \mathcal{F}_L(z)\,\beta^L\ , \tag{11}$$

where (α_K, β^L) is a real, symplectic basis of $H^3(Y)$ satisfying

$$\int_Y \alpha_K\wedge\beta^L = \delta_K^L\ , \qquad \int_Y \alpha_K\wedge\alpha_L = 0 = \int_Y \beta^K\wedge\beta^L\ . \tag{12}$$

Both $Z^K(z)$ and $\mathcal{F}_L(z)$ are holomorphic function of the moduli z and furthermore $\mathcal{F}_L(z) = \partial_{Z^L}\mathcal{F}(Z(z))$ is the derivative of a holomorphic prepotential $\mathcal{F}(Z(z)))$. Manifolds with a Kähler metric whose Kähler potential are in this way entirely determined by a holomorphic prepotential are termed special Kähler manifolds [72–76].

As we will see in Sect. 2.4 the Kähler-form J can be complexified as $J_c = J + iB$ where B the NS two-form of type II string theories. This in turn introduces complex Kähler deformations t^a which arise as the expansion of J_c

$$J_c = J + iB = t^a\omega_a\ , \qquad \omega_a \in H^{(1,1)}(Y)\ . \tag{13}$$

The associated complexified moduli space $\mathcal{M}_{\rm k}^{h^{(1,1)}}$ spanned by the coordinates t^a also is a special Kähler manifold with a Kähler potential and prepotential $F(t)$ given by

$$K_k = -\ln\mathcal{K}\ , \qquad \mathcal{K} = \mathcal{K}_{abc}v^av^bv^c\ , \qquad F(t) = \mathcal{K}_{abc}t^at^bt^c\ , \tag{14}$$

where $\mathcal{K}_{abc} = \int\omega_a\wedge\omega_b\wedge\omega_c$ are topological intersection numbers.

2.3 Mirror symmetry

Mirror symmetry is not yet a symmetry but rather the conjecture about a not yet rigorously defined space of Calabi-Yau threefolds [77]. It has been established on a subspace of Calabi-Yau manifolds [78] and is a very useful concept in order to compute certain couplings in the effective action. It states that for 'every' Calabi-Yau Y there exists a mirror manifold $\tilde{Y}$ with reversed Hodge numbers, i.e.

$$h^{1,1}(Y) = h^{1,2}(\tilde{Y})\ , \qquad h^{1,2}(Y) = h^{1,1}(\tilde{Y})\ . \tag{15}$$

In terms of the Hodge diamond (5) this corresponds to a reflection along the diagonal or in other words the third cohomology $H^{(3)} = H^{(3,0)}\oplus H^{(2,1)}\oplus H^{(1,2)}\oplus H^{(0,3)}$ is interchanged with the even cohomologies $H^{(\rm even)} = H^{(0,0)}\oplus H^{(1,1)}\oplus H^{(1,2)}\oplus H^{(3,3)}$.

Furthermore, the respective (complexified) moduli spaces of (9) are identified under mirror symmetry

$$\mathcal{M}_{\rm cs}^{h^{(1,2)}}(Y) \equiv \mathcal{M}_{\rm k}^{h^{(1,1)}}(\tilde{Y})\ , \qquad \mathcal{M}_{\rm k}^{h^{(1,1)}}(Y) \equiv \mathcal{M}_{\rm cs}^{h^{(1,2)}}(\tilde{Y})\ . \tag{16}$$

This in turn means that the underlying prepotentials are identical

$$\mathcal{F}(Y) \equiv F(\tilde{Y})\ , \qquad F(Y) \equiv \mathcal{F}(\tilde{Y})\ . \tag{17}$$

This fact has been used to compute instanton corrections to the prepotential F of the Kähler moduli (14) which only in the large volume approximation is a cubic polynomial.

In type II string theory mirror symmetry manifests itself by the equivalence of the two different type II string theories, called type IIA and type IIB, in mirror symmetric background or in other words the following equivalence holds

$$\text{IIA in background } \mathcal{M}_4 \times Y \quad \equiv \quad \text{IIB in background } \mathcal{M}_4 \times \tilde{Y}\ . \tag{18}$$

Therefore one can focus the attention on one of the two string theories and infer couplings of the other one by mirror symmetry. However, depending on the precise question it might be easier to ask it either in IIA or IIB.

2.4 Kaluza-Klein reduction of type IIB on Y

Let us now discuss the Kaluza-Klein reduction of type IIB on Y in more detail following [20,31,35,79]. The massless bosonic spectrum of type IIB in $D = 10$ consists of the dilaton $\hat{\phi}$, the metric $\hat{g}$ and a two-form $\hat{B}_2$ in the NS-NS sector and the axion $\hat{l}$, a two-form $\hat{C}_2$ and a four-form $\hat{C}_4$ in the R-R sector.[2] Using the notation of differential forms the type IIB low energy effective action in the $d = 10$ Einstein frame is given by [3]

$$\begin{aligned} S^{(10)}_{IIB} = & -\int \Big(\frac{1}{2}\hat{R} * \mathbf{1} + \frac{1}{4} d\hat{\phi} \wedge * d\hat{\phi} + \frac{1}{4} e^{-\hat{\phi}} \hat{H}_3 \wedge * \hat{H}_3\Big) \\ & -\frac{1}{4}\int \Big(e^{2\hat{\phi}} d\hat{l} \wedge * d\hat{l} + e^{\hat{\phi}} \hat{F}_3 \wedge * \hat{F}_3 + \frac{1}{2}\hat{F}_5 \wedge * \hat{F}_5\Big) - \frac{1}{4}\int \hat{C}_4 \wedge \hat{H}_3 \wedge \hat{F}_3\ , \end{aligned} \tag{19}$$

where $*$ denotes the Hodge-$*$ operator and the field strengths are defined as

$$\begin{aligned} \hat{H}_3 &= d\hat{B}_2\ , \qquad \hat{F}_3 \ = \ d\hat{C}_2 - \hat{l} d\hat{B}_2\ , \\ \hat{F}_5 &= d\hat{C}_4 - \frac{1}{2} d\hat{B}_2 \wedge \hat{C}_2 + \frac{1}{2}\hat{B}_2 \wedge d\hat{C}_2\ . \end{aligned} \tag{20}$$

The five-form field strength $\hat{F}_5$ additionally satisfies the self-duality condition $\hat{F}_5 = *\hat{F}_5$ which is imposed at the level of the equations of motion.

In the background (1) the ten-dimensional background metric is block-diagonal or in other words the line element to takes the form

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + g_{i\bar{\jmath}} dy^i d\bar{y}^{\bar{\jmath}}\ , \tag{21}$$

where $g_{\mu\nu}, \mu, \nu = 0, \ldots, 3$ is a Minkowski metric and $g_{i\bar{\jmath}}$ is the Calabi-Yau metric.

The next step is to expand all ten-dimensional fields into eigenforms forms on Y. However since we are only interested in the light $d = 4$ degrees of freedom we only keep the zero modes in this expansion. For the Calabi-Yau metric this has already been done in eqs. (7) and (8). The type IIB gauge potentials appearing in the Lagrangian (19) are similarly expanded in terms of harmonic forms on Y according to

$$\begin{aligned} \hat{B}_2 &= B_2(x) + b^a(x)\,\omega_a\ , \qquad \hat{C}_2 \ = \ C_2(x) + c^a(x)\,\omega_a\ , \quad a = 1, \ldots, h^{(1,1)}\ , \\ \hat{C}_4 &= D_2^a(x) \wedge \omega_a + V^K(x) \wedge \alpha_K - U_K(x) \wedge \beta^K + \rho_a(x)\,\tilde{\omega}^a\ , \quad K = 0, \ldots, h^{(1,2)}. \end{aligned} \tag{22}$$

As we already indicated the ω_A are harmonic $(1,1)$-forms while the $\tilde{\omega}^a$ are harmonic $(2,2)$-forms which form a basis of $H^{(2,2)}(Y)$ dual to the $(1,1)$-forms ω_a. (α_K, β^L) is the symplectic basis of $H^{(3)}(Y)$ introduced in (12). The four-dimensional fields appearing in the expansion (22) are the scalars $b^a(x)$, $c^a(x)$ and $\rho_a(x)$,

[2] The hats '^' denote ten-dimensional fields.

Table 1 $N = 2$ multiplets for Type IIB supergravity compactified on a Calabi-Yau manifold.

gravity multiplet	1	$(g_{\mu\nu}, V^0)$
vector multiplets	$h^{(2,1)}$	(V^k, z^k)
hypermultiplets	$h^{(1,1)}$	(v^a, b^a, c^a, ρ_a)
double-tensor multiplet	1	(B_2, C_2, ϕ, l)

the one-forms $V^K(x)$ and $U_K(x)$ and the two-forms $B_2(x), C_2(x)$ and $D_2^a(x)$. The self-duality condition of $\hat{F}_5$ eliminates half of the degrees of freedom in $\hat{C}_4$ and one conventionally chooses to eliminate D_2^a and U_K in favor of ρ_a and V^K. Finally, the two type IIB scalars $\hat{\phi}, \hat{l}$ also appear as scalars in $D = 4$ and therefore the hats are dropped henceforth. Altogether these fields assemble into $N = 2$ multiplets which are summarized in Table 1.

The $N = 2$ low energy effective action is computed by inserting (20) – (22) into the action (19) and integrating over the Calabi-Yau manifold. Furthermore, the two antisymmetric tensors B_2 C_2 can be dualized to scalar fields so that the double-tensor multiplet can be treated as an extra hypermultiplet. After an appropriate Weyl rescaling the resulting effective action then is of the standard $N = 2$ form given by [73, 80, 81]

$$S_{IIB}^{(4)} = \int -\frac{1}{2} R * \mathbf{1} + \frac{1}{4} \mathrm{Re}\, \mathcal{M}_{KL} F^K \wedge F^L + \frac{1}{4} \mathrm{Im}\, \mathcal{M}_{KL} F^K \wedge * F^L$$
$$-g_{k\bar{l}}\, \mathrm{d}z^k \wedge * \mathrm{d}\bar{z}^{\bar{l}} - h_{AB}\, \mathrm{d}q^A \wedge * \mathrm{d}q^B \,, \tag{23}$$

where $F^K = dV^K$ and $\mathcal{M}(z)$ are z-dependent gauge couplings functions which can be expressed in terms of the holomorphic prepotential $\mathcal{F}(z)$ (see [81] for an explicit formula.). $g_{k\bar{l}}$ is the special Kähler metric introduced in (10) and the q^A collectively denote all $4(h^{(1,1)} + 1)$ scalars in hypermultiplets . $h_{\hat{A}\hat{B}}$ is the metric on the space $\mathcal{M}_Q$ spanned by the q^A which $N = 2$ constrains to be quaternionic [73, 80, 81]. For Calabi-Yau compactifications of type II theories the quaternionic manifold is of a special form called 'dual quaternionic manifold' [82] which is entirely characterized by the holomorphic prepotential $F(t)$ introduced in (14). The explicit form of $h_{\hat{A}\hat{B}}$ can be found in [79, 83].

Let us summarize. If only vector and hypermultiplets are present the scalar manifold $\mathcal{M}$ of the $N = 2$ theory is the product of a quaternionic manifold $\mathcal{M}_Q$ spanned by the scalars q^A in the hypermultiplets and a special Kähler manifold $\mathcal{M}_{SK}$ spanned by the scalars z^k in the vector multiplets

$$\mathcal{M} = \mathcal{M}_Q^{4(h^{(1,1)}+1)} \times \mathcal{M}_{SK}^{2h^{(1,2)}} \,. \tag{24}$$

For Calabi-Yau compactifications this space has a submanifold which is the product of two special Kähler manifolds and which coincide with the geometrical moduli space (9). In this case the $N = 2$ moduli space $\mathcal{M}$ of (24) is entirely determined by two prepotential $\mathcal{F}(z), F(t)$ both of which are exactly known due to mirror symmetry.

2.5 Kaluza-Klein reduction of type IIA on $\tilde{Y}$

For completeness let us also briefly discuss Calabi-Yau compactifications of type IIA following [35, 84]. The massless bosonic spectrum in $d = 10$ has an identical NS-sector containing the dilaton $\hat{\phi}$, the ten-dimensional metric $\hat{g}$ and the two-form $\hat{B}_2$ while the RR sector contains a one-form $\hat{A}_1$ and three-forms $\hat{C}_3$. The ten-dimensional type IIA supergravity action in the Einstein frame given by [3]

$$S_{IIA}^{(10)} = \int -\tfrac{1}{2} \hat{R} * \mathbf{1} - \tfrac{1}{4} d\hat{\phi} \wedge * d\hat{\phi} - \tfrac{1}{4} e^{-\hat{\phi}} \hat{H}_3 \wedge * \hat{H}_3 - \tfrac{1}{2} e^{\frac{3}{2}\hat{\phi}} \hat{F}_2 \wedge * \hat{F}_2$$

Table 2 $N = 2$ multiplets for Type IIA supergravity compactified on a Calabi-Yau manifold.

gravity multiplet	1	$(g_{\mu\nu}, A^0)$
vector multiplets	$h^{(1,1)}$	(A^a, t^a)
hypermultiplets	$h^{(2,1)}$	$(z^k, \xi^k, \tilde{\xi}_k)$
tensor multiplet	1	$(B_2, \phi, \xi^0, \tilde{\xi}_0)$

$$-\tfrac{1}{2}e^{\frac{1}{2}\hat{\phi}}\hat{F}_4 \wedge *\hat{F}_4 - \tfrac{1}{2}\Big[\hat{B}_2 \wedge d\hat{C}_3 \wedge d\hat{C}_3 \ - (\hat{B}_2)^2 \wedge d\hat{C}_3 \wedge d\hat{A}_1\Big] , \tag{25}$$

where the field strengths are defined as

$$\hat{H}_3 = d\hat{B}_2 \ , \quad \hat{F}_2 = d\hat{A}_1 \ , \quad \hat{F}_4 = d\hat{C}_3 - \hat{A}_1 \wedge \hat{H}_3 \ . \tag{26}$$

Compactifying this theory on the mirror threefold $\tilde{Y}$ one obtains again an $N = 2$ theory in $d = 4$ where the zero modes assemble into massless $N = 2$ multiplets and the effective action is of the form (23). The deformations of the metric are expanded exactly as in eqs. (7) and (8) while the expansion of the ten-dimensional gauge potentials reads

$$\begin{aligned} \hat{A}_1 &= A^0(x) \ , \qquad \hat{B}_2 \ = \ B_2(x) + b^a(x)\,\omega_a \ , \quad a \ = \ 1, \ldots, h^{(1,1)} \ , \\ \hat{C}_3 &= C_3(x) + A^a(x) \wedge \omega_a + \ \xi^K(x)\,\alpha_K - \tilde{\xi}_K(x)\,\beta^K \ , \quad K \ = \ 0, \ldots, h^{(2,1)} \ . \end{aligned} \tag{27}$$

Here $b^a, \xi^K, \tilde{\xi}_K$ are four-dimensional scalars, A^0, A^A are one-forms and B_2 is a two-form. Together these fields assemble into $N = 2$ multiplets as summarized in Table 2. We see that the role of the Kähler deformations t^a and the complex structure deformation z^k is exactly reversed in that t^a is now a member of vector multiplets while the z^k reside in hypermultiplets.

The effective action is again of the form (23) with the only difference that the role of the two prepotentials is reversed. $\mathcal{F}(z)$ characterizes the quaternionic manifold $\mathcal{M}_Q$ while $F(t)$ is the prepotential of the special Kähler manifold $\mathcal{M}_{SK}$. As a consequence the two effective actions coincide for a mirror pair of compactification manifold $Y, \tilde{Y}$ in agreement with (18).

3 Background fluxes

3.1 General discussion

If localized sources such as D-branes are present it is possible to turn on background fluxes on the Calabi-Yau manifold [9, 14–16, 18–45]. We postpone the discussion of D-branes until Sect. 6 and in this section consider the effect of background fluxes separately.

Generically background fluxes arise from integrating a p-form field strength F_p over a p-cycle γ_p^I in Y

$$\int_{\gamma_p^I \in Y} F_p = e_I \neq 0 \ . \tag{28}$$

In order to keep the Bianchi identity and the equation of motion intact one insists that $dF_p = 0 = d^\dagger F_p$ holds. This implies that the background fluxes e_I have to be constant. Equivalently one can expand F_p in terms of harmonic forms ω_p^I with constant coefficients e_I

$$F_p \ = \ e_I\,\omega_p^I \ , \qquad \omega_p \in H^p(Y) \tag{29}$$

such that the ω_p are Poincare dual to the cycle γ_p^I.

Due to a Dirac quantization condition the e_I are quantized in string theory. However in the low energy/large volume approximation we are considering here they appear as continuous parameters which deform the low energy supergravity. If one keeps the e_I as small perturbations the light spectrum does not change. Instead the low energy supergravity turns into a gauged or massive supergravity where the fluxes e_I appear as additional gauge couplings or as mass parameters. As a consequence a potential is generated which at least partially lifts the vacuum degeneracy of string theory. Furthermore at the minimum of this potential supersymmetry is generically spontaneously broken.

3.2 Fluxes in IIB

Let us be slightly more specific and consider background fluxes in IIB compactifications. In this case one can turn on three-form flux for $G_3 \equiv F_3 - \tau H_3$ where $\tau = l + i\mathrm{e}^{-\phi}$. Expanded into the symplectic basis one has

$$G_3 \;=\; m^I(\tau)\,\alpha_I + e_I(\tau)\,\beta^I \;, \tag{30}$$

where

$$e_I(\tau) \;=\; e_I^{\mathrm{RR}} - \tau\, e_I^{\mathrm{NS}} \;, \qquad m^I(\tau) \;=\; m^{\mathrm{RR}\,I} - \tau\, m^{\mathrm{NS}\,I} \;. \tag{31}$$

Altogether these are $2(h^{(1,2)}+1)$ RR-flux parameters and $2(h^{(1,2)}+1)$ NS-flux parameters.

The electric fluxes gauge a translational isometry of the quaternionic manifold $\mathcal{M}_Q$ in that the ordinary derivatives are replaced by covariant derivatives

$$\partial_\mu q^{1,2} \to D_\mu q^{1,2} = \partial_\mu q^{1,2} + e_I^{\mathrm{NS,RR}} A_\mu^I \;. \tag{32}$$

Here $q^{1,2}$ denote the dual scalars of the two space-time two-forms B_2 and C_2 which indeed are scalar fields in the hypermultiplets. For the magnetic fluxes the situation is slightly more involved in that B_2 and C_2 become massive with m^I being related to the mass parameters [35,85,86]. In both cases the induced scalar potential reads [24]

$$V(z,\tau) = -(\bar{e} - \bar{\mathcal{M}}\cdot\bar{m})_K (\mathrm{Im}\mathcal{M})^{-1KL}(e - \mathcal{M}\cdot m)_L \;, \tag{33}$$

where $\mathcal{M}(z)$ is the matrix of gauge couplings appearing in (23).

3.3 Fluxes in IIA and mirror symmetry

In type IIA compactified on the mirror Calabi-Yau $\tilde{Y}$ one can turn on the RR-fluxes [35,45]

$$F_2 \;=\; -\tilde{m}^{\mathrm{RR}\,a}\omega_a \;, \qquad F_4 \;=\; \tilde{e}_a^{\mathrm{RR}}\,\tilde{\omega}^a \;, \tag{34}$$

and the NS-fluxes

$$H_3 \;=\; \tilde{m}^{\mathrm{NS}\,I}\alpha_I - \tilde{e}_I^{\mathrm{NS}}\,\beta^I \;. \tag{35}$$

Two additional RR-flux arises from the dual of the four-dimensional space-time three-form C_3 in (27) and from the mass parameter of the ten-dimensional massive IIA supergravity [87]. Thus altogether in type IIA we have $2(h^{(1,1)}+1)$ RR-fluxes and $2(h^{(1,2)}+1)$ NS-fluxes.

An interesting question is the fate of mirror symmetry in the presence of fluxes. Just by counting the flux-parameters we immediately see that in the RR-sector the numbers perfectly match. In this case one also finds perfect agreement of the corresponding effective actions if one identifies the fluxes. Or in other words one finds [35,45]

$$\mathcal{L}^{\mathrm{IIB}}(Y,e,m) \;\equiv\; \mathcal{L}^{\mathrm{IIA}}(\tilde{Y},\tilde{e},\tilde{m}) \;, \qquad e = \tilde{e}, \quad m = \tilde{m} \;. \tag{36}$$

However, for NS-fluxes the situation is considerably more complicated. In this case there is no obvious mirror symmetry since in both theories the three-form H_3 is expanded in terms of $H^{(3)}$ and thus $2(h^{(1,1)}+1)$ flux parameters are missing on both sides. Since we are in the NS-sector these missing fluxes can only come from the internal metric or in other words must arise from a geometrical concept. Technically one needs a NS two-form and a NS four-form which complexify the RR-fluxes (34) which then could map to the complex type IIB three-form G_3 (30) under mirror symmetry. It has been suggested in [88] to compactify on a 'non-Calabi-Yau' manifold $\widehat{Y}$ where an NS-four-form arises from the non-integrability of the complex structure. This proposal was made more concrete in ref. [89] where $\widehat{Y}$ was identified as a 'half-flat manifold' considered before in the mathematical literature [68,90]. Therefore we pause and discuss such manifold in the next section before we turn to orientifold compactifications in Sect. 5.

4 Manifolds of $SU(3)$-structure

4.1 Generalities

In the study of space-time backgrounds of the form (1) one needs to distinguish two conditions. First of all for phenomenological reasons one is interested to choose Y_6 in such a way that the effective four-dimensional theory has the minimal amount of supersymmetry. Therefore, as reviewed in Sect. 2.1, one needs to demand that Y_6 admits a globally defined spinor or equivalently one needs to choose Y_6 to be a manifold with $SU(3)$ structure. If one further insists that this supersymmetry is unbroken an additional condition has to be imposed. Since all spinorial quantities vanish in a ground state which preserves four-dimensional Poincaré invariance, one has to examine the supersymmetry transformation of the spinors (which are bosonic quantities) and in particular the supersymmetry transformation of the gravitino Ψ_M. It reads

$$\delta\Psi_M = \nabla_M \eta + \sum_p (\gamma \cdot F_p)_M \eta + \dots \ , \tag{37}$$

where η is the parameter of the supersymmetry transformations and we have written the contribution of all p-form field strength appropriately contracted with (anti-symmetrized) products of γ-matrices symbolically as $(\gamma \cdot F_p)_M$. For the argument here the precise form of these terms is irrelevant but they can be found for example in [91]. What we see immediately from (37) is that if all background fluxes vanish unbroken supersymmetry requires the existence of a covariantly constant spinor η or in other words demands that Y_6 is a Calabi-Yau manifold. If on the other hand the background fluxes are non-zero one has two choices. Either one still insists on keeping some fraction of the supercharges unbroken. This requires $\nabla_M \eta \neq 0$ or in other words the geometry back-reacts to the presence of the fluxes. If one does not require the existence of an unbroken supercharges the fluxes and/or $\nabla_M \eta$ can break supersymmetry spontaneously. In this section we do not specify exactly which case occurs but consider the generic situation that η exists but is not covariantly constant $\nabla_M \eta \neq 0$. This has been studied recently from different points of view for example in refs. [89,92–106].

In general manifolds which admit a G-invariant tensor or spinor are called 'manifolds with G-structure' in the mathematical literature [65–70]. Even though generically $\nabla_M \eta$ does not vanish with respect to the Levi-Civita connection on such manifolds one can show that there always is a different connection with torsion which satisfies $\nabla^{(T)}\eta = 0$. Here we only focus on manifolds with $SU(3)$ structure for which J and Ω defined in eqs. (2) and (3) always exist due to the existence of an invariant η. However, in general $\mathcal{J}$ is merely an almost complex structure in that it still satisfies $\mathcal{J}^2 = -\mathbf{1}$ but the associated Nijenhuis-tensor no longer vanishes. Similarly both J and Ω are no longer closed but instead they obey [68]

$$\begin{aligned} \mathrm{d}J &= \tfrac{3i}{4}\left(W_1\bar{\Omega} - \bar{W}_1\Omega\right) + W_4 \wedge J + W_3 \ , \\ d\Omega &= W_1 J^2 + W_2 \wedge J + \bar{W}_5 \wedge \Omega \ , \end{aligned} \tag{38}$$

and

$$W_3 \wedge J \;=\; W_3 \wedge \Omega \;=\; W_2 \wedge J^2 \;=\; 0\,, \tag{39}$$

where the W_α are five different torsion classes which can be characterized by their $SU(3)$ representation. W_1 is a zero-form, W_4, W_5 are one-forms, W_2 is a two-form and W_3 is a three-form. Generically manifolds with $SU(3)$ structure are neither complex, nor Kähler, nor Ricci-flat. Only for a particular choice of the torsion such that some of the W_α vanish one has manifolds with additional properties. For example Calabi-Yau manifolds are manifolds of $SU(3)$ structure where all five torsion classes vanish $W_\alpha = 0$.

4.2 Compactifications on manifolds with $SU(3)$ structure

One can perform a Kaluza-Klein reduction in a space-time background where Y_6 is not a Calabi-Yau manifold but instead a manifold with torsion following [89, 98, 107]. The subtlety is that in order to make sense of this reduction one has to treat the torsion as a perturbation of a Calabi-Yau manifold. This is necessary in order to ensure that the light spectrum does not change and exactly as in the case of background fluxes only a potential is generated. Locally this can always be done and since we are working in a supergravity approximation this is valid within this approximation. So the picture is as follows. One starts from a Calabi-Yau manifold and 'deforms' it by turning on torsion keeping the light spectrum of modes unchanged. Technically this means that one chooses exactly the same basis of forms as for the Calabi-Yau manifold to Kaluza-Klein expand the ten-dimensional fields. The difference is that not all of them are harmonic anymore due to (38).

The manifolds for which this program has been carried out in [89, 107, 123] are not the most general $SU(3)$ manifolds but they satisfy the additional property

$$\mathrm{d}\,(J \wedge J) \;=\; 0 \quad \Leftrightarrow \quad W_4 = W_5 = 0\,. \tag{40}$$

The manifolds which are mirror symmetric to Calabi-Yau compactifications of type II theories with background fluxes are further constrained to obey [89]

$$\mathrm{d}\,\mathrm{Im}\,\Omega \;=\; 0 \quad \Leftrightarrow \quad \mathrm{Im}\,(W_1 \oplus W_2) = 0\,, \tag{41}$$

and they are called 'half-flat' manifolds in the mathematical literature [68]. In this case the 'missing' NS 4-form is $F_4^{\mathrm{NS}} \sim \mathrm{d}\,\mathrm{Re}\,\Omega$ which when expanded in a basis of $(2,2)$ forms provides for the mirror of the electric fluxes. However, the mirror of the magnetic fluxes have not been properly identified so far. One proposal is that one needs to further enlarge the concept of compactifications and also allow for the possibility of non-commutative or other structures [108–110].

5 $N = 1$ Calabi-Yau orientifolds in IIB

Let us now turn turn to the discussion of Calabi-Yau orientifold compactifications [7, 9–12, 15, 16, 111]. Their relevance arises from the fact that they have negative tension and are often necessary ingredients to ensure the consistency of a compactifications. In order to cancel gravitational and electro-magnetic tadpoles on a compact manifold in the presence of D-branes and/or background fluxes, objects with negative tension have to be included [9]. As we have reviewed in Sect. 2 starting from type II string theory in $D = 10$ one obtains an $N = 2, D = 4$ theory by compactifying on a Calabi-Yau threefold Y. This $N = 2$ is further broken to $N = 1$ by introducing BPS D-branes and/or orientifold planes (but no fluxes).

In this section we review the modification which occur for Calabi-Yau orientifold compactification of type IIB string theory and summarize the resulting $N = 1, d = 4$ low energy effective action in the presence of background fluxes following [15]. For the effective action of type IIA and a discussion of mirror symmetry in orientifold compactifications we refer the reader to [16].

In orientifolds of string theory one mods out by orientation reversal of the string world-sheet Ω_p together with an 'internal' symmetry σ which acts solely on Y but leaves the $d = 4$ Minkowskian space-time untouched. Maintaining $N = 1$ supersymmetry requires σ to be an isometric and holomorphic involution of Y [10,12]. The Calabi-Yau Kähler form J is left invariant by the action of σ but it can act non-trivially on the holomorphic three-form Ω. Depending on the transformation of Ω two different symmetry operations $\mathcal{O}$ are possible [10,12]. One can have either

$$\mathcal{O}_{(1)} = (-1)^{F_L}\Omega_p\,\sigma^* \;, \qquad \sigma^*\Omega = -\Omega \;, \qquad \Rightarrow \quad O3/O7 - \text{planes} \;, \tag{42}$$

or

$$\mathcal{O}_{(2)} = \Omega_p\,\sigma^* \;, \qquad \sigma^*\Omega = \Omega \;, \qquad \Rightarrow \quad O5/O9 - \text{planes} \;. \tag{43}$$

Ω_p is the world-sheet parity, F_L is the space-time fermion number in the left-moving sector and σ^* denotes the action of σ on forms (the pull-back of σ). The first case leads to the possibility of $O3$- or $O7$-planes while the second case allows $O5$- or $O9$-planes. In order to compute the spectrum and the effective action one does not have to specify a particular Calabi-Yau manifold but instead can focus on the entire class of threefolds which admit an isometric and holomorphic involution obeying (42) or (43). Exactly as in Sect. 2 spectrum and effective action can be expressed in terms of geometrical and topological quantities of the orientifold.

5.1 The spectrum

The orientifold projections (42), (43) truncate the $N = 2$ spectrum of Sect. 2 and reassemble the surviving fields in $N = 1$ multiplets [12,15].[3] In the four-dimensional compactified theory only states invariant under the projection are kept. Ω_p is the world sheet parity transformation under which the type IIB fields $\hat\phi, \hat g$ and $\hat C_2$ are even while $\hat B_2, \hat l, \hat C_4$ are odd. F_L is the 'space-time fermion number' in the left moving sector and therefore $(-1)^{F_L}$ leaves the NS-NS fields $\hat\phi, \hat g, \hat B_2$ invariant but changes the sign of the RR fields $\hat l, \hat C_2, \hat C_4$. For O3/O7-planes (42) implies that the invariant states have to obey

$$\begin{array}{rclcrcl} \sigma^*\hat\phi &=& \hat\phi \;, & \qquad & \sigma^*\hat l &=& \hat l \;, \\ \sigma^*\hat g &=& \hat g \;, & & \sigma^*\hat C_2 &=& -\hat C_2 \;, \\ \sigma^*\hat B_2 &=& -\hat B_2 \;, & & \sigma^*\hat C_4 &=& \hat C_4 \;. \end{array} \tag{44}$$

In addition, σ^* is not arbitrary but required to satisfy

$$\sigma^*\Omega = -\Omega \;. \tag{45}$$

Since σ is a holomorphic involution the cohomology groups $H^{(p,q)}$ (and thus the harmonic (p,q)-forms) split into two eigenspaces under the action of σ^*

$$H^{(p,q)} = H^{(p,q)}_+ \oplus H^{(p,q)}_- \;. \tag{46}$$

$H^{(p,q)}_+$ has dimension $h^{(p,q)}_+$ and denotes the even eigenspace of σ^* while $H^{(p,q)}_-$ has dimension $h^{(p,q)}_-$ and denotes the odd eigenspace of σ^*. The Hodge $*$-operator commutes with σ^* since σ preserves the orientation and the metric of the Calabi-Yau manifold and thus the Hodge numbers obey $h^{(1,1)}_\pm = h^{(2,2)}_\pm$. Holomorphicity of σ further implies $h^{(2,1)}_\pm = h^{(1,2)}_\pm$ while the property (45) leads to $h^{(3,0)}_+ = h^{(0,3)}_+ = 0$, $h^{(3,0)}_- = h^{(0,3)}_- = 1$. Furthermore, the volume-form which is proportional to $\Omega \wedge \bar\Omega$ is invariant under σ^* and thus one has $h^{(0,0)}_+ = h^{(3,3)}_+ = 1, h^{(0,0)}_- = h^{(3,3)}_- = 0$.

[3] Truncating an $N = 2$ spectrum consistently to $N = 1$ can also be discussed entirely from a supergravity point of view without ever making reference to a Calabi-Yau manifold or its orientifold [112,113].

The four-dimensional invariant spectrum is found by using the Kaluza-Klein expansion given in eqs. (7), (8) and (22) keeping only the fields which in addition obey (44) and (45). We see immediately that both $d = 4$ scalar fields arising from $\hat{\phi}$ and $\hat{l}$ remain in the spectrum and as before we denote them by ϕ and l. Since σ is a holomorphic isometry it leaves both the metric and the complex structure and thus also the Kähler form J invariant. As a consequence only $h_+^{(1,1)}$ Kähler deformations v^α remain in the spectrum arising from

$$J = v^{a_+}(x)\,\omega_{a_+}\ , \qquad a_+ = 1,\dots,h_+^{(1,1)}\ , \tag{47}$$

where ω_{a_+} denotes a basis of $H_+^{(1,1)}$. Similarly, from eq. (8) one sees that the invariance of the metric together with (45) implies that the complex structure deformations kept in the spectrum correspond to elements in $H_-^{(1,2)}$ and (8) is replaced by

$$\delta g_{ij} = \frac{i}{||\Omega||^2}\,\bar{z}^{k_-}(\bar{\chi}_{k_-})_{i\bar{\imath}\bar{\jmath}}\,\Omega^{\bar{\imath}\bar{\jmath}}{}_j\ , \qquad k_- = 1,\dots,h_-^{(1,2)}\ , \tag{48}$$

where $\bar{\chi}_{k_-}$ denotes a basis of $H_-^{(1,2)}$.

From eqs. (44) one learns that in the expansion of $\hat{B}_2, \hat{C}_2$ only odd elements survive while for $\hat{C}_4$ only even elements are kept. Therefore the expansion (22) is replaced by

$$\begin{aligned} \hat{B}_2 &= b^{a_-}(x)\,\omega_{a_-}\ , \qquad \hat{C}_2 \ = \ c^{a_-}(x)\,\omega_{a_-}\ , \quad a_- = 1,\dots,h_-^{(1,1)}\ , \\ \hat{C}_4 &= D_2^{a_+}(x)\wedge\omega_{a_+} + V^{K_+}(x)\wedge\alpha_{K_+} + U_{K_+}(x)\wedge\beta^{K_+} + \rho_{a_+}(x)\tilde{\omega}^{a_+},\ K_+ = 1,\dots,h_+^{(1,2)}\ , \end{aligned} \tag{49}$$

where ω_{a_-} is a basis of $H_-^{(1,1)}$, $\tilde{\omega}^{a_+}$ is a basis of $H_+^{(2,2)}$ which is dual to ω_{a_+}, and $(\alpha_{K_+}, \beta^{K_+})$ is a real, symplectic basis of $H_+^{(3)} = H_+^{(1,2)} \oplus H_+^{(2,1)}$. As for Calabi-Yau compactifications imposing the self-duality on $\hat{F}_5$ eliminates half of the degrees of freedom in the expansion of $\hat{C}_4$. For the one-forms V^{K_+}, U_{K_+} this corresponds to the choice of electric versus magnetic gauge potentials. On the other hand choosing the two forms $D_2^{a_+}$ or the scalars ρ_{a_+} determines the structure of the $N = 1$ multiplets to be either a linear or a chiral multiplet. Note that the two $D = 4$ two-form B_2 and C_2 present in the $N = 2$ compactification (see (22)) have been projected out and in the expansion of $\hat{B}_2$ and $\hat{C}_2$ only scalar fields appear.

Altogether the fields assembles into $N = 1$ multiplets summarized in Table 3 [12]. As we already mentioned we can replace $h_+^{(1,1)}$ of the chiral multiplets by linear multiplets.

Compared to the $N = 2$ spectrum of the Calabi-Yau compactification given in Table 1 we see that the graviphoton 'left' the gravitational multiplet while the $h^{(2,1)}$ $N = 2$ vector multiplets decomposed into $h_+^{(2,1)}$ $N = 1$ vector multiplets plus $h_-^{(2,1)}$ chiral multiplets. Furthermore, the $h^{(1,1)} + 1$ hypermultiplets lost half of their physical degrees of freedom and are reduced into $h^{(1,1)} + 1$ chiral multiplets. This is consistent

Table 3 $N = 1$ spectrum of $O3/O7$-orientifold compactification.

gravity multiplet	1	$g_{\mu\nu}$
vector multiplets	$h_+^{(2,1)}$	V^{K_+}
chiral multiplets	$h_-^{(2,1)}$	z^{k_-}
	1	(ϕ, l)
	$h_-^{(1,1)}$	(b^{a_-}, c^{a_-})
chiral/linear multiplets	$h_+^{(1,1)}$	(v^{a_+}, ρ_{a_+})

Table 4 $N = 1$ spectrum of $O5/O9$-orientifold compactification.

gravity multiplet	1	$g_{\mu\nu}$
vector multiplets	$h_{-}^{(2,1)}$	$V^{K_{-}}$
chiral multiplets	$h_{+}^{(2,1)}$	$z^{k_{+}}$
	$h_{+}^{(1,1)}$	$(v^{a_{+}}, c^{a_{+}})$
chiral/linear multiplets	$h_{-}^{(1,1)}$	$(b^{a_{-}}, \rho_{a_{-}})$
	1	(ϕ, C_2)

with the theorem of [112,114] where it was shown that any Kähler submanifold of a quaternionic manifold can have at most half of its (real) dimension.

The exact same analysis can be carried out for the second projection (43). The details can be found in [15] and here we only summarize the results in Table 4.

Compared to the spectrum of the first projection given in Table 3 we see that the vectors and complex structure deformations have switched their role with respect to the decomposition in $H^{(3)}$. Furthermore, different real fields combine into the complex scalars of the chiral/linear multiplets or in other words the complex structure on the moduli space has changed. Now (v, c) and (b, ρ) combine into chiral multiplets whereas before (v, ρ) and (b, c) formed the chiral multiplets. Note that the complex structure which combines (v, b) and which is natural from the $N = 2$ point of view does not appear.

5.2 $d = 4, N = 1$ effective Lagrangian

The low energy effective action for the orientifold compactifications can be obtained from the $N = 2$ action (23) by imposing the truncation discussed in the previous section. The resulting $N = 1$ action can then be displayed in the standard supergravity form where it is expressed in terms of a Kähler potential K, a holomorphic superpotential W and the holomorphic gauge-kinetic coupling functions f as follows [115]

$$S^{(4)} = -\int \tfrac{1}{2}R{*}\mathbf{1} + K_{A\bar{B}}DM^A \wedge {*}D\bar{M}^{\bar{B}} + \tfrac{1}{2}\mathrm{Re}f_{KL}\, F^K \wedge {*}F^L + \tfrac{1}{2}\mathrm{Im}f_{KL}\, F^K \wedge F^L + V\, . \quad (50)$$

Here $F^K = dV^K$ and the M^A collectively denote all complex scalars in the theory and $K_{A\bar{B}}$ is a Kähler metric satisfying $K_{A\bar{B}} = \partial_A \bar{\partial}_{\bar{B}} K(M, \bar{M})$. The scalar potential is expressed in terms of the Kähler-covariant derivative $D_A W = \partial_A W + (\partial_A K)W$ and includes appropriate D-terms

$$V = e^K \big(K^{I\bar{J}} D_I W D_{\bar{J}} \bar{W} - 3|W|^2\big) + \tfrac{1}{2}\, (\mathrm{Re}\ f)^{-1\ KL} D_K D_L\, . \quad (51)$$

Exactly as in $N = 2$ the variables which appear naturally in the Kaluza-Klein reduction are not necessarily the right variables to put the effective action into the form (50). Instead one again has to find the correct complex structure on the space of scalar fields.

The complex structure deformations z are good Kähler coordinates since they are the coordinates of a special Kähler manifold already in $N = 2$. For the remaining fields the definition of the Kähler coordinates is not so obvious. For O3/O7-planes one finds [15,37,116]

$$\tau = l + ie^{-\phi}\, , \qquad G^{a_-} = c^{a_-} - \tau b^{a_-}\, , \quad (52)$$
$$T_{a_+} = \frac{3i}{2}\rho_{a_+} + \frac{3}{4}\mathcal{K}_{a_+b_+c_+}v^{b_+}v^{c_+} - \frac{3i}{4(\tau - \bar{\tau})}\mathcal{K}_{a_+b_-c_-}G^{b_-}(G - \bar{G})^{c_-}\, ,$$

where

$$\mathcal{K}_{a_+b_+c_+} = \int \omega_{a_+} \wedge \omega_{b_+} \wedge \omega_{c_+} \ , \qquad \mathcal{K}_{a_+b_-c_-} = \int \omega_{a_+} \wedge \omega_{b_-} \wedge \omega_{c_-} \ . \tag{53}$$

In these variables the Kähler potential reads

$$\begin{aligned} K &= K_{\rm cs}(z,\bar z) + K_k(\tau, T, G) \ , \\ K_{\rm cs} &= -\ln\Big[- i \int \Omega(z) \wedge \bar\Omega(\bar z)\Big] \ , \\ K_k &= -\ln\big[- i(\tau - \bar\tau)\big] - 2\ln\Big[\tfrac{1}{6}\mathcal{K}(\tau, T, G)\Big] \ , \end{aligned} \tag{54}$$

where

$$\mathrm{Vol}(Y) = \tfrac{1}{6}\mathcal{K} = \tfrac{1}{6}\mathcal{K}_{a_+b_+c_+} v^{a_+} v^{b_+} v^{c_+} \ . \tag{55}$$

$\mathcal{K}$ should be understood as a function of the Kähler coordinates (τ, T, G) which enter by solving (52) for v^{a_+} in terms of (τ, T, G). Unfortunately this solution cannot be given explicitly and therefore $\mathcal{K}$ is known only implicitly via $v^{a_+}(\tau, T, G)$.

However, in the dual picture, where instead of the scalars ρ_{a_+} in the expansion (49) one keeps the two-forms $D_2^{a_+}$, the Kähler deformations v^{a_+} are the lowest components of linear multiplets containing as bosonic components $(v^{a_+}, D_2^{a_+})$. In this case one can give explicitly the metric for the linear multiplets and the somewhat involved definition of T_{a_+} in (52) can be understood as the superspace relation which expresses the dualization between chiral and linear multiplets [15, 117, 118].

The Kähler potential (54) is again block diagonal in that complex structure deformations z do not mix with the other scalars. Thus, the moduli space has the form

$$\mathcal{M} = \mathcal{M}_{\rm cs}^{h_-^{(1,2)}} \times \mathcal{M}_{\rm k}^{h^{(1,1)}+1} \ , \tag{56}$$

where each factor is a Kähler manifold and $\mathcal{M}_{\rm cs}^{h_-^{(1,2)}}$ even is a special Kähler manifold.

Although not immediately obvious from its definition K_k obeys a no-scale-type condition in that it satisfies [119]

$$\frac{\partial K_k}{\partial M^A}\,(K_k^{-1})^{A\bar B}\,\frac{\partial K_k}{\partial \bar M^{\bar B}} = 4 \ . \tag{57}$$

This relation is responsible for a positive semi-definite scalar potential.

Without background fluxes there is no superpotential or any D-term induced. However, including background fluxes for G_3 exactly as in Calabi-Yau compactifications one has

$$G_3 = m^{K_-}\alpha_{K_-} - e_{K_-}\beta^{K_-} \ , \qquad K_- = 0, \ldots, h_-^{(1,2)} \ , \tag{58}$$

with $2(h_-^{(1,2)} + 1)$ complex flux parameters

$$m^{K_-} = m^{\mathrm{RR}\,K_-} - \tau\, m^{\mathrm{NS}\,K_-} \ , \qquad e_{K_-} = e^{\mathrm{RR}}_{K_-} - \tau\, e^{\mathrm{NS}}_{K_-} \ . \tag{59}$$

Including these fluxes into the Kaluza-Klein reduction induces a superpotential (but no D-term) given by [9, 15, 21, 23, 24, 37, 38]

$$W = \int_Y \Omega \wedge G_3 \ . \tag{60}$$

The order parameters for supersymmetry breaking are the F-terms $F_A = D_A W$ and possible D-terms. For the superpotential (60) one finds unbroken supersymmetry i.e. $F_A = 0$ for $G_3 \in H_-^{(2,1)}$. For $G_3 \in H_-^{(0,3)}$ one finds a broken supersymmetry $F_{T_{a_+}} \neq 0$ in Minkowski space, i.e. with $V = 0$. On the hand for $G_3 \in H_-^{(3,0)} \oplus H_-^{(1,2)}$ one obtains only unstable vacuum solutions.

6 Space-time filling D3- and D7-branes in type IIB orientifolds

The next step is to include space-time filling D-branes into the compactification [41,51–54,56–61]. Here we confine our attention to D3- and D7-branes in type IIB orientifolds and review the effective action following [54,58]. Let us start with D3-branes.

6.1 D3-branes

A space-time filling D3-brane is a point in the Calabi-Yau orientifolds and on its four-dimensional world volume lives a $U(1)$ gauge theory together with three gauge neutral chiral multiplets $\zeta^i, i = 1, 2, 3$. They can be viewed as the positions of the D3-branes inside the Calabi-Yau orientifold (see fig. 2). A non-Abelian generalization is constructed from a stack of N D3-branes which gives rise to a $U(N)$ gauge theory and a set of chiral matter fields ζ^i, $(i = 1, 2, 3)$ in the adjoint representation of $U(N)$ [120].

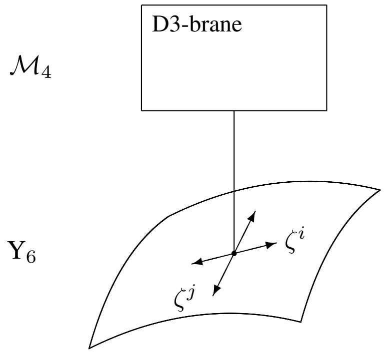

Fig. 2 Space-time filling D3-branes.

The dynamics of a D-branes is governed by the Dirac-Born-Infeld action S_{DBI} together with a Chern Simons action S_{CS}. For a generic Dp-brane they are given by [3]

$$
\begin{aligned}
S^{\mathrm{sf}}_{\mathrm{DBI}} &= -T_p \int_{\mathcal{W}} d^{p+1}\xi\, e^{-\hat{\phi}} \sqrt{-\det\left[\varphi^*(g+B) + 2\pi\alpha' F\right]}\ , \\
S_{\mathrm{CS}} &= \mu_p \int_{\mathcal{W}} \varphi^*\Big(\sum_q C^{(q)} e^B\Big)\, e^{2\pi\alpha' F}\ ,
\end{aligned}
\tag{61}
$$

where T_p is the tension, μ_p is the RR-charge of the Dp-brane and F is the gauge field strength. In this case the integrals in (61) are taken over the $(p+1)$-dimensional world-volume $\mathcal{W}$ of the Dp-brane which is embedded in the ten dimensional space-time manifold $\mathcal{M}$ via the map $\varphi : \mathcal{W} \hookrightarrow \mathcal{M}$. In order to preserve $N = 1$ supersymmetry the D-brane has to satisfies a BPS condition. For a D3-brane this amounts to the fact that the tension T_3 is equal to the RR-charge $T_3 = \mu_3$.[4]

Adding both terms to ten-dimensional type IIB bulk action and performing again a Kaluza-Klein reduction in a low energy and small ζ^i expansion one derives again a $d = 4$ low energy effective action. It has $N = 1$ supersymmetry and can be written in the in standard supergravity form (50). One finds the Kähler potential [54]

$$
K = K_{\mathrm{cs}}(z, \bar{z}) + K_k(\tau, T, G, z, \zeta)\ ,
$$

[4] Furthermore D-branes cannot be included into the bulk theory arbitrarily. Instead in order to obtain a consistent theory the tadpole cancellation conditions for branes and orientifold planes must be satisfied [11,121]. In the following we always assume that these conditions are fulfilled.

$$K_{\rm cs} = -\ln\Big[-i\int \Omega(z)\wedge\bar\Omega(\bar z)\Big]\ , \tag{62}$$
$$K_k = -\ln\big[-i(\tau-\bar\tau)\big] - 2\ln\Big[\tfrac{1}{6}\mathcal{K}(\tau,T,G,z,\zeta)\Big]\ ,$$

where $\mathcal{K}$ is again given by (55). Thus, in terms of the Kaluza-Klein variables the Kähler potential coincides with the K of the orientifolds given in (54). However, the definition of the chiral Kähler coordinates changes and (52) is replaced by

$$\tau = l + ie^{-\phi}\ , \qquad G^{a_-} = c^{a_-} - \tau b^{a_-}\ , \tag{63}$$
$$T_{a_+} = \tfrac{3i}{2}\rho_{a_+} + \tfrac{3}{4}\mathcal{K}_{a_+b_+c_+}v^{b_+}v^{c_+} - \tfrac{3i}{4(\tau-\bar\tau)}\mathcal{K}_{a_+b_-c_-}G^{b_-}(G-\bar G)^{c_-}$$
$$+\tfrac{3}{2}i\mu_3\ell^2(\omega_{a_+})_{i\bar\jmath}\,\mathrm{Tr}\,\zeta^i\big(\bar\zeta^{\bar\jmath} - \tfrac{i}{2}\bar z^{\hat{a}_-}(\bar\chi_{a_-})^{\bar\jmath}_l\zeta^l\big) + h.c.\ ,$$

where $\omega_{a_+}\in H^{(1,1)}_+$, $\chi_{a_-}\in H^{(1,2)}_-$. Using the modified definition of T_{a_+} one again has to solve for v^{a+} in terms of the chiral variables T,τ,G and z. Inserted into K_k of (62) then results in a $\mathcal{K}(\tau,T,G,z,\zeta)$. We see from (63) that the complex structure moduli z^k couple to the matter fields ζ^i and thus the moduli space no longer is a direct product. The Kähler potential (62) is still the sum of two terms but both terms now depend on z.

As an explicit example consider the situation $h^{(1,1)}_+ = 1,\ G=0$. In this case (63) can be solved for v and one obtains as the Kähler potential

$$K_k = -\ln\big[-i(\tau-\bar\tau)\big] - 3\ln\big[T+\bar T + \omega_{i\bar\jmath}Tr\zeta^i(\bar\zeta^{\bar\jmath} - \bar z^k(\chi_k)^{\bar\jmath}_l\zeta^l) + h.c.\big] \tag{64}$$

Using the Kaluza-Klein reduction one also determines the gauge kinetic function of the non-Abelian gauge theory of the D3-brane to be

$$f\sim\tau\ . \tag{65}$$

Finally the superpotential is found to be

$$W = W_{\rm flux}(\tau,z) + \tfrac{1}{3}Y_{ijk}\,Tr\zeta^i\zeta^j\zeta^k\ , \qquad Y_{ijk} = \Omega_{ijk}(z)\ , \tag{66}$$

where $W_{\rm flux}(\tau,z)$ is given in (60).

6.2 Space-time filling D7-brane wrapped on 4-cycle S^Λ

Let us now instead of D3-branes add space-time filling D7-brane into the theory. In this case four dimensions of the D7-branes have to be internal and wrap on a 4-cycle S^Λ inside the Calabi-Yau orientifold [56–61].[5]

The massless bosonic spectrum resulting from the wrapped D7-brane consists of a four-dimensional $U(1)$ gauge field $A_\mu(x)$ and Wilson line moduli fields $a_\alpha(x)$ both arising from the eight-dimensional world-volume gauge field. Furthermore, fluctuations of the internal cycle S^Λ lead to 'matter fields' $\zeta^i(x)$ which arise from a normal coordinate expansion of the D7-brane. In the limit of small D7-brane fluctuations ζ^i and small complex structure deformations z these fields can be treated independently. As a consequence the 'matter fields' ζ^i appear as an expansion into two-forms of S^Λ of type $(2,0)$ [58].

The effective action is again derived by a Kaluza-Klein reduction as performed in [58]. The Dirac-Born-Infeld and Chern-Simons action given in (61) have the identical form with μ_3 replaced by μ_7 and a worldvolume $\mathcal{W} = \mathcal{M}_4\times S^\Lambda$. The resulting low energy effective action can be written in the standard

[5] The four cycle S^Λ includes both the cycle the D7-brane wraps and its image with respect to the orientifold involution σ.

$N = 1$ form (50) where the appropriate chiral Kähler coordinates are found to be

$$\begin{aligned} S &= \tau - \mu_7 \mathcal{L}_{i\bar{\jmath}} \zeta^i \bar{\zeta}^{\bar{\jmath}} \,, \qquad \tau = l + ie^{-\phi} \,, \qquad G^{a_-} = c^{a_-} - \tau b^{a_-} \,, \\ T_{a_+} &= \frac{3i}{2} \left(\rho_{a_+} - \tfrac{1}{2} \mathcal{K}_{a_+ b_- c_-} c^{b_-} b^{c_-} \right) + \tfrac{3}{4} \mathcal{K}_{a_+ b_+ c_+} v^{b_+} v^{c_+} \\ &\quad - \frac{3i}{4(\tau - \bar{\tau})} \mathcal{K}_{a_+ b_- c_-} G^{b_-} (G^{c_-} - \bar{G}^{c_-}) + 3i\mu_7 \ell^2 \mathcal{C}_{a_+}^{\alpha\bar{\beta}} a_\alpha \bar{a}_{\bar{\beta}} \,, \end{aligned} \tag{67}$$

with $\ell = 2\pi\alpha'$ and where $\mathcal{L}_{i\bar{\jmath}}, \mathcal{C}_\alpha^{\alpha\bar{\beta}}$ are intersection numbers on the four-cycle defined in [58]. In terms of these Kähler coordinates the Kähler potential for the low energy effective supergravity action is found to be

$$K = K_{\text{cs}}(z) + K_k(S, G, T, \zeta, a) \tag{68}$$

$$K_k = -\log\left[-i\left(S - \bar{S}\right) - 2i\mu_7 \mathcal{L}_{i\bar{\jmath}} \zeta^i \bar{\zeta}^{\bar{\jmath}}\right] - 2\log\left[\tfrac{1}{6}\mathcal{K}(S, G, T, \zeta, a)\right] \,,$$

where $K_{\text{cs}}(z)$ is given in (62) and $\mathcal{K}(S, G, T, \zeta, a)$ is obtained by solving (67) for v^{a_+} exactly as before.

For the holomorphic gauge coupling function one finds

$$f \sim T_\Lambda \,, \tag{69}$$

where T_Λ includes the Kähler modulus v^Λ which parameterizes the size of the four-cycle the D7-brane wraps. The superpotential has been computed for example in [57].

Apart from the superpotential there also is a D-term potential which arises from the non-vanishing D-term

$$D = \frac{12\mu_7 \ell}{\mathcal{K}} \int_{S_P} J \wedge B \,. \tag{70}$$

By appropriately adjusting $B = b^{a_-} \omega_{a_-}$ this D-term can always be made to vanish which just corresponds to the BPS-condition for the D7-brane.

Apart from the background in the bulk discussed in Sect. 3 one can also consider turning on fluxes on the D7-brane. This requires that the integral $\int_{\gamma_2} F_2$ is non-vanishing where F_2 is the 'internal' field strength of the D7-gauge boson. These fluxes generate additional contributions to the D-term and also a superpotential. More details can be found in [58,61].

7 Soft supersymmetry breaking

Now that we have discussed D-branes in Calabi-Yau orientifolds we can add bulk and brane fluxes and discuss the effect of the spontaneous supersymmetry breaking for the matter fields ζ which live on the brane.

From a phenomenological point of view one is interested in hierarchical supersymmetry breaking such that the scale of supersymmetry breaking $m_{3/2}$ is much lower than the Planck scale M_{Pl} preferably in the 100GeV – 1TeV region. In this limit any spontaneously broken supergravity appears to leading order in $m_{3/2}/M_{\text{Pl}}$ as a softly broken, globally supersymmetric theory which is characterized by a Kähler potential K, a superpotential W^{eff}, a gauge kinetic function f and a set of soft supersymmetry breaking terms [122–126]. Before we turn to the induced soft terms for D3 and D7-branes let us first discuss them from a purely supergravity perspective.

7.1 Supergravity perspective

Following [124,125] it is convenient to expand the Kähler potential and the superpotential in a power series in the matter fields ζ

$$K(M, \bar{M}, \zeta, \bar{\zeta}) = \hat{K}(M, \bar{M}) + Z_{i\bar{\jmath}}(M, \bar{M})\, \zeta^i \bar{\zeta}^{\bar{\jmath}} + \left(\tfrac{1}{2}\, H_{ij}(M, \bar{M})\, \zeta^i \zeta^j + \text{h.c.}\right) + \dots \,,$$

$$W(M,\zeta) = \hat{W}(M) + \tfrac{1}{2}\,\tilde{\mu}_{ij}(M)\,\zeta^i\zeta^j + \tfrac{1}{3}\,\tilde{Y}_{ijk}(M)\,\zeta^i\zeta^j\zeta^k + \ldots\,. \tag{71}$$

Inserted into the effective action (50) and keeping the leading order in $m_{3/2}/M_{\rm Pl}$ results in the potential

$$V^{(\rm eff)} = \tfrac{1}{2}D^2 + |\partial_i W^{(\rm eff)}|^2 + m^2_{i\bar{j}}\zeta^i\bar{\zeta}^{\bar{j}} + \big(\tfrac{1}{3}A_{ijk}\zeta^i\zeta^j\zeta^k + \tfrac{1}{2}B_{ij}\zeta^i\zeta^j + \text{h.c.}\big) \tag{72}$$

where $W^{(\rm eff)}, m^2_{i\bar{j}}, A_{ijk}$ and B_{ij} can be expressed as geometrical quantities on the moduli space spanned by the moduli M^A. Their precise form can be found in [124]. In addition a soft gaugino mass $m_{\tilde{g}}$ is induced given by

$$m_{\tilde{g}} = F^A\partial_A \ln g^{-2} \tag{73}$$

where F^A is the non-vanishing F-term which breaks supersymmetry spontaneously.

With these preliminaries let us turn to the concrete situation of spontaneous supersymmetry breaking by background fluxes and discuss the structure of the soft terms.

7.2 Matter on D3-branes

Let us first discuss the soft terms when the Standard Model lives on D3-branes and supersymmetry is spontaneously by the three-form flux G_3 which is discussed in [52–54]. From a phenomenological point of view this case is not all that promising. As we discussed below (60) if $G_3 \in H_-^{(2,1)}$ then supersymmetry is unbroken and all soft terms vanish. If $G_3 \in H_-^{(0,3)}$ one finds a no-scale type supersymmetry breaking by $F_{T_{a_+}} \neq 0$ with $V = 0$. In this case also all soft terms vanish which is sometimes called 'strict' no scale supersymmetry breaking. For $G_3 \in H_-^{(3,0)} \oplus H_-^{(1,2)}$ one finds an unstable $V > 0$ and the discussion of soft terms is not completely sensible.

7.3 Matter on D7-branes

The situation changes once the Standard Model lives SM on D7-branes [56,57,59–61]. Let us again focus on on the case where $G_3 \in H_-^{(0,3)}$ and supersymmetry is broken by $F_{T_{a_+}} \neq 0$. Due to the different Kähler potential (68) and the different gauge kinetic function (69) the soft terms significantly change. The gaugino and scalar masses are no longer zero and one finds

$$m_g \sim m_{3/2}\,, \quad m^2 \sim m_{3/2}\,, \quad A \sim Y \tag{74}$$

8 Conclusions/open problems

In this lecture we reviewed some aspects of spontaneous supersymmetry breaking by background fluxes. We showed that this can also be achieved by compactification on manifolds with $SU(3)$ structure. We reviewed the computation of the $N = 1$ effective action for Calabi-Yau orientifolds including background fluxes and space-time filling $D3/D7$-branes. We briefly discussed the resulting soft supersymmetry breaking terms.

There remains to be a number of urgent open questions. In particular one should include a warped space-time or more generally the back-reaction of the geometry properly into the analysis. It would be worthwhile to also extend the phenomenological consideration to orientifolds with $SU(3)$-structure. Furthermore the effect of quantum correction both perturbatively and non-perturbatively should be taken into account.

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