Phenomenological Aspects of String Theory

Jan Louis
Martin–Luther–Universität Halle–Wittenberg,
FB Physik, D-06099 Halle, Germany

In these lectures we review some of the phenomenological implications of the recent developments in understanding non-perturbative aspects of string theory.

Dedicated to the Memory of Michael Lichtfeldt

1 Introduction

Since the seventies string theory has been discussed as a possible candidate for a theory which unifies all known particle interactions including gravity. Until recently, however, string theory has only been known in its perturbative regime. That is, the (particle) excitations of a string theory are computed in the free theory ($g_{\rm s} = 0$), while their scattering processes are evaluated in a perturbative series for $g_{\rm s} \ll 1$.[1] The string coupling constant $g_{\rm s}$ is a free parameter of string theory but for $g_{\rm s} = \mathcal{O}(1)$ no method of computing the spectrum or the interactions had been known. This situation dramatically changed during the past years. For the first time it became possible to go beyond the purely perturbative regime and to compute some of the non-perturbative properties of string theory.[2] The central point of these developments rests on the idea that the strong-coupling limit of a given string theory can be described in terms of another, weakly coupled, 'dual theory'. This dual theory can take the form of either a different string theory, or the same string theory with a different set of perturbative excitations, or a new theory termed M-theory.

Since string theory is a candidate for a unified theory of all interactions it has always been a primary goal to identify the Standard Model of Particle Physics as the low energy limit of string theory. The massless spectrum of string theory can indeed accommodate families of chiral fermions transforming in appropriate representations of a non-Abelian gauge group as well as Higgs bosons necessary for the electro-weak symmetry breaking. Furthermore, most ground states of string theory studied so far are supersymmetric and have a universal gauge coupling constant at the leading order which is in very good agreement with the electroweak precision data of this decade.[3] However, a more quantitative agreement with the Standard Model has so far not been achieved. The main obstacles seem to be a missing mechanism for spontaneously breaking supersymmetry at a scale hierarchically lower than the Planck scale $M_{\rm Pl}$, the implementation of a Higgs mechanism in string theory which generates small masses of the light states and finally the lifting of an enormous vacuum degeneracy of string ground states. It is commonly believed that these deficiencies are due to our lack of understanding the non-perturbative structure of string theory. In this respect it is of interest to ask what are the phenomenological implications of the recent developments and this is the subject of these lectures.

2 Perturbative String Theory

2.1 The perturbative expansion.

In string theory the fundamental objects are one-dimensional strings which, as they move in time, sweep out a 2-dimensional worldsheet Σ.[1] Strings can be open or closed and their worldsheet is embedded in some D-dimensional target space which is identified with a Minkowskian spacetime. States in the target space appear as eigenmodes of the string and their scattering amplitudes are generalized by appropriate scattering amplitudes of strings. These scattering amplitudes are built from a fundamental vertex, which for closed strings is depicted in Fig. 1. It represents the splitting of a string or the joining of

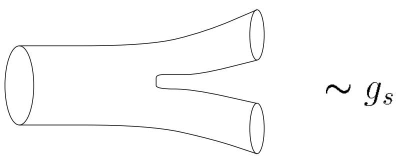

Figure 1: The fundamental closed string vertex.

two strings and the strength of this interaction is governed by a dimensionless string coupling constant $g_{\rm s}$. Out of the fundamental vertex one composes all possible closed string scattering amplitudes $\mathcal{A}$, for example the four-point amplitude shown in Fig. 2. The expansion in the topology of the Riemann surface (i.e. the number of holes in the surface) coincides with a power series expansion in the string coupling constant formally written as

$$\mathcal{A} = \sum_{n=0}^{\infty} g_{\rm s}^{-\chi}\, \mathcal{A}^{(n)} \ , \tag{1}$$

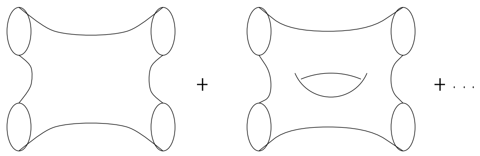

Figure 2: The perturbative expansion of string scattering amplitudes. The order of g_s is governed by the number of holes in the world sheet.

where $\mathcal{A}^{(n)}$ is the scattering amplitude on a Riemann surface of genus n and $\chi(\Sigma)$ is the Euler characteristic of the Riemann surface

$$\chi(\Sigma) = \frac{1}{4\pi}\int_\Sigma R^{(2)} = 2 - 2n - b \ . \tag{2}$$

$R^{(2)}$ is the curvature scalar on Σ and b the number of boundaries of the Riemann surface. (For the four-point amplitude of Fig. 2 one has $b = 4$.)[a]

In all string theories there is a massless scalar field ϕ called the dilaton which couples to $R^{(2)}$ and therefore its vacuum-expectation value determines the size of the string coupling. One finds[4,1]

$$g_\mathrm{s} \ = \ e^{\langle\phi\rangle} \ . \tag{3}$$

g_s is a free parameter since ϕ is a flat direction (a modulus) of the effective potential. Thus, string perturbation theory is defined in that region of the parameter space (which is also called the moduli space) where $g_\mathrm{s} < 1$ and the tree-level amplitude (genus-0) is the dominant contribution with higher-loop amplitudes suppressed by higher powers of g_s. Until 1995 this was the only regime accessible in string theory.

2.2 The spacetime spectrum of the string.

The propagation of a free string ($g_\mathrm{s} = 0$) is governed by the 2-dimensional action

$$S_\mathrm{free} = -\frac{1}{4\pi\alpha'}\int_\Sigma d\sigma d\tau\, \partial_i X^\mu(\sigma,\tau)\, \partial^i X^\nu(\sigma,\tau)\ \eta_{\mu\nu} \ , \tag{4}$$

where ∂_i denotes $\partial/\partial\sigma$ and $\partial/\partial\tau$. Here σ parameterizes the spatial direction on Σ while τ denotes the 2-dimensional time coordinate. The coordinates of the D-dimensional target spacetime in which the string moves, are represented

[a] For open strings different diagrams contribute at the same order of the string loop expansion.[1]

by X^μ, with $\mu = 0, \ldots, D-1$; in terms of the 2-dimensional field theory they appear as D scalar fields. For S to be dimensionless α' has dimension $[\text{length}]^2 \sim [\text{mass}]^{-2}$. Thus α' sets the mass scale M_s of string theory and up to numerical factors which we supply later one has $M_s^{-2} \sim \alpha'$. The masses of all perturbative string states are multiples of M_s. Demanding that string theory contains Einstein gravity as its low-energy limit relates M_s and $M_{\rm Pl}$. By comparing for example physical graviton-graviton scattering amplitudes in both theories one finds for closed string theories in D dimensions [5]

$$M_s \sim M_{\rm Pl}\, g_s^{\frac{2}{D-2}} \tag{5}$$

where we dropped numerical proportionality factors.

The equations of motion of the action (4) are given by

$$(\partial_\tau^2 - \partial_\sigma^2) X^\mu = 0 \ , \tag{6}$$

with the solutions

$$X^\mu = X_L^\mu(\sigma+\tau) + X_R^\mu(\sigma-\tau) \ . \tag{7}$$

A closed string satisfies the boundary condition $X^\mu(\sigma) = X^\mu(\sigma + 2\pi)$, which does not mix X_L^μ and X_R^μ and leaves them as independent solutions. This splitting into left (L) and right (R) moving fields has the consequence that upon quantizing the 2-dimensional field theory, also the Hilbert space splits into a direct product $\mathcal{H} = \mathcal{H}_L \otimes \mathcal{H}_R$ where $\mathcal{H}_L(\mathcal{H}_R)$ contains states built from oscillator modes of $X_L(X_R)$. These states also carry a representation of the D-dimensional target space Lorentz group and thus can be identified as perturbative states in spacetime of a given spin and mass.[b]

In open string theory one has a choice to impose at the end of the open string either Neumann (N) boundary conditions, $\partial_\sigma X^\mu = 0$, or Dirichlet (D) boundary conditions, $X^\mu = \text{constant}$. The boundary conditions mix left- and right-movers and the product structure of the closed string is not maintained. As a consequence a perturbative spectrum of states is built from a single Hilbert space. Neumann boundary conditions leave the D-dimensional Lorentz invariance unaffected. Dirichlet boundary conditions, on the other hand, lead to very different types of objects and a very different set of states (D-branes) in spacetime.[6] In this case the end of an open string is constrained to only move in a fixed spatial hyper-plane. This plane must be regarded as a dynamical object with degrees of freedom induced by the attached open string. A careful analysis shows that the corresponding states in spacetime are not

[b] These are perturbative states since the quantization procedure is a perturbation theory around the free string theory with $g_s = 0$.

Table 1: The closed-string theories, their worldsheet supersymmetry and the maximal possible spacetime dimension.

closed string theories	worldsheet supersymmetry	$D_{\max}$
bosonic string	$(0,0)$	26
superstring	$(1,1)$	10
heterotic string	$(0,1)$	10

part of the perturbative spectrum but rather correspond to non-perturbative solitonic type excitations[c]. It is precisely these states which dramatically affect the properties of string theory in its non-perturbative regime.

The spacetime properties of a string theory significantly change once one introduces supersymmetry on the worldsheet. There are independent left- and right-moving supercharges Q_L, Q_R, so that in general one can have p supercharges Q_L and q supercharges Q_R; this is also termed (p,q) supersymmetry. A supersymmetric version of the action (4) requires the presence of Majorana-Weyl worldsheet fermions χ^μ with appropriate couplings; for example a scalar supermultiplet of $(1,0)$ supersymmetry contains the fields $\big(X_L(\sigma+\tau), \chi_L(\sigma+\tau)\big)$. Depending on the amount of worldsheet supersymmetry one defines the different *closed* string theories: the bosonic string, the superstring and the heterotic string (see Table 1).

For *open* string theories the left- and right-moving worldsheet supercharges are not independent. One can either have a bosonic open string (with no worldsheet supersymmetry) or an open superstring with one supercharge which is a linear combination of Q_L and Q_R. The latter string theory is called type-I. It contains (unoriented) open and closed strings with SO(32) Chan-Paton factors coupling to the ends of the open string.

A unitary S-matrix in spacetime requires conformal invariance on the worldsheet and this imposes a restriction on the maximal number of spacetime dimensions and the spacetime spectrum. All supersymmetric string theories necessarily have $D \leq 10$ and they are particularly simple in their maximal possible dimension $D = 10$.[d]

In $D = 10$ there are only five consistent spacetime supersymmetric string

[c] They are non-perturbative in that their mass (or rather their tension for higher-dimensional D-branes) goes to infinity in the weak coupling limit $g_s \to 0$.

[d] For closed strings an additional constraint arises from the requirement of modular invariance of one-loop amplitudes which results in an anomaly-free spectrum of the corresponding low-energy effective theory.[7] For open strings anomaly cancellation is a consequence of the the absence of tadpole diagrams.[1]

theories: type-IIA, type-IIB, heterotic $E_8 \times E_8$ (HE8), heterotic SO(32) (HSO) and the type-I SO(32) string. The first two string theories have 32 supercharges and a unique massless multiplet for each case. The other three string theories all have 16 supercharges. In this case, the supersymmetric representation theory alone does not completely determine the spectrum. The gravitational multiplet is unique, but the gauge group representation of the vector multiplets is only fixed if also anomaly cancellation is imposed.

2.3 Vacuum Cleaning

String phenomenology focuses on the low energy limit of string theory and asks to what extent the Standard Model emerges as this low energy effective theory. Thus, only those string theories have to be considered which can possibly accommodate the Standard Model. Furthermore, each string theory has a huge number of ground states and again only the phenomenologically viable ones are of interest here. This process of choosing (by hand) a subset of all string theories and within a string theory only a subspace of the space of ground states is sometimes termed "vacuum cleaning".[e]

The criteria for this selection process are somewhat ambiguous but the following necessary conditions should hold: [f]

1. $D = 4$
   The spacetime should have four flat Minkowski dimensions.
2. $SU(3) \times SU(2) \times U(1) \subset G$
   The gauge group G should be big enough to contain the gauge group of the Standard Model and account for its fermion content.
3. $n_g \geq 3$
   The number of light chiral generations n_g should be at least three.

In addition to 1.–3. one further condition is usually imposed:

4. $N = 1$ spacetime supersymmetry
   The low energy limit should be $N = 1$ supersymmetric.

[e]This terminology was coined by L. Dixon.

[f]One might contemplate to impose additional constraints. For example, one could demand the gauge group to be precisely the gauge group of the SM $G = SU(3) \times SU(2) \times U(1)$ or the number of light generations to be exactly three $n_g = 3$. However, none of these two is obviously true since they strongly depend on the physics which governs the energy range between the weak scale M_{weak} and M_{Pl}. Similarly, demanding no fast proton decay or a reasonable fermion mass hierarchy is difficult to impose without further knowledge of the physics just above M_{weak}.

This last condition is much more questionable than the first three. After all there are no experimental signs for supersymmetry yet. However, it seems difficult to understand how a hierarchy of scales $M_{\rm weak}/M_{\rm Pl} \ll 1$ can be generated and kept stable without something like supersymmetry. Furthermore, among the known, consistent string backgrounds almost all display low energy supersymmetry. It is for these two reasons that most phenomenological investigations in string theory have concentrated on supersymmetric backgrounds and we follow here the same assumption.[g] However, it might be worthwhile at some point to relax condition 4 and study non-supersymmetric ground states in more detail.[8] Once we accept condition 4 we eventually have to face the problem of how to break supersymmetry near $M_{\rm weak}$. This question will occupy section 2.7.

The bosonic string (open or closed) is tachyonic and cannot accommodate spacetime fermions. Thus it does not obey conditions 2,3 and will be immediately discarded. The superstring is tachyon-free and does have spacetime fermions in its massless spectrum. However, Dixon, Kaplunovsky and Vafa showed that the particular fermion representation of the Standard Model can never appear in the massless spectrum.[9] Therefore also the perturbative superstring has been discarded. Finally, the heterotic string and the type I string have no obvious deficiency and have been extensively studied. In fact until recently it was the heterotic string which was the prime target of string phenomenology.[10,11] On the one hand it is easier to accommodate chiral representations in the massless spectrum of the heterotic string and on the other hand the construction of consistent four-dimensional ground states is considerably simpler in the heterotic string. Only recently (and for reasons which we review later on) the phenomenological properties of the type I string have been investigated.[12]

2.4 The Low Energy Effective Action

The spacetime spectrum of a string theory contains a finite number of massless modes, which we denote as L, and an infinite number of massive modes H. The mass of H is an (integer) multiple of M_s. To derive the low energy effective action $\mathcal{L}_{\rm eff}(L)$ which only depends on the light modes L one considers scattering processes of L with external momenta p much smaller than M_s, i.e. $p^2/M_s^2 \ll 1$. A systematic procedure for computing $\mathcal{L}_{\rm eff}(L)$ has been developed and is often referred to as the S-matrix approach.[5,13,14] One computes the S-matrix elements for a given string vacuum as a perturbative power series in g_s. At

[g] In fact one needs precisely $N = 1$ supersymmetry since theories with $N > 1$ have difficulties accomodating the chiral structure of the Standard Model.

the lowest order (tree level) an S-matrix element typically has a pole in the external momentum which corresponds to the exchange of a massless mode L. The finite part is a power series in p^2/M_s^2 and corresponds to the exchange of the whole tower of massive H-modes. $\mathcal{L}_{\rm eff}$ is then constructed to reproduce the string S-matrix elements in the limit $p^2/M_s^2 \ll 1$ with S-matrix elements constructed entirely from the effective field theory of the L-modes. In this low energy effective theory the exchange of the H-modes in the string scattering is replaced by an effective interaction of the L-modes. For a four-point amplitude this procedure is schematically sketched in figure 3. The first row denotes the

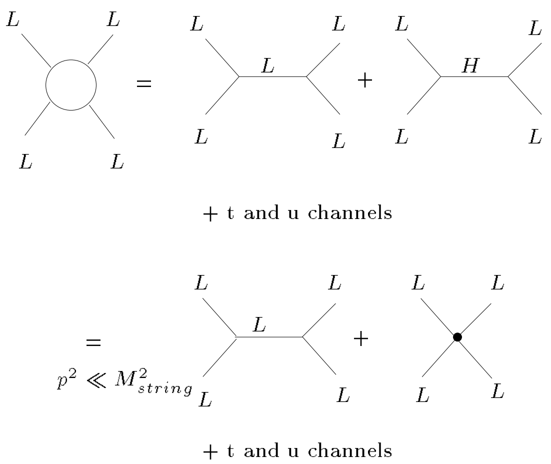

Figure 3: The S-matrix approach.

string scattering amplitude and its separation in a 'pole piece' (exchange of a massless mode) and the finite piece (exchange of the heavy modes). The second row indicates ordinary field-theoretical Feynman diagrams computed from the effective Lagrangian. The pole piece is reproduced by the same exchange of the massless modes while the finite part is identified with an effective interaction. Using this procedure $\mathcal{L}_{\rm eff}$ can be systematically constructed as a power series in both p^2/M_s^2 and g_s. The power of p^2 counts the number of spacetime derivatives in $\mathcal{L}_{\rm eff}$; at order $(p^2/M_s^2)^0$ one finds the effective potential while the order (p^2/M_s^2) corresponds to the two-derivative kinetic terms.[h]

[h] Instead of using the S-matrix approach one can alternatively construct the effective action by computing the β-functions of the two-dimensional σ-model and interpreting them as the equations of motion of string theory. The effective action is then constructed to reproduce these equations of motion.[4]

The selection criteria 1–4 already significantly reduce the number of string vacua for which the low energy effective theory has to be computed. Further simplification of the S-matrix approach comes from the use of all symmetries a string vacuum might have in that one does not have to compute separately S-matrix elements which are related by a symmetry.[i] One only has to determine those couplings in the effective theory which are not related by general coordinate transformations, gauge transformations and $N=1$ supersymmetry. Therefore, let us recall the bosonic terms of the most general gauge invariant supergravity Lagrangian with only chiral and vector multiplets and no more than two derivatives [15,16]

$$\begin{aligned}\mathcal{L} &= -\sqrt{g}\Big(\frac{1}{2\kappa^2}R + G_{I\bar{J}}\, D_m\bar{\phi}^{\bar{J}} D^m\phi^I + V(\phi,\bar{\phi}) \\ &\quad + \sum_a\big(\frac{1}{4g_a^2}(F_{mn}F^{mn})_a + \frac{\theta_a}{32\pi^2}(F\tilde{F})_a\big) \\ &\quad + \ \text{fermionic terms}\Big) \ , \end{aligned} \tag{8}$$

where $\kappa^2 = 8\pi M_{\rm Pl}^{-2}$. Supersymmetry imposes constraints on the couplings of $\mathcal{L}$ in eq. (8). The metric $G_{I\bar{J}}$ of the manifold spanned by the complex scalars ϕ^I is necessarily a Kähler metric and therefore obeys

$$G_{I\bar{J}} = \frac{\partial}{\partial\phi^I}\frac{\partial}{\partial\bar{\phi}^{\bar{J}}}\, K(\phi,\bar{\phi}) \ , \tag{9}$$

where $K(\phi,\bar{\phi})$ is the Kähler potential. It is an arbitrary real function of ϕ and $\bar{\phi}$. The gauge group G is in general a product of simple group factors G_a labelled by an index a, i.e.

$$G = \prod_a G_a \ . \tag{10}$$

With each factor G_a there is an associated gauge coupling g_a which can depend on the ϕ^I. However, supersymmetry constrains the possible functional dependence and demands that the (inverse) gauge couplings g_a^{-2} are the real part of holomorphic functions $f_a(\phi)$ called the gauge kinetic functions. The imaginary part of the $f_a(\phi)$ are (field-dependent) θ-angles. One finds

$$\begin{aligned} g_a^{-2} &= \mathrm{Re} f_a(\phi) \ , \\ \theta_a &= -8\pi^2\,\mathrm{Im} f_a(\phi) \ . \end{aligned} \tag{11}$$

[i] Except to check the consistency of the procedure.

The scalar potential $V(\phi, \bar{\phi})$ is also determined by a holomorphic function, the superpotential $W(\phi)$

$$V(\phi, \bar{\phi}) = e^{\kappa^2 K} \Big(D_I W G^{I\bar{J}} \bar{D}_{\bar{J}} \bar{W} - 3\kappa^2 |W|^2 \Big), \tag{12}$$

where $D_I W := \frac{\partial W}{\partial \phi^I} + \kappa^2 \frac{\partial K}{\partial \phi^I} W$. To summarize, $\mathcal{L}$ is completely determined by three functions of the chiral multiplets, the real Kähler potential $K(\phi, \bar{\phi})$, the holomorphic superpotential $W(\phi)$ and the holomorphic gauge kinetic functions $f_a(\phi)$ and it is these functions which have to be computed in string theory.

In string theory these couplings receive contributions at the tree level, perturbative quantum corrections and non-perturbative quantum corrections. However, the holomorphicity of $W(\phi)$ and $f_a(\phi)$ lead to two perturbative non-renormalization theorems: $W(\phi)$ receives no perturbative corrections [17] while $f_a(\phi)$ is only corrected at one-loop order but has no further perturbative corrections.[18] Altogether one has

$$\begin{aligned} W &= W^{(0)} + W^{(\mathrm{NP})} , \\ f &= f^{(0)} + f^{(1)} + f^{(\mathrm{NP})} , \\ K &= \sum_{n=0}^{\infty} K^{(n)} + K^{(\mathrm{NP})} , \end{aligned} \tag{13}$$

where the superscript (NP) indicates possible non-perturbative corrections.

2.5 Heterotic String Vacua in $D = 4$ with $N = 1$ Supersymmetry

Heterotic string vacua in $D = 4$ are constructed from conformal field theories (CFT) with central charge $(c, \bar{c}) = (c_{\mathrm{st}}, \bar{c}_{\mathrm{st}}) + (c_{\mathrm{int}}, \bar{c}_{\mathrm{int}}) = (4, 6) + (22, 9)$.[1,10] The left moving internal $c_{\mathrm{int}} = 22$ CFT together with the right moving spacetime $\bar{c}_{\mathrm{st}} = 6$ CFT gives rise to (non-Abelian) gauge bosons of a gauge group G. For $D = 4$ the constraint from modular invariance is much less stringent as for $D = 10$ and many gauge groups other than $E_8 \times E_8$ or $SO(32)$ are allowed. Generically, the gauge group has a product structure $G = \prod_a G_a$ where the size of G is not arbitrary but bounded by the central charge c_{int}

$$\mathrm{rank}(G) \leq 22 \ . \tag{14}$$

The right moving $\bar{c}_{int} = 9$ CFT can support spacetime supercharges if it is invariant under additional (global) worldsheet supersymmetries.[j] In particular,

[j] Strictly speaking there also is a condition on the (worldsheet) $U(1)$ charge of the primary states.[19] Alternatively, the conditions for spacetime supersymmetry can be stated in terms of generalized Riemann identities of the partition function.[20]

$N = 1$ spacetime supersymmetry requires a (global) $(0,2)$ supersymmetry of the $\bar{c}_{int} = 9$ CFT.[19]

The previous discussion related the amount of spacetime supersymmetry to properties of the internal CFT in particular to the amount of worldsheet supersymmetry. A subset of these CFT can be associated with a compact manifold on which the ten-dimensional heterotic string is compactified. Such compact manifolds have to be six-dimensional Ricci-flat Kähler manifolds with holonomy group $SU(3)$ termed Calabi–Yau threefolds.

The number of $\bar{c}_{int} = 9$ CFT with $(0,2)$ supersymmetry is huge and has not been classified yet. As a consequence the space of heterotic string vacua is large. Furthermore, within each string vacuum there is a continuous degeneracy parameterized by the flat directions of $V_{\rm eff}$. These are the dilaton ϕ, the axion a (the dual of the antisymmetric tensor $B_{\mu\nu}$) and a set of moduli T^i.[k] Instead of mapping out the details of the space of heterotic string vacua many investigations concentrated on generic properties which are shared by all (or almost all) vacua. In these lectures we also follow this approach and in particular discuss the unification of the gauge couplings and the issue of supersymmetry breaking.

2.6 Gauge Coupling Unification

In the perturbative heterotic string the gauge complings g_a are universal at leading order and given by $g_a^{-2} = k_a g_{\rm s}^{-2}$, where k_a denotes the level of the Kac-Moody algebra of the left moving internal $c_{\rm int} = 22$ CFT.[21] Including one-loop corrections one finds [22,23]

$$g_a^{-2}(\mu) = \frac{k_a}{g_{\rm s}^2} + \frac{b_a}{8\pi^2} \ln \frac{M_s}{\mu} + \Delta_a(T^i) \ , \tag{15}$$

where the one-loop coefficients of the β-function obey $b_a = \sum_r n_r T_a(r) - 3T_a(G)$ with the normalization $Tr_r T^a T^b = T(r)\,\delta^{ab}$ of the gauge group generators T^a in the representation r. In order to make contact with the formulas of the previous section one combines the dilaton with the dual axion a of the antisymmetric tensor into a complex superfield $S = e^{-2\phi} + i\,a$ such that in perturbation theory

$$\begin{aligned} f_a &= k_a S + f_a^{(1)}(T) \ , \\ \Delta_a(T,\bar{T}) &= {\rm Re} f_a^{(1)}(T) + \mathcal{A}(T,\bar{T}) \ . \end{aligned} \tag{16}$$

[k] The physical Yukawa couplings, for example, do depend on the T^i (or rather their vaccum expectation values) and thus the computation of the fermion masses requires a mechanism which lifts these flat directions.

Contrary to naive expectation $\Delta_a(T,\bar{T})$ is not a harmonic function of the moduli but due to infrared effects aquires a non-harmonic term $\mathcal{A}$ known as the holomorphic anomaly.[23,24,25,10]

M_s is the string scale which is related to $M_{\rm Pl}$ and $g_{\rm s}$ as in (5) with the precise numerical coefficients [22]

$$M_s = \frac{2\cdot 3^{-\frac{3}{4}}\, e^{\frac{1}{2}(1-\gamma)}}{\sqrt{2\pi\alpha'}} = \frac{3^{-\frac{3}{4}}\, e^{\frac{1}{2}(1-\gamma)}}{4\pi}\, g_{\rm s} M_{\rm Pl} \approx g_{\rm s}\cdot 5\cdot 10^{17} GeV\ . \qquad (17)$$

The string scale is roughly one order of magnitude bigger than the phenomenologically preferred GUT-scale $M_{\rm GUT} \approx 3\cdot 10^{16} GeV$. At this scale the experimentally measured gauge couplings of the Standard Model unify with $g_{\rm GUT}^2 \approx g_{\rm s}^2 \approx \frac{4\pi}{23}$ under the assumption that right above $M_{\rm weak}$ the particle spectrum of the Standard Model is replaced by the spectrum of the supersymmetric Standard Model.[26] Thus, the perturbative heterotic string does reproduce the experimental situation of a unified gauge coupling. However, the unification occurs not quite at the right scale. The problem is that M_s is not an independent parameter in the heterotic string but tied to $g_{\rm s}$ and $M_{\rm Pl}$ via (17). The mismatch between M_s and $M_{\rm GUT}$ needs an explanation but the fact that it comes so close is one of the attractive model independent features of the perturbative heterotic string.

In the past a number of attempts to overcome this mismatch have been suggested and we briefly review some of them here. One of the early suggestions has been that maybe the compactification scale of Calabi-Yau manifolds can be chosen lower than M_s and therefore serve as $M_{\rm GUT}$. However, within the perturbative heterotic string this suggestion is problematic.[27] Since this argument partially breaks down in non-perturbative string theory let us go through it in slightly more detail.

Compactification of the ten-dimensional effective field theory on a Calabi–Yau threefold yields a relation between the string coupling $g_{\rm s}^{(4)}$ in the four-dimensional action and the string coupling $g_{\rm s}^{(10)}$ in the ten-dimensional action which involves the volume V_6 of the Calabi–Yau threefold

$$(g_{\rm s}^{(4)})^{-2} = (g_{\rm s}^{(10)})^{-2}\, V_6\, l_{\rm s}^{-6}\ , \qquad (18)$$

where $l_{\rm s} \equiv \sqrt{\alpha'}$ is the string length. The volume V_6 is in principle an independent scale in the problem, the compactification scale. The perturbative decompactification limit sends $V_6\, l_{\rm s}^{-6} \to \infty$ and demands that the string coupling stays in the perturbative regime, ie. $g_{\rm s}^{(10)}$ is kept fixed and small. Eq. (18) then implies in this limit $g_{\rm s}^{(4)} \to 0$. On the other hand, the measured gauge couplings do not allow an arbitrarily small gauge coupling as a consequence of (15). Instead one roughly has to have

$$\frac{1}{23} = \frac{g^2_{\rm GUT}}{4\pi} \approx \frac{(g_{\rm s}^{(4)})^2}{4\pi} = \frac{(g_{\rm s}^{(10)})^2}{4\pi}\frac{l_{\rm s}^6}{V_6} < \frac{l_{\rm s}^6}{4\pi V_6} \quad \text{for} \quad g_{\rm s}^{(10)} < 1 \ . \tag{19}$$

We learn that V_6 cannot be arbitrarily big but has to obey $V_6 l_{\rm s}^{-6} < \frac{23}{4\pi} \approx 2$ which implies $V_6 = \mathcal{O}(l_{\rm s}^6)$. Thus V_6 cannot be used as an independent scale or tuned to be $M_{\rm GUT}$.

The same problem in another disguise can be seen from eq. (15). One might hope to find string vacua where $\Delta_a(T^i)$ is large.[28,11] If $\Delta_a(T^i)$ has the form

$$\Delta_a(T^i) = -\frac{b_a}{8\pi^2}\ln\delta(T^i) + \tilde{\Delta}_a(T^i) \tag{20}$$

one would have a "redefinition of the GUT-scale"

$$g_a^{-2}(\mu) = \frac{k_a}{g_{\rm s}^2} + \frac{b_a}{8\pi^2}\ln\frac{M_{\rm GUT}}{\mu} + \tilde{\Delta}_a(T^i) \ , \tag{21}$$

with $M_{\rm GUT} = \frac{M_{\rm s}}{\delta(T^i)}$. In order to have $M_{\rm GUT} \approx 3\cdot 10^{16} GeV$ one needs $\delta = 20$ or $\ln\delta \approx 3$. Thus the mismatch of scales puts a strong contraint on sign, coefficient and size of $\Delta_a(T^i)$. However, generically one finds $\ln\delta = O(1)$ which is just another way to observe the perturbative decompactification limit.

As an alternative szenario one can envisage a GUT-group $G_{\rm GUT}$ at $M_{\rm Pl}$ which breaks by an appropriate Higgs mechanism at $M_{\rm GUT}$ to $G_{\rm SM}$. However, now one is in need of an explanation why this breaking occurs precisely at $M_{\rm GUT}$.

2.7 Supersymmetry Breaking and Stabilizing the Dilaton

Let us now turn to the question of lifting the vacuum degeneracy and supersymmetry breaking which is another and more serious problem shared by all perturbative heterotic string vacua. In particular we need to address the following points:

- What determines the vacuum expectation values $\langle S\rangle$ and $\langle T^i\rangle$ of the dilaton and moduli fields and what are their values? As a consequence of (15) one needs for the dilaton

$$\langle {\rm Re} S\rangle \simeq g_s^{-2} \simeq \frac{23}{4\pi} \ . \tag{22}$$

For the case of a geometrical compactification the moduli T^i parameterize the size and shape of the Calabi-Yau threefold. Thus, as a consequence of (19) one needs to arrange

$$\langle T^i \rangle = O(l_{\rm s}^2) \ . \tag{23}$$

- What is the mechanism for supersymmetry breaking and at what scale does the breaking occur? The naturalness problem of the Standard Model and the unification of the gauge couplings result in the theoretical prejudice of unbroken supersymmetry almost all the way down to $M_{\rm weak}$.

- Independently of the previous points one needs an explanation of the hierarchy $\frac{M_{\rm weak}}{M_{\rm Pl}}$.

None of these issues has a satisfactory answer within the perturbative heterotic string and thus the hope has been that non-perturbative effects come to rescue. Without a non-perturbative formulation of the heterotic string it is difficult to address non-perturbative properties and in fact only those of the effective field theory can be sensibly studied.[29,30] These field-theoretic effects certainly do occur in string theory but to what extent they dominate over 'stringy' non-perturbative contributions remains open.[l]

One considers an asymptotically free supersymmetric gauge theory which becomes strong at the scale

$$\Lambda \, = \, M_{\rm Pl} \, e^{-\frac{8\pi^2}{bg^2}} \ . \tag{24}$$

Thus a hierarchy $\frac{\Lambda}{M_{\rm Pl}} \ll 1$ is generated if g and/or b are small. In addition supersymmetry can be broken and the scale of the breaking (often parameterized by the gravitino mass $m_{3/2}$) is found to be

$$m_{3/2} \approx \frac{\Lambda^3}{M_{\rm Pl}^2} \ . \tag{25}$$

Thus for $\Lambda \sim 10^{13} - 10^{14} GeV$ one obtains $m_{3/2} \sim 10^1 - 10^3 GeV$ which is the 'desired' mass scale.[m] In string theory asymptotically free gauge theories do exist but their gauge couplings g are necessarily tied to the dilaton $g = g_s$ and the unification of all gauge couplings (15) implies $g^{-2} = \frac{23}{4\pi}$.

[l] Conversely, under the assumption that the field-theoretic effects are the dominant non-perturbative contribution the following analysis is legitimate.

[m] Once supersymmetry is broken it can induce radiatively the breakdown of the electroweak symmetry by a supersymmetric version of the Coleman–Weinberg mechanism.[31]

The strong gauge forces generate a potential which looks like

$$V \cong \frac{|\Lambda(S)|^6}{M_{\rm Pl}^2} \ . \tag{26}$$

Inserting (24) one finds that the minimum of V occurs at $g_{\rm s} = 0, \langle S\rangle \to \infty$ (see fig. 4) unacceptable for realistic phenomenology. This is a generic problem of all heterotic string vacua and is known as the dilaton problem.[32]

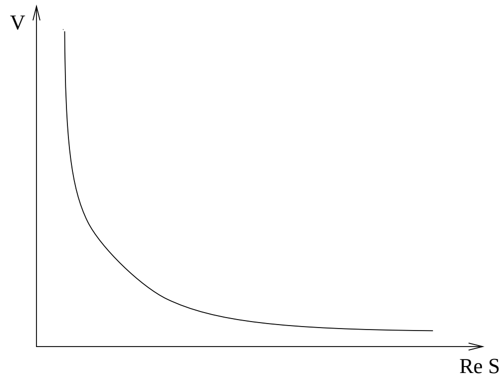

Figure 4: The dilaton potential

It is surprisingly difficult to contemplate solutions of the dilaton problem. Within the perturbative heterotic string essentially only one scenario has been proposed.[33] One considers two (or more) confining gauge groups each with different one-loop corrections to the gauge couplings (this situation does exist in string theory). The appropriate condensation scale for each group factor reads

$$\Lambda_a = M_{\rm Pl}\, e^{-\frac{8\pi^2}{b_a}(S+f_a^{(1)}(T^i))}. \tag{27}$$

Self-consistent expansion in $\Lambda/M_{\rm Pl}$ gives at leading order

$$V \approx \frac{1}{M_{\rm Pl}^2}\left|\Lambda_1^3 + \Lambda_2^3\right|^2 \tag{28}$$

with a minimum at $|\Lambda_1| = |\Lambda_2|$. Inserting (27) one obtains

$$\langle {\rm Re} S\rangle \approx \frac{b_1 b_2}{b_2 - b_1}\left(\frac{f_1^{(1)}}{b_1} - \frac{f_2^{(1)}}{b_2}\right) \ , \tag{29}$$

where in our conventions the size of $f^{(1)}$ is generically $f^{(1)} = O(\frac{b}{24\pi^2})$. Estimating the order of magnitudes one has $4\pi\langle\text{Re}S\rangle = O(\frac{b}{6\pi})$. Thus $4\pi\langle\text{Re}S\rangle = 23$ can only be arranged if

i) b is large ($b \approx 400$),

ii) $f^{(1)}$ is large ($f^{(1)} \approx 2$),

iii) one 'fine-tunes' $b_1 \simeq b_2$.

The first two options are impossible in the perturbative heterotic string. $b \approx 400$ is incompatible with rank(G) ≤ 22 and $f^{(1)} \approx 2$ is in conflict with the consistency condition $T^i = O(l_s^2)$.[n] Only the last option (iii) seems barely possible in the perturbative heterotic string. For example $G_1 = SU(8), G_2 = SU(9), f^{(1)} \approx \frac{2}{9}, \text{Re}T \approx 2$ does not contradict any perturbative constraint. However, arranging $4\pi\text{Re}S = 23$ is only one requirement and one needs to simultaneously generate the hierarchy $\Lambda \sim 10^{13} - 10^{14} GeV$. As we already noted this demands b and/or g to be small. Inserting the appropriate numbers one finds that within the perturbative heterotic string it is almost impossible to arrange the correct g_{GUT} and simultaneously generate a hierarchy.

2.8 Feature of the Type I String

In type I strings one encounters different relations among the physical parameters

$$\begin{aligned} (g_I^{(4)})^{-2} &\sim (g_I^{(10)})^{-1}\,(\frac{V_6}{l_I^6})\ , \\ M_{\text{Pl}}^2 &\sim l_I^{-2}\,(g_I^{(4)})^{-4}\,(\frac{l_I^6}{V_6}) \end{aligned} \tag{30}$$

where l_I is the string length of type I. The crucial difference compared to the heterotic string is the additional factor of $(\frac{l_I^6}{V_6})$ which is absent in eq. (5).[34,35] Thus the type I string scale $M_I \sim l_I^{-1}$ is not tied to M_{Pl} and can be freely adjusted if $(\frac{l_I^6}{V_6})$ is appropriately tuned. Conversely, it is now possible to choose $V^{-1/6} \sim M_{\text{GUT}}$ and adjust l_I to give the correct M_{Pl}.

In fact, it is also possible to consider $l_I^{-1} = \mathcal{O}(M_{\text{weak}})$ and not be in apparent conflict with any experimental results.[35,36] This possibility has recently received some attention and is currently investigated.

[n] As we will see shortly both options are available in the non-perturbative heterotic string.

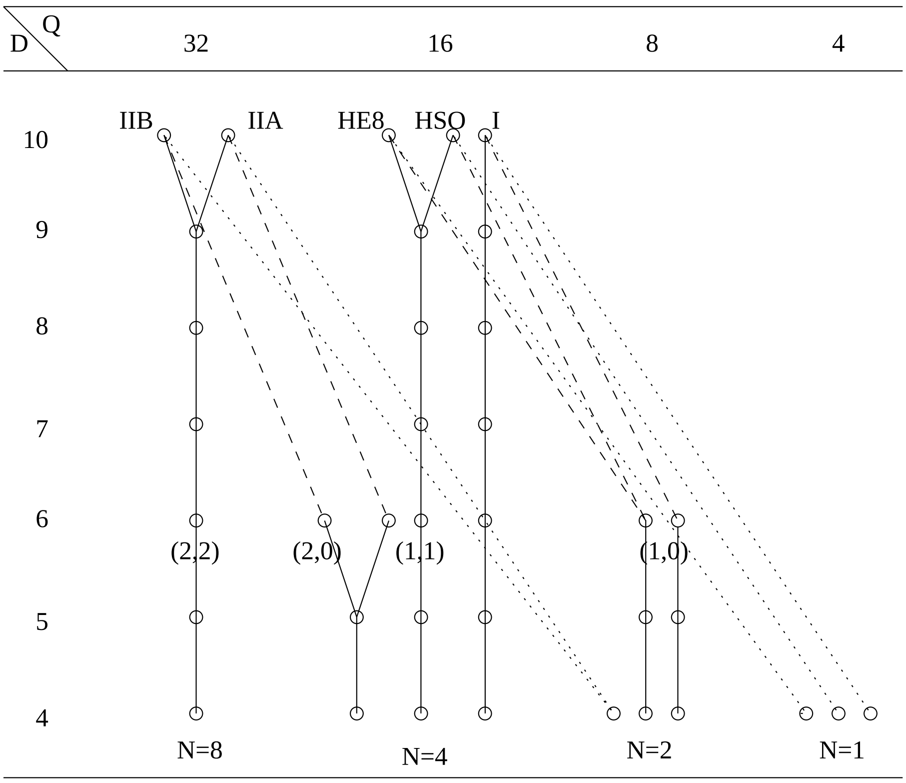

Figure 5: Calabi-Yau compactifications of the 10-dimensional string theories. The solid line $(-)$ denotes toroidal compactification, the dashed line $(--)$ denotes $K3$ compactifications and the dotted line $(\cdots)$ denotes Y_3 compactifications. Whenever two compactifications (two lines) terminate in the same point, the two string theories are related by a perturbative duality. (A line crossing a circle is purely accidental and has no physical significance.)

3 Non-perturbative Developments

Since 1995 string theory has seen spectacular progress in that for the first time it has been possible to also control a subset of the interaction in the strong coupling regime. This is due to the observation that many (if not all) of the perturbatively distinct string theories are related when all quantum corrections are taken into account.[2] In particular it has been observed that often the strong coupling regime of one string theory can be mapped to the weak coupling regime of another, perturbatively different string theory. This situation is termed duality among string theories and it offers the compelling picture that the known perturbative string theories are merely different regions in the moduli space of one underlying theory termed ‘M-theory’.

The precise nature of the strong-coupling limit sensitively depends on the number of (Minkowskian) spacetime dimensions and the amount of supersymmetry. Supersymmetry has played a major role in the recent developments in two respects. First of all, it is difficult (and it has not been satisfactorily accomplished) to rigorously prove a string duality, since it necessitates a full non-perturbative formulation, which is not yet available. Nevertheless it has been possible to perform nontrivial checks of the conjectured dualities for quantities or couplings whose quantum corrections are under (some) control. It is a generic property of supersymmetry that it protects a subset of the couplings and implies a set of nonrenormalization theorems. The recent developments heavily rely on the fact that the mass (or tension) of BPS-multiplets is protected and that holomorphic couplings obey a nonrenormalization theorem. Thus, they can be computed in the perturbative regime of string theory and, under the assumption of unbroken supersymmetry, reliably extrapolated into the non-perturbative region. It is precisely for these BPS-states and holomorphic couplings that the conjectured dualities have been successfully verified.

Second of all, for a given spacetime dimension D and a given representation of supersymmetry there can exist perturbatively different string theories. For example, the heterotic SO(32) string in $D = 10$ and the type-I string in $D = 10$ share the same supersymmetry, but their interactions are different in perturbation theory. However, once non-perturbative corrections are taken into account, it is believed that the two theories are identical and merely different perturbative limits of the same underlying quantum theory. A similar phenomenon is encountered with other string theories in different dimensions and the moduli space of string theory is much smaller than was previously assumed.

In fig. 5 the five 10-dimensional string theories (IIA, IIB, I, heterotic $E_8 \times E_8$, heterotic $SO(32)$) together with their geometrical compactifications on tori T^n, K3-surfaces and Calabi-Yau threefolds are displayed. In fig. 6 their respective strong coupling limits are depicted.

We are particularly interested in the strong coupling limit of string theories with $D = 4$ and $N = 1$ supersymmetry. They are related to M-theory compactified on Calabi–Yau threefolds times an interval $\mathrm{M}/Y_3 \times S^1/Z_2$ or F-theory compactified on elliptic Calabi–Yau fourfolds F/Y_4. Let us discuss these two cases in turn.

3.1 M-theory

It turns out that not all strong-coupling limits are governed by a perturbatively different string theory. Instead it is possible that the strong-coupling limit of a given theory is something entirely new, not any of the other string theories.[37] The prime example of this situation is the strong-coupling limit of the type-IIA theory in $D = 10$. It has a Kaluza-Klein BPS spectrum with masses

$$M^{\mathrm{KK}} \sim \frac{|n|}{g_{\mathrm{s}}} \, , \tag{31}$$

where n is an arbitrary integer. These KK-states are not part of the perturbative type-IIA spectrum since they become heavy in the weak-coupling limit $g_{\mathrm{s}} \to 0$. However, in the strong-coupling limit $g_{\mathrm{s}} \to \infty$ they become light and can no longer be neglected in the effective theory. This infinite number of light states (which can be identified with D-particles of type-IIA string theory, or extremal black holes of IIA supergravity) signals that the theory effectively decompactifies where g_{s} is related to the radius R_{11} of a new (11-th) dimension[37]

$$R_{11} \sim l_{11}\, g_{\mathrm{s}}^{\frac{2}{3}} \, . \tag{32}$$

l_{11} is the characteristic length scale of the 11th dimension which is related to the 11-dimensional Planck scale via $\kappa_{11}^2 \sim l_{11}^9$. Supersymmetry is unbroken in this limit and hence the KK-states assemble in supermultiplets of the 11-dimensional supergravity. Since there is no string theory which has 11-dimensional supergravity as the low-energy limit, the strong-coupling limit of type-IIA string theory has to be a new theory, called M-theory, which cannot be a theory of (only) strings.[o]

[o]There exists a conjecture according to which the degrees of freedom of M-theory are captured in $\mathrm{U}(N)$ supersymmetric matrix models in the $N \to \infty$ limit.[38] These matrix models have been known for some time[39] and were also known to describe supermembranes[40] in the lightcone gauge.[41] The same quantum-mechanical models describe the short-distance dynamics of N D-particles, caused by the exchange of open strings.[42,6,43]

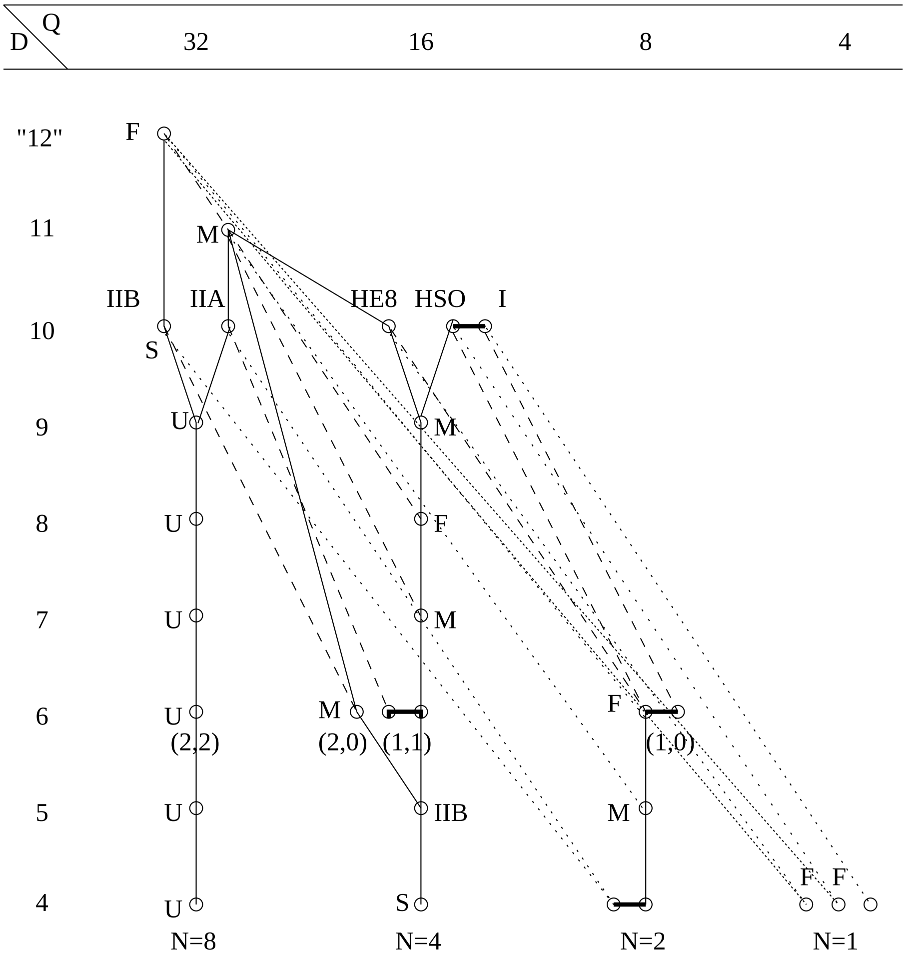

Figure 6: The distinct string theories and their strong-coupling limits. As in Fig. 5 the solid line ($-$) denotes toroidal compactifications, the dashed line ($--$) denotes $K3$ compactifications and the dotted line ($\cdots$) denotes Y_3 compactifications. The fine-dotted line (···) denotes Y_4 compactifications while the horizontal bar (▬) indicates a string-string duality. The theories marked with a 'U' ('S') have a U–duality (S–duality); the strong-coupling limit of the theories marked by 'M' ('F') are controlled by M-theory (F-theory). The two orbifold compactifications $\mathrm{M}/T^1/Z_2$ and $\mathrm{M}/T^5/Z_2$ are denoted by a solid line.

A second and maybe even more surprising result shows that also the strong-coupling limit of the heterotic $E_8 \times E_8$ string is captured by M-theory. In this case, 11-dimensional supergravity is not compactified on a circle but rather on a Z_2 orbifold of the circle.[44] In this case there is an E_8 gauge factor on each hyperplane at the end of the interval. Just as in the type-IIA case one has $R_{11} \sim g_s^{2/3}$ and thus weak coupling corresponds to small R_{11} and the two 10-dimensional hyperplanes sit close to each other; in the strong-coupling limit the two 10-dimensional hyperplanes move far apart (to the end of the world).

3.2 Compactifications of M-theory

Let us first record the relations of the 11-dimensional length scale l_{11} and the string scale l_s[37]

$$l_s \sim l_{11}\,(g_s^{(10)})^{-\frac{1}{3}}\ . \qquad (33)$$

As we already indicated in eq. (18) the string coupling of a compactified theory with D spacetime dimensions is related to the 10-dimensional string coupling by a volume factor

$$(g_s^{(D)})^{-2} = (g_s^{(10)})^{-2}\, V_{10-D}\, l_s^{D-10} \sim (g_s^{(10)})^{\frac{4-D}{3}}\, V_{10-D}\, l_{11}^{D-10}\ , \qquad (34)$$

where the last equation used eq. (33). In particular for $D = 4$ $g_s^{(10)}$ drops out of this relation and one has

$$(g_s^{(4)})^{-2} = V_6 l_{11}^{-6}\ . \qquad (35)$$

The regime which is governed by M-theory corresponds to taking the limit $R_{11} \to \infty$, $g_s^{(10)} \to \infty$, $l_s \to 0$ with l_{11} fixed. From eq. (35) we learn that in $D = 4$ one has

$$g_s^{(4)} \quad \text{fixed}\ , \qquad V_6 l_s^{-6} \to \infty\ . \qquad (36)$$

Thus the M-theory regime does not correspond to a strongly coupled string theory in $D = 4$ rather it governs the large volume limit at fixed $g_s^{(4)}$. It was precisely this limit which was not allowed in the perturbative regime where we derived $V_6 l_s^{-6} \to \infty$ with $g_s^{(10)}$ fixed and $g_s^{(4)} \to 0$.

3.3 Gauge Coupling Unification revisited

It is now possible to use V_6 as an independent scale since it is no longer constrained to be of order $O(l_s^6)$. [34] We have

$$\frac{g_{\rm GUT}^2}{4\pi} = \frac{(g_s^{(10)})^2}{4\pi}\frac{l_s^6}{V_6} = \frac{l_{11}^6}{4\pi V_6}. \qquad (37)$$

Similarly, the 4-dimensional Planck scale can also be expressed in terms of the M-theory scales via

$$\kappa_{(4)}^{-2} = \kappa_{(11)}^{-2} V_6 R_{11}^{-1} \ . \tag{38}$$

Thus, one can choose V_6 as the GUT scale $M_{\rm GUT}^2 \sim V_6^{-\frac{1}{3}}$, tune $l_{\rm s}$ to adjust $g_{\rm GUT}$ and tune $g_{\rm s}^{(10)}$ (or R_{11}) to adjust $M_{\rm Pl}$. This improved situation is related to the fact that in the M-theory regime 3 parameters V_6, l_{11}, R_{11} are at our disposal. In the perturbative regime we could only tune $M_{\rm s}$ and $g_{\rm s}$ while V_6 was constrained to roughly coincide with $M_{\rm s}$. So we lost one 'prediction' (which was slightly off) at the expense of having a new parameter.

The question arises to what extent V_6 can be arbitrarily large. This issue is discussed in the literature[34,45,35,36] and we only focus on one particular aspect here.[p] For Calabi-Yau compactifications the gauge group is necessarily $G = E_8 \times E_6$. The difference of the two gauge couplings is given by[47,25]

$$\begin{aligned} \Delta_{E_6} - \Delta_{E_8} &= \frac{12}{16\pi^2}\, F_1 \ , \\ \text{where} \quad F_1 &\equiv \frac{1}{2}\int \frac{d^2\tau}{\tau_2} {\rm Tr}'_{R,R} \left((-1)^{F-\bar F}\, F\bar F\, q^{L-\frac{3}{8}} \bar q^{\bar L-\frac{3}{8}} \right)_{(9,9)} \end{aligned} \tag{39}$$

In the large volume limit one has

$$V_6 l_{\rm s}^{-6} \to d_{ijk}\, {\rm Re}T^i\, {\rm Re}T^j\, {\rm Re}T^k \qquad \text{for} \quad T^i \to \infty \ , \tag{40}$$

where the T^i are the dimensionless Kähler-moduli of the Calabi-Yau threefold. In this limit the one-loop corrections of the gauge couplings depend linearly on T^i [48]

$$\Delta_a \to c_{ai}\, {\rm Re}T^i \ , \quad f_a^{(1)} \to c_{ai}\, T^i \quad \text{for} \quad T^i \to \infty \ , \tag{41}$$

where

$$c_{E_6 i} - c_{E_8 i} = \frac{1}{8\pi^2} \int_{Y_3} k_i \wedge c_2 > 0 \ . \tag{42}$$

The k_i are a basis of $H^{(1,1)}$ and c_2 is the second Chern class. As we discussed in the previous section $V_6 l_{\rm s}^{-6}$ large (or equivalently ${\rm Re}T^i$ large) with fixed $g_{\rm s}^{(4)}$ (or equivalently fixed S) is the regime governed by M-theory. In our notation this amounts to

$$f_a = S + f_a^{(1)}(T) \stackrel{{\rm Re}T\to\infty}{\to} S + c_{ai} T^i \ . \tag{43}$$

[p] Other phenomenological issues of the M-theory regime are discussed in the literature.[46]

Depending on the sign of c_{ai} one can have the following situations

$$\begin{array}{lll} c_{ai} > 0 & \rightarrow & g_a \rightarrow 0 \quad \text{for} \quad \mathrm{Re}T^i \rightarrow \infty \\ c_{ai} < 0 & \rightarrow & g_a \rightarrow \infty \quad \text{for} \quad \mathrm{Re}T^i \rightarrow \infty \\ c_{ai} = 0 & \rightarrow & g_a = \text{const.} \quad \text{for} \quad \mathrm{Re}T^i \rightarrow \infty . \end{array} \tag{44}$$

Thus there is a critical volume whenever

$$f_a = 0 \tag{45}$$

occurs. This corresponds to infinitely strong *gauge* coupling not strong string coupling. For Calabi–Yau threefolds we have

$$c_{E_6 i} - c_{E_8 i} = \frac{1}{8\pi^2} \int_{Y_3} k_i \wedge c_2 > 0 \tag{46}$$

which implies $f_{E_6} > f_{E_8}$ or in other words the E_8 is always more strongly coupled. In each string vacuum one thus has to check if $V_6^{-\frac{1}{3}} \sim M_{\mathrm{GUT}}^2$ is accessible and no singularity in the gauge couplings already occurs at smaller volume. Generically one finds a singularity in f_{E_8} before reaching M_{GUT}.

3.4 F-theory

The strong coupling limit of type-IIB theory in 10 spacetime dimensions is believed to be governed by type-IIB itself. This is accomplished by an exact $\mathrm{SL}(2,Z)$ quantum symmetry which is a generalization of a strong-weak coupling duality. This fact led Vafa to propose that the type-IIB string could be viewed as the toroidal compactification of a twelve-dimensional theory, called F-theory.[50] Apart from having a geometrical interpretation of the $\mathrm{SL}(2,Z)$ symmetry this proposal led to the construction of new, non-perturbative string vacua in lower spacetime dimensions. In order to preserve the $\mathrm{SL}(2,Z)$ quantum symmetry the compactification manifold cannot be arbitrary but has to be what is called an elliptic fibration. That is, the manifold is locally a fibre bundle with a two-torus T^2 over some base B but there are a finite number of singular points where the torus degenerates. As a consequence nontrivial closed loops on B can induce an $\mathrm{SL}(2,Z)$ transformation of the fibre. This implies that the dilaton is not constant on the compactification manifold, but can have $\mathrm{SL}(2,Z)$ monodromy.[51]. It is precisely this fact which results in nontrivial (non-perturbative) string vacua inaccessible in string perturbation theory.

F-theory can be compactified on elliptic Calabi–Yau manifolds and each of such compactifications is conjectured to capture the non-perturbative physics of an appropriate string vacuum. One finds:

- The IIB string in $D = 10$ can be viewed as F-theory compactified on T^2 with a frozen Kähler modulus.
- F-theory compactified on an elliptic $K3$ yields an 8-dimensional vacuum with 16 supercharges which is quantum equivalent to the heterotic string compactified on T^2.[50,52]
- F-theory compactified on an elliptic Calabi–Yau threefold has 8 unbroken supercharges and is quantum equivalent to the heterotic string compactified on $K3$.[50]
- Finally, the heterotic string compactified on a Calabi–Yau threefold Y_3 is quantum equivalent to F-theory compactified on an elliptic Calabi–Yau fourfold.[53] Calabi-Yau fourfolds are Calabi-Yau manifolds of complex dimension four and holonomy group SU(4).

3.5 Phenomenological Aspects of F-theory in $D = 4$

Since compactification of F-theory on Calabi–Yau fourfolds is not yet well understood also the phenomenological investigations are only at the beginning. Let us point out a few features which seem to emerge from the study of non-perturbative heterotic string vacua.

- One finds that generically new massless gauge bosons appear which have no tree level coupling to the dilaton or in other words they have $k_{(\mathrm{NP})} = 0$.[54] The total gauge group is thus a sum

$$G = G_{(\mathrm{P})} + G_{(\mathrm{NP})}, \tag{47}$$

  where $G_{(\mathrm{NP})}$ denotes the non-perturbative factors. The gauge couplings for these factors are governed by some moduli other than the dilaton, ie. $g_{(\mathrm{NP})}^{-2} = \mathrm{Re}T + \ldots$. Thus g_a^{-2} is no longer universal at tree level but only the perturbative gauge factors of $G_{(\mathrm{P})}$ are universal. Furthermore, $\mathrm{rank}(G)$ is no longer bounded by (14) but can be almost arbitrarily big. The current record gauge group has $\mathrm{rank}(G) = 302896$ with 251 simple factors and $SO(7232)$ being the biggest of them.[55]

  Thus, there seems to be no problem in such vacua to have a hidden sector with $b \simeq 400$; for example $SO\,(140)$ could nicely do the job. Furthermore, one can have many gauge factors participating in the stabilization of the dilaton and hierarchical supersymmetry breaking. Now it is easy to have a different sector which breaks supersymmetry and another sector which stabilizes the dilaton and the moduli.[56]

However, now also the gauge couplings of the Standard Model are no longer necessarily universal and one needs a mechanism to ensure it. This might point towards a GUT-unification within string theory.

- A different issue concerns non-perturbative corrections to the superpotential. Due to the non-renormalization theorem one only has $W = W^{(0)} + W^{(\mathrm{NP})}$ but within the perturbative heterotic string the corrections to $W^{(\mathrm{NP})}$ have to be of the form $W^{(\mathrm{NP})} \sim e^{-f(S,T)}$. However, an asymptotically free gauge factor of $G_{(\mathrm{NP})}$ generates terms of the form $W^{(\mathrm{NP})} \sim e^{-\gamma f(T)}$ with no dilaton dependence. In this case $W^{(\mathrm{NP})}$ is indistinguishable from $W^{(0)}$ or in other words non-perturbative effects can 'compete' with perturbative effects due to their unusual dilaton dependence.

  This fact has been used to 'explain' the conifold singularities of the tree level superpotential.[57,58] In all Calabi–Yau compactifications one has

$$W^{(0)} = Y_{ijk} Q^i Q^j Q^k + \dots \,, \tag{48}$$

  where Q^i are charged matter multiplets and the Y_{ijk} are their moduli dependent Yukawa couplings. The moduli dependence is always singular on subspaces of the moduli space, ie.

$$Y_{ijk} \sim \frac{1}{z(T)} \,, \tag{49}$$

  where $z(T)$ has a zero at a conifold singularity. Singularities of holomorphic, not renormalized quantities should have a physical mechanism behind them. It has been suggested that a non-perturbative gauge group with a quantum modified moduli space confines and generates the conifold singularity.[57,58]

Dedication

During the school I learned about the death of my colleague and friend Michael Lichtfeldt. Improving the education of physics students was one of his main goals which he followed with utmost dedication and enthusiasm. I hope these lectures are a contribution in his spirit. I feel deep sorrow that I am no longer able to discuss issues in the education of physicists with him.

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