Spontaneous $N = 2 \to N = 1$ Supersymmetry Breaking and the Super-Higgs Effect in Supergravity

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To My Parents and My Family

Contents

Chapter 1

Introduction

1.1 Why Supersymmetry ?

Today the Standard Model has become a successful theory describing physics at subnuclear scales which has been tested by many collider experiments to a high level of accuracy [1, 2]. The Higgs bosons predicted by the Standard Model has not been directly observed by todays experiments.

Despite its great success, there still remains several serious problems, such as the arbitrariness of the particle spectrum and gauge group, the large number of free parameters, and maybe the most severe one is the inability to turn on gravity described by the general theory of relativity. These suggest that the Standard Model is not the final answer of nature but rather an effective description valid up to the electroweak scale of order $\mathcal{O}(100\,GeV)$. Thus, the Standard Model has to be extended.

Various efforts have been made over the last two decades to go beyond the standard model and correspondingly, solve the above problems. The most prominent one and still promising until now is the *supersymmetric extension* of the Standard Model which is reviewed, for example, in [3].[1] It has $N = 1$ global supersymmetry because extended supersymmetries ($N \geq 2$) cannot accommodate the chiral structure of the Standard Model. As supersymmetry is not observed in nature, it must be broken at low energy if it is to play any role at all. This leads to a mass split between bosonic and fermionic partners of the supersymmetry breaking scale. The determination of this scale should explain why the supersymmetric partners of the Standard Model particles could be heavy enough to escape detection in accelerator experiments around the electroweak scale $\sim 100\,GeV$ so far. One of interesting aspects of this theory is that all three gauge couplings unify at a scale $\sim 10^{16}\,GeV$, see *e.g.* [4].

There are various ways to break supersymmetry, however only two of them are of phenomenological interest, namely, supersymmetry has to be either spontaneously or softly broken. Since the supersymmetric extension of the Standard Model only has a global supersymmetry, spontaneous breaking poses a new problem, namely the presence of a massless fermion called *Goldstone fermion*. This is a consequence of the supersymmetric Goldstone theorem, see *e.g.* [5]. So if the global supersymmetry is spontaneously broken, the supersymmetric extension of the standard model would be ruled out. Thus, the only way out is a soft breaking of global supersymmetry. This can be done by adding non-supersymmetric terms to the theory which do not generate

[1] See next section for a discussion of supersymmetry.

any quadratic divergences [6]. In addition, there is an alternative way to motivate the relevance of softly broken supersymmetric theories. Ultimately, one has to couple the supersymmetric standard model to gravity. This in turn requires the promotion of global supersymmetry to a local supersymmetry which is called *supergravity*.[2] Furthermore, spontaneous local supersymmetry breaking in the limit $M_{Planck} \to \infty$ but with the gravitino mass remains fixed, yields the soft supersymmetry breaking terms [7]. This motivates many theorists today to study supergravity as a candidate beyond the Standard Model.

Let us turn to extended supersymmetric theories. Since these theories cannot accommodate the chiral structure, it seems that the extended supersymmetries are not phenomenologically interesting. Furthermore, *the no go theorem* which states that any supersymmetric theory with N supersymmetries either all or none of them are spontaneously broken, demands that extended supersymmetric theories must be broken at the same supersymmetry breaking scale which is phenomenologically impossible. However, in the last decade few examples have been appeared which show that these theories can spontaneously be broken to $N = 1$ [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19].[3] These examples indicate that the no go theorem can be avoided and in addition, yield a hope for phenomenological studies. Still the resulting $N = 1$ theories cannot accommodate the chiral structure because their parental theories are extended supersymmetric theories. Nevertheless, extended supersymmetries remain an interesting study, in particular to see how one can evade the no go theorem and study the general aspects of their breaking. In this thesis, we address some general aspects of spontaneous breaking $N = 2 \to N = 1$ in supergravity as an example of spontaneous breaking of extended supersymmetric theories.

1.2 What is Supersymmetry ?

In this section we briefly consider the structure of rigid (global) supersymmetry in four dimensional Minkowski space. The interested reader is referred to the literature for further details [20, 21, 22, 5].

By definition, supersymmetry transforms bosons into fermions and vice versa. In order to realize of such transformations one introduces *supersymmetry generators* (or supercharges) Q, acting as:

$$Q|\text{boson}\rangle = |\text{fermion}\rangle \ , \quad Q|\text{fermion}\rangle = |\text{boson}\rangle \ , \tag{1.1}$$

where we have split up the Hilbert space into bosonic states $|\text{boson}\rangle$, and fermionic states $|\text{fermion}\rangle$. Such a boson-fermion symmetry has far-reaching consequences. First it affects the statistic of the transformed state and changes it by a half-unit. Thus, the supersymmetry generators themselves have spin one-half and form spinor representations of the Lorentz group, contrary to the usual generators of symmetry transformation which have integer spin. The second important implication of such transformation is that every particle has a *superpartner*. The notation for the bosonic superpartners of the known fermions are labeled by the prefix 's-' (*e.g.* slepton, squark), whereas the fermionic superpartners of the known bosons are denoted by the suffix '-ino' (*e.g.* gaugino, gravitino). The members of a supersymmetric theory are arranged in a so

[2]See section 1.3 for a discussion of supergravity

[3]See also section 1.4.

called *supermultiplet* which has the same number of bosonic and fermionic degrees of freedom.

Such supercharges Q which form spinor representations of the Lorentz group satisfy an *anticommutation* relation [23]

$$\{A, B\} \equiv AB + BA \quad , \tag{1.2}$$

and moreover, do not contradict the theorem of Coleman and Mandula [24], which states that for every non-trivial relativistic field theory , under some very mild assumption all the symmetries of the S-matrix commute with the generators of the Poincaré group. This is because the essential assumption that they make, is that the symmetry generators form a closed algebra under *commutation* relations, thus restricting themselves to Lie groups of symmetry transformation.

It was proven in [25] that a set of commutation and anticommutation relations between Poincaré generators and supercharges (usually called *Poincaré superalgebra*) is the only graded Lie algebra of symmetries of the S-matrix consistent with relativistic quantum field theory. Furthermore, the Poincaré superalgebra together with other generators of the Lie group G which is the symmetry of the S-matrix form an algebra which admits a $\mathbb{Z}_2$ graded structure. Such an algebra is usually called *supersymmetry algebra*. To see the meaning of this graded structure, let us first call the generators which satisfy the commutation relation (Lie algebra) even and the supercharges Q to be odd. Then these even and odd generators must satisfy the rules:

$$\begin{aligned} [\text{even}, \text{even}] &= \text{even} \quad , \\ \{\text{odd}, \text{odd}\} &= \text{even} \quad , \\ [\text{even}, \text{odd}] &= \text{odd} \quad . \end{aligned} \tag{1.3}$$

To make it clear, let us denote P_a the four momentum and J_{ab} the Lorentz group generators respectively, with $a = 0, ..., 3$ and in addition there are some supercharges $Q^{\widehat{A}}$, where $\widehat{A} = 1, ..., N$. Therefore the expression of the supersymmetry algebra which has the $\mathbb{Z}_2$ graded structure (1.3) is the following:

$$\begin{aligned} [J_{ab}, J_{cd}] &= -\mathrm{i}\big(\eta_{bc}\, J_{ad} + \eta_{ad}\, J_{bc} - \eta_{bd}\, J_{ac} - \eta_{ac}\, J_{bd}\big) \quad , \\ [J_{ab}, P_c] &= \mathrm{i}\big(\eta_{ac}\, P_b - \eta_{bc}\, P_a\big) \quad , \\ \{Q^{\widehat{A}}, Q^{\widehat{B}}\} &= -2\,(\gamma C)^a P_a\, \delta^{\widehat{A}\widehat{B}} - 4\, C\, Z^{\widehat{A}\widehat{B}} \quad , \\ [P_a, P_b] &= [P_a, Q^{\widehat{A}}] = 0 \quad , \\ [J_{ab}, Q^{\widehat{A}}] &= -\frac{\mathrm{i}}{2}\gamma_{ab}\, Q^{\widehat{A}} \quad , \end{aligned} \tag{1.4}$$

where $Q^{\widehat{A}}$ are four spinors, C is a charge conjugation matrix defined by

$$C\, \gamma_a\, C^{-1} = -\gamma_a^{\mathrm{T}} \quad , \tag{1.5}$$

with the superscript T stands for the transpose, γ_a are the Dirac matrices, $2\gamma_{ab} \equiv [\gamma_a, \gamma_b]$, and the metric $2\eta_{ab} = \{\gamma_a, \gamma_b\} = 2\,\mathrm{diag}(+1, -1 - 1, -1)$.[4] In (1.4) we have also introduced the antisymmetric quantities $Z^{\widehat{A}\widehat{B}}$ called central charges and they commute with all the generators defined above. Due to their antisymmetry, it is easy to see that

[4]See also appendix A.

the central charges are trivially zero if there is only one supercharge. This minimal supersymmetry in four dimensions is called $N = 1$ supersymmetry. On the other hand if there are more than one supercharge present in a theory, then it is called *extended* supersymmetry.

Furthermore, since the mass squared operator defined as

$$M^2 = P^a P_a \quad , \tag{1.6}$$

commutes with all generators of the supersymmetry algebra (1.4), then the mass squared is a supersymmetric invariant.[5] Hence in Minkowski space all the particles within the same supermultiplet have to be degenerate in mass. There are two types of irreducible representation: the massive and massless representations. As we are going to see they have a rather different structure and need a separate study. In addition, we restrict ourselves in this section to study the massive representation without the central charges, *i.e.* $Z^{\widehat{A}\widehat{B}} = 0$, while the massless representation satisfies trivially this requirement.

Before proceeding further to the massless and massive representations of the supersymmetry algebra (1.4), let us first use the fact that a four spinor is reducible which means that the supercharges $Q^{\widehat{A}}$ can be decomposed into two Weyl spinors

$$Q^{\widehat{A}}_{\pm} = \frac{1}{2}(1 \pm \gamma_5) Q^{\widehat{A}} \quad . \tag{1.7}$$

Then it follows that the anticommutation relation in (1.4) reduces into

$$\begin{aligned} \{Q^{\widehat{A}}_{+}, \bar{Q}^{\widehat{B}}_{-}\} &= 2\,\sigma^a P_a\, \delta^{\widehat{A}\widehat{B}} \quad , \\ \{Q^{\widehat{A}}_{+}, \bar{Q}^{\widehat{B}}_{+}\} &= \{Q^{\widehat{A}}_{-}, \bar{Q}^{\widehat{B}}_{-}\} = 0 \quad , \end{aligned} \tag{1.8}$$

where $\bar{Q}^{\widehat{A}} \equiv Q^{\widehat{A}\dagger}\gamma_0 = Q^{\widehat{A}\mathrm{T}}\, C$ and we have chosen the following basis for γ-matrix:

$$\gamma^a = \begin{pmatrix} 0 & \sigma^a \\ \bar{\sigma}^a & 0 \end{pmatrix} , \quad \gamma_5 = \begin{pmatrix} \mathbb{1} & 0 \\ 0 & -\mathbb{1} \end{pmatrix} , \quad C = \begin{pmatrix} -\mathrm{i}\sigma^2 & 0 \\ 0 & \mathrm{i}\sigma^2 \end{pmatrix} , \tag{1.9}$$

with

$$\sigma^a = (\mathbb{1}, \sigma^x) , \quad \bar{\sigma}^a = (\mathbb{1}, -\sigma^x) , \tag{1.10}$$

where $x = 1, 2, 3$ and σ^x are the standard Pauli matrices.[6]

We shall first analyze the massless case, $P^a P_a = 0$. Using Lorentz boost we can always go to the frame where $P^a = m(1, 0, 0, 1)$. The commutation relations (1.8) show that the only non-zero supercharges are $Q^{\widehat{A}}_{+2}$ and its conjugate $\bar{Q}^{\widehat{A}}_{-2}$. Furthermore, $Q^{\widehat{A}}_{+2}(Q^{\widehat{A}}_{-2})$ raise (lower) the spin of a state by a half-unit. Thus, the particle spectrum in a multiplet can be constructed by acting with $Q^{\widehat{A}}_{+2}$'s on the vacuum states $|\lambda\rangle$ where λ denotes the helicity. Below, we list some examples for $N = 1, 2, 4$.

$$N = 1 \ : \quad \left\{ \begin{array}{cc} 2|0\rangle \, , & |\pm 1/2\rangle \\ |\pm 1/2\rangle \, , & |\pm 1\rangle \end{array} \right.$$

[5] By definition, an element which commutes with all elements of a Lie algebra is called *Casimir element*. In our case, the mass squared operator M^2 and the central charges $Z^{\widehat{A}\widehat{B}}$ are indeed the Casimir elements of the supersymmetry algebra (1.4).

[6] See appendix A.

$$N = 2 \ : \quad \begin{cases} |\mp 1/2\rangle \ , & 4|0\rangle \ , \quad |\pm 1/2\rangle \\ 2|0\rangle \ , & 2|\pm 1/2\rangle \ , \quad |\pm 1\rangle \end{cases} \tag{1.11}$$

$$N = 4 \ : \quad 6|0\rangle \ , \quad 4|\pm 1/2\rangle \ , \quad |\pm 1\rangle$$

Thus, for $N = 1$, the representation contains a Majorana spinor and two real scalars (a complex scalar) called *scalar multiplet*, or a massless vector and a Majorana spinor called *vector multiplet*. For the $N = 2$ hypermultiplet we have two Majorana spinors with four real scalars (two complex scalars). This representation has the same particle content as two copies of the $N = 1$ scalar multiplet. The $N = 2$ vector multiplet contains a massless vector, two Majorana spinors and a complex scalar. Note that this multiplet contains two $N = 1$ multiplets, namely a vector and a scalar multiplet in the adjoint representation of the gauge group.[7] Finally, the $N = 4$ massless multiplet it accommodates a massless vector, four Majorana fermions, and six real scalars. In addition, no scalar multiplet is possible in this case.

Now, we can then study the massive case, $P^a P_a = m^2$. We can always go to the rest frame where $P^a = (m, 0, 0, 0)$. Let us first consider the $N = 1$ case. The particle spectrum in a multiplet is built up by acting with $Q_+^{\widehat{A}}$'s on the vacuum states $|\Omega_j\rangle$ which has spin j, with $j \geq 0$. For $j = 0$, the ground state is a spin singlet (a singlet vacuum), while for $j > 0$, it belongs to a $(2j + 1)$-dimensional representation of the group $SU(2)$. This leads to a multiplet with spins

$$(j) \oplus (j + 1/2) \oplus (j - 1/2) \oplus (j) \quad . \tag{1.12}$$

In the following we give some examples for $N = 1, 2$.[8]

$$\begin{gathered} N = 1 \ : \quad \begin{cases} j = 0 \ ; & 2|0\rangle \ , \quad |1/2\rangle \\ j = 1/2 \ ; & |0\rangle \ , \quad 2|1/2\rangle \ , \quad |1\rangle \\ j = 1 \ ; & |1/2\rangle \ , \quad 2|1\rangle \ , \quad |3/2\rangle \end{cases} \\ N = 2 \ : \quad \begin{cases} j = 0 \ ; & 5|0\rangle \ , \quad 4|1/2\rangle \quad |1\rangle \\ j = 1/2 \ ; & 4|0\rangle \ , \quad 6|1/2\rangle \ , \quad 4|1\rangle \quad |3/2\rangle \\ j = 1 \ ; & |0\rangle \ , \quad 4|1/2\rangle \ , \quad 6|1\rangle \ , \quad |3/2\rangle \quad |2\rangle \end{cases} \end{gathered} \tag{1.13}$$

We see that in the case $N = 1, j = 0$, the massive multiplet has the same particle content as in massless multiplet. For $j = 1/2$, the multiplet contains a real scalar field, two Majorana spinors, and a gauge bosons. Of particular interest to us is the $j = 1$ multiplet which consist of a Majorana spinor, two gauge fields and a spin-$\frac{3}{2}$ field called Rarita-Schwinger field. This multiplet is called $N = 1$ *massive spin-$\frac{3}{2}$ multiplet* and futhermore, as we will see in section 3.2 and 4.1, plays an important role in studying spontaneous $N = 2 \to N = 1$ supersymmetry breaking.

In the $N = 2$ case, no massive hypermultiplet exists and moreover, the situation becomes complicated as j increased. For $j \geq 1$, the theories also involve a massive

[7] Thus, when extending the theory from $N = 1$ to $N = 2$, the additional supersymmetry requires the introduction of two more $N = 1$ scalar multiplets which are called mirror gauge and mirror matter [26].

[8] In the massive representation, the anticommutation (1.8) may also be cast in a form which exhibits the $SU(2) \times USp(2N)$ symmetry. This symmetry group is useful because states of a given spin transform irreducibly under $USp(2N)$, see [20, 22].

spin-2 boson (called graviton) which has been ruled out by today's experiments. Thus, the $j \geq 1$ multiplets are unphysical.

To summarize, we have discussed the supersymmetry algebra in Minkowski space and constructed its massless and massive representations. In all cases, these finite collections of particles have the same number of bosonic and fermionic degrees of freedom. In addition, the members of a massive supermultiplet have to be degenerate in mass since the squared mass operator $P^a P_a$ is supersymmetric Casimir operator.

Next, we discuss the possibility of supersymmetry representation on the fields. For that purpose, we consider a simplest case, namely the $N = 1$ massless scalar multiplet which is often called *the chiral multiplet.* The free Lagrangian of this chiral multiplet is given by

$$L = \frac{1}{2}\partial_a z\,\partial^a \bar{z} + \frac{\mathrm{i}}{2}\bar{\zeta}_+\gamma^a\partial_a\zeta_- \quad , \tag{1.14}$$

where $(z, \bar{z}, \zeta)$ are a complex scalar, its complex conjugate, and a single Majorana fermion whose supersymmetry transformations leave invariant (1.14) can be written down in

$$\begin{aligned} \delta z &= 2\bar{\epsilon}_+\,\zeta_+ \ , & \delta\bar{z} &= 2\mathrm{i}\bar{\epsilon}_-\,\zeta_- \ , \\ \delta\zeta_+ &= -\mathrm{i}\gamma^a\partial_a z\,\epsilon_- \ , & \delta\zeta_- &= -\mathrm{i}\gamma^a\partial_a\bar{z}\,\epsilon_+ \ , \end{aligned} \tag{1.15}$$

where $\zeta_\pm = \frac{1}{2}(\mathbb{1}\pm\gamma_5)\zeta$. Here ϵ is the parameter of supersymmetry transformation which is constant in this case. For this reason supersymmetry with constant transformation parameters is usually called *rigid (global)* supersymmetry.[9] The above free model is called the massless Wess-Zumino multiplet [27] and can also be generalized in a supersymmetric way to include a mass and interaction terms. Furhermore, looking at the scalar kinetic term, one can check that the scalars in rigid $N = 1$ theory form a complex manifold which is *Kähler* [28].[10] This feature plays a prominent role in studying the supersymmetric σ-model for general scalar interaction.

To see whether the supersymmetry transformations (1.15) form a representation of the anticommutation relation in (1.4), we have to perform two subsequent supersymmetry variations:

$$[\delta_1, \delta_2]z = -2\mathrm{i}\,\bar{\epsilon}_{+2}\gamma^a\epsilon_{-1}\,\partial_a z \quad , \tag{1.16}$$

and analogously for $\bar{z}$.[11] For ζ, however, we also find a term proportional to the fermionic field equation. So the multiplet (z, ζ) forms a representation of the superalgebra, only when impose the fermionic field equation. In general, we speak of an *on-shell* multiplet when it forms a representation of the supersymmetry algebra, provided the equations of motion hold. If we add a new complex scalar F and change the transformation rules for the multiplet (z, ζ, F) as follows:

$$\begin{aligned} \delta z &= \bar{\epsilon}_+\zeta_+ \quad , \\ \delta\zeta_+ &= -\mathrm{i}\gamma^a\partial_a z\,\epsilon_- + F\epsilon_+ \quad , \\ \delta F &= -\mathrm{i}\bar{\epsilon}_-\gamma^a\partial_a\zeta_+ \quad , \end{aligned} \tag{1.17}$$

[9] In other words, ϵ is a Killing spinor which is the solution of the Killing spinor equation $\partial_\mu\epsilon = 0$. However, in anti-de Sitter space a Killing spinor is no longer constant spinor, see appendix B.

[10] See also [29] for a review.

[11] Note that the supersymmetry transformation δ can be expressed in term of supercharge Q via $\delta = \bar{\epsilon}Q$. So the change from anticommutator to commutator is due to the exchange of fermions.

then the supersymmetry algebra closes without the need for equations of motion and we speak of an *off-shell* representation. An off-shell Lagrangian is found by adding to (1.14) the term $\frac{1}{2}F\bar{F}$. Note that the field equations of F and $\bar{F}$ are algebraic. When resubstituted into the action, we retrieve (1.14) for the component of the on-shell multiplet. Such non-propagating fields that are needed to close the supersymmetry algebra without reference of the action, are called *auxiliary* fields.[12]

Finally, we want to mention that the supersymmetric field theories also exist for all multiplets (1.11) and (1.13) [30]. Thus, we can discuss them in the similar way as the $N = 1$ chiral theory described above. The generalization of such theories with non-constant supersymmetry transformation parameters is the main subject of the next section.

1.3 Supergravity

Now, we turn to multiplets which contain a spin-2 boson called graviton and discuss their corresponding field theories. It turns out that in order to have consistent supersymmetric field theories, the supersymmetry transformation parameters should not be constant but rather depend explicitly on the spacetime coordinates.[13] Such theories are manifestly local supersymmetric and referred to as *supergravity* theories.

As was shown in the previous sections, by its very nature supersymmetry implies the presence of both bosonic and fermionic fields carrying integer and half-integer, respectively. The graviton, the particle described by the spacetime metric has spin 2. The gravitino, the particle associated with fermionic gauge field of supersymmetry, has spin 3/2. Massless spin-3/2 particles are described by the Rarita-Schwinger action. In N-extended supergravity the number of gravitino must be equal to N. All other particles carry spin less than 3/2. They are described by vector, spinor, and spinless (scalar) fields.

For realistic supergravity in four dimensions, this number of supercharges cannot exceed 8. Hence, the possibility of supergravity in four dimensions is the interval $1 \leq N \leq 8$. In the table 1.1 below we list some massless examples which are gravitational multiplet together their vector- and hypermultiplet for $N = 1, 2, 3, 4$. For $N > 4$, no matter multiplets are possible, and all particles, including the spin-1 gauge fields, belong to the gravitational multiplet.

[12]In fact, the addition of auxiliary fields is related to the counting of bosonic and fermionic degrees of freedom in an off-shell supersymmetry representation. For the case at hand, a real scalar still has one degree of freedom, while a Majorana spinor has four instead of two degrees of freedom. Thus we have four bosonic and fermionic degrees of freedom.

[13]In addition, these parameters are not Killing spinors.

N	gravitational multiplet	vector multiplet	matter multiplet
1	$[(2),\, (\frac{3}{2})\,]$	$[(1),\, (\frac{1}{2})\,]$	$[(\frac{1}{2}),\, 2(0)\,]$
2	$[(2),\, 2(\frac{3}{2}),\, (1)\,]$	$[(1),\, 2(\frac{1}{2}),\, 2(0)\,]$	$[2(\frac{1}{2}),\, 4(0)\,]$
3	$[(2),\, 3(\frac{3}{2}),\, 3(1),\, (\frac{1}{2})\,]$	$[(1),\, 4(\frac{1}{2}),\, 6(0)\,]$	none
4	$[(2),\, 4(\frac{3}{2}),\, 6(1),\, 4(\frac{1}{2}),\, 2(0)\,]$	$[(1),\, 4(\frac{1}{2}),\, 6(0)\,]$	none

Table 1.1: Some examples of local supersymmetric theories for $N = 1, 2, 3, 4$

In $N = 1$ supergravity, the gravitational multiplet is composed of a graviton and a gravitino. This multiplet can also be coupled to the $N = 1$ vector- and chiral multiplets. In general $N = 1$ supergravity, local supersymmetry demands the scalar fields forming a Kähler manifold, a manifold with a closed fundamental two-form, with an additional line bundle on it. Such manifold is called a Hodge-Kähler manifold.[14] In addition, the scalar potential is in general is not positive definite.

Beautiful structures emerge for the geometry of the scalar fields in $N = 2$ theories. The scalar manifold is the product of two different manifold that belong to special class of Kähler and quaternionic Kähler geometries.[15] The word 'special' for the Kähler geometries indicates that their Kähler potential is no longer an arbitrary real function but determined in terms of a holomorphic prepotential. It also has an extra symplectic structure related to the duality transformation of the vectors. In general, these special Kähler manifolds are not homogeneous spaces. Yet there is a subclass of homogeneous special manifolds. These manifolds have been classified for homogeneous noncompact symmetric spaces in [31] which can be expressed in terms of coset manifolds, see appendix C.[16]

The second geometries are the quaternionic Kähler manifolds. All these manifolds are Einstein spaces of constant (non-zero) curvature. Furthermore, $N = 2$ supergravity requires negatively curved (and typically noncompact) manifolds [33, 34]. The only known homogeneous noncompact cases are the symmetric spaces which have been studied in mathematical literature [35, 36, 37].[17] All of them are given in appendix C.

Unlike $N = 1$ theory, the scalar potential in N-extended supergravity theories is caused by gauging the supersymmetric σ-model with respect to Killing vectors which generate isometries on the scalar manifold. This general extended theory is referred to as *gauged N-extended supergravity theory.* Of particular interest for us is the $N = 2$ theory.

[14]See appendix D and references given there.

[15]See next section and references given there.

[16]The homogeneous but nonsymmetric Kähler geometries which arise as manifolds of the scalars in vector multiplet have also been determined in [32].

[17]The homogeneous nonsymmetric quaternionic Kähler spaces has also been found in [36].

1.4 Partial Supersymmetry Breaking

In this section, we discuss the possibility of breaking supersymmetry. First of all, we recall the no-go theorem which was first discussed in [38]. Let us consider the rigid N-extended supersymmetry algebra, namely the anticommutator (1.8) in the rest frame

$$\{Q_{+}^{\widehat{A}}, \bar{Q}_{-}^{\widehat{B}}\} = 2H\,\delta^{\widehat{A}\widehat{B}} \quad , \tag{1.18}$$

and therefore for any fixed $\widehat{A}$,

$$||Q_{+}^{\widehat{A}}|0\rangle||^2 = 2\langle 0|H|0\rangle \quad , \tag{1.19}$$

where H is the Hamiltonian and $|0\rangle$ is a vacuum state. Now, if there exists a $\widehat{A}_0$ such that the vacuum is not annihilated by the corresponding supercharges, or in other words supersymmetry is spontaneously broken, then the left hand side (1.19) is strictly positive and so is the vacuum expectation value of H. Then, for any other supersymmetry generator $Q^{\widehat{A}}$, the left hand side of (1.19) is also strictly positive and $Q^{\widehat{A}}$ is spontaneusly broken as well. So, the no-go theorem states that in a N-extended global supersymmetric theory, either all or no supercharges are spontaneusly broken. From this line of reasoning, one might think that partial breaking is impossible.

Fortunately, this no-go theorem has two significant loopholes. The first is that, technically-speaking, spontaneously-broken supercharges do not exist. Indeed, in a spontaneously broken theory, one only has the right to consider the algebra of the *currents*. For the case at hand, the current algebra can be modified as follows [8],

$$\int d^3\vec{x}_1 \{\bar{J}_{0-}^{\widehat{A}}(x_1), J_{\mu+}^{\widehat{B}}(x)\} = 2\sigma^{\nu}\, T_{\nu\mu}(x)\,\delta^{\widehat{A}\widehat{B}} + \sigma_{\mu}\, C^{\widehat{A}\widehat{B}} \quad , \tag{1.20}$$

where $J_{\mu\pm}^{\widehat{B}}(x)$ are the supercurrents, $T_{\nu\mu}(x)$ is the stress energy tensor, and $C^{\widehat{A}\widehat{B}}$ are constants. The addition of this constant term is valid since it is only a central extension, and thus the Jacobi identity is still satisfied. For unbroken supersymmetry $C^{\widehat{A}\widehat{B}} = 0$ because integrating the left hand side of (1.20) over three-space varibles $\vec{x}$ would prevent the infinite contribution and then, one retrieves the supersymmetry algebra (1.18). However, for the broken case this is not the case. This discrepancy prevents the current algebra (1.20) from being integrated into the supersymmetry algebry (1.18), so the no-go theorem is evaded. There are, by now, few examples of partial supersymmetry breaking which exploit this first loophole. This phenomena first emerged in string theory, which is manifestly global supersymmetry discussed in [8,9]. Several years later, this feature also appeared in four dimensional global supersymmetric theories [10,11,12].

The second loophole is that the Hilbert space we are dealing with has a positive definite metric, which fails in the case of local supersymmetry.[18] Therefore, one cannot conclude that if the right hand side of (1.19) is zero, then the supercharges annihilate the vacuum state as it was shown in [13, 14]. There are also only few examples which take advantage of this second loophole. For example, in Minkowski (flat) backgrounds it was studied in [13,16,17,18,19] whereas in curved backgrounds, namely anti-de Sitter backgrounds, only two examples are known [14, 15]. These facts leave many important questions open. First and foremost, one would like to know how the general story of partial supersymmetry breaking in supergravity theories looks like without considering any specific model.

[18] For example, in canonically quantized supergravity the metric of the Hilbert space is indefinite, see [39] and references therein.

1.5 Topic and Organization of the Thesis

After this long discussion about supersymmetry and its partial breaking, we come finally to the main concern of the thesis. Here, we want to mention that the examples in supergravity use the second loophole to facilitate partial breaking $N = 2 \to N = 1$. However, only four of them which was studied in [16, 17, 18, 19] are interesting for us to consider. These models have Minkowskian ground states and moreover, are discussed only for a particular class of the scalar manifold of the $N = 2$ supergravity theory. Thus, it is of interest to uncover the general story of spontaneous $N = 2 \to N = 1$ breaking. There are various questions one can address which can be roughly be structured as follows [40]:

(i) What are the necessary conditions for a given ground state which preserves $N = 1$ supersymmetry. As we are going to see these are equivalent to geometrical conditions on the scalar manifold which is spanned by the scalar fields in vector- and hypermultiplets.

(ii) What are the solutions to the geometrical conditions found in (i).

(iii) What are the consistency conditions on the couplings of the original $N = 2$ theory in order to have the low energy effective $N = 1$ action which describes the interactions below the supersymmetry breaking scale. These can also be stated as geometrical properties of the scalar manifold.

In this thesis we mainly study ground states which respect the full Lorentz invariance. These turn out to be flat Minkowski spaces or spaces of constant curvature (de Sitter and anti-de Sitter). For the latter case, only anti-de Sitter spaces is a consistent solution with residual $N = 1$ supersymmetry in the ground states.

This thesis is organized as follows. In chapter 2 we review several facts about $N = 2$ supergravity in four dimensions. Some useful formulae needed for our analysis are discussed. We start by discussing the properties of the scalar manifolds and at the end, we discuss the gauged $N = 2$ supergravity theory including the scalar potential and its first derivative. The main part of this thesis related to the questions (i) and (ii) is chapter 3 and 4. In chapter 3 we discuss $N = 2 \to N = 1$ supersymmetry breaking in Minkowski backgrounds. At the beginning, we give the simple example which has been studied in [16]. Next, we describe a general picture of $N = 2 \to N = 1$ supersymmetry breaking and then derive the necessary condition for the occurence of such breaking where the no-go theorem is evaded. The Higgs and the super-Higgs effects is also discussed. Furthermore, we extend the case to the curved backgrounds, namely anti-de Sitter spaces in chapter 4. In this chapter, we apply the same procedure as in Minkowski spaces. Chapter 5 contains our conclusion. In additon, four appendices are included. The first assembles our notation and conventions. In the second, we give a discussion about anti-de Sitter supersymmetry which is used for our analysis in chapter 4. The third gives some additional facts about $N = 2$ supergravity. In the fourth appendix we review $N = 1$ supergravity and further discuss $N = 1 \to N = 0$ supersymmetry breaking.

We finally want to mention that the results in chapter 3 and 4 will be published in [41].

Chapter 2

$N = 2$ Gauged Supergravity

In this section we briefly recall several facts about gauged $N = 2$ supergravity in four dimensions [42] and introduce the notations used in our analysis. We assemble our notation in appendix A and give some additional properties of $N = 2$ supergravity in appendix C. This chapter is organized as follows. First, we describe the particle content in $N = 2$ supergravity and then discuss geometries spanned by the scalar fields. At the end we discuss the Lagrangian of the gauged $N = 2$ supergravity and its scalar potential.

2.1 Multiplets of $N = 2$ Supergravity

A physical spectrum of the $N = 2$ supergravity is composed of a gravitational multiplet, n_V vector multiplets, and n_H hypermultiplets. These multiplets consist of the following component fields:

- *a gravitational multiplet*

  $$(e^a_\mu, \psi^A_\mu, A^0_\mu) \ , \qquad a, \mu = 0, ..., 3, \qquad A, B = 1, 2 \, .$$

  This multiplet contains the vielbein e^a_μ, the $SU(2)$ doublet of gravitinos ψ^A_μ, and the graviphoton A^0_μ.

- n_V *vector multiplets*

  $$(A^i_\mu, \lambda^{iA}, z^i) \ , \qquad i = 1, ..., n_V \, .$$

  Each vector multiplet contains a gauge boson A^i_μ, a doublet of gauginos λ^{iA}, and a complex scalar field z^i.

- n_H *hypermultiplets*

  $$(\zeta^\alpha, q^u) \ , \qquad \alpha = 1, ..., 2n_H \ , \qquad u = 1, ..., 4n_H \, .$$

  Each hypermultiplet contains a doublet of spinors, that is the hyperini ζ^α and four real scalar fields $q^u (u = 1, ..., 4n_H)$.

The scalar fields can be viewed as the coordinates of a scalar manifold $\mathcal{M}$, $N = 2$ supersymmetry imposes that this manifold factorizes

$$\mathcal{M} = \mathcal{M}_V \otimes \mathcal{M}_H \qquad , \tag{2.1}$$

where $\mathcal{M}_V$ has real dimension $\dim_{\mathbb{R}} \mathcal{M}_V = 2n_V$ and is spanned by the z^i while $\mathcal{M}_H$ has real dimension $\dim_{\mathbb{R}} \mathcal{M}_H = 4n_H$ and is spanned by the q^u. Furthermore, $\mathcal{M}_V$ has to be a Hodge-(special) Kähler manifold whereas $\mathcal{M}_H$ is constrained to be a quaternionic Kähler manifold.[1]

2.2 Special Kähler Geometry

In the vector multiplet the complex scalars span a Hodge-(special) Kähler manifold whose the holonomy is subgroup of $U(1) \times U(n_V)$. $\mathcal{M}_V$ is a Kähler manifold with metric $g_{i\bar{j}}(z, \bar{z}) = \partial_i \partial_{\bar{j}} \mathcal{K}_V(z, \bar{z})$ where the Kähler potential $\mathcal{K}_V$ obeys an additional constraint [44,45]. This constraint states that the $\mathcal{K}_V$ is not an arbitrary real function but determined in terms of a holomorphic prepotential $\mathcal{F}$ according to

$$\mathcal{K}_V(z, \bar{z}) = -\ln(\mathrm{i}\bar{X}^\Lambda \mathcal{F}_\Lambda - \mathrm{i}X^\Lambda \bar{\mathcal{F}}_\Lambda) \;, \qquad \Lambda = 0, ..., n_V \;, \tag{2.2}$$

where $\mathcal{F}(X)$ is a holomorphic prepotential of degree two, i.e. $X^\Lambda \mathcal{F}_\Lambda = 2\mathcal{F}(X)$ with $\mathcal{F}_\Lambda \equiv \frac{\partial \mathcal{F}}{\partial X^\Lambda}$ and the X^Λ are $(n_V + 1)$ holomorphic function of the z^i. In addition, the pair $(X^\Lambda, \mathcal{F}_\Lambda)$ transforms as

$$\begin{pmatrix} X^\Lambda \\ \mathcal{F}_\Lambda \end{pmatrix} \rightarrow \begin{pmatrix} \widetilde{X}^\Lambda \\ \widetilde{\mathcal{F}}_\Lambda \end{pmatrix} = e^f M \begin{pmatrix} X^\Lambda \\ \mathcal{F}_\Lambda \end{pmatrix} \qquad , \tag{2.3}$$

where $f \in \mathbb{C}$ and $M \in Sp(2n_V + 2, \mathbb{R})$. In addition, the transformation (2.3) leaves the metric $g_{i\bar{j}}$ invariant.

Furthermore, as $(X^\Lambda, \mathcal{F}_\Lambda)$ transform under symplectic transformation (2.3), one can always choose a basis of holomorphic section where no prepotential $\mathcal{F}(X)$ exists. Let us discuss this aspect in detail. First, we set $f = 0$ and parameterize the matrix M as

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \qquad . \tag{2.4}$$

Moreover, A, B, C, D, are $(n_V + 1) \times (n_V + 1)$ matrices which satisfy the symplectic condition

$$\begin{aligned} A^{\mathrm{T}}C - C^{\mathrm{T}}A &= 0 \quad , \\ B^{\mathrm{T}}D - D^{\mathrm{T}}B &= 0 \quad , \\ A^{\mathrm{T}}D - C^{\mathrm{T}}B &= \mathbb{1} \quad . \end{aligned} \tag{2.5}$$

Then using $\mathcal{F}_\Lambda = \mathcal{F}_{\Lambda\Sigma} X^\Sigma$ the new basis $(\widetilde{X}^\Lambda, \widetilde{\mathcal{F}}_\Lambda)$ has the form

$$\begin{aligned} \widetilde{X}^\Lambda &= (A^\Lambda_\Sigma + B^{\Lambda\Delta}\mathcal{F}_{\Delta\Sigma})X^\Sigma \quad , \\ \widetilde{\mathcal{F}}_\Lambda &= (C_{\Lambda\Sigma} + D^\Delta_\Lambda \mathcal{F}_{\Delta\Sigma})X^\Sigma \quad , \end{aligned} \tag{2.6}$$

[1] In rigid $N = 2$ supersymmetric theories the hypermultiplet scalars span a *hyper-Kähler* manifold [43,29].

and the holomorphic prepotential has form $2\widetilde{\mathcal{F}}(\widetilde{X}) = \widetilde{X}^\Lambda \widetilde{\mathcal{F}}_\Lambda$.

Now we discuss the possible non-existence of $\mathcal{F}(X)$. If we start with some special coordinates $(X^\Lambda, \mathcal{F}_\Lambda)$, it is possible to find the new basis $\widetilde{X}^\Lambda$ in that the mapping $X^\Lambda \to \widetilde{X}^\Lambda$ is not invertible. This happens whenever the $(n_V+1)\times(n_V+1)$ matrix $(A^\Lambda_\Sigma + B^{\Lambda\Delta}\mathcal{F}_{\Delta\Sigma})$ is not invertible, *i.e.* its determinant vanishes. This does not mean that $(\widetilde{X}^\Lambda, \widetilde{\mathcal{F}}_\Lambda)$ are not good symplectic section since the symplectic matrix M is always invertible. It simply means that $\widetilde{\mathcal{F}}_\Lambda \neq \widetilde{\mathcal{F}}_{\Lambda\Sigma}\widetilde{X}^\Sigma$ and furthermore, $\widetilde{\mathcal{F}}_\Lambda \neq \frac{\partial \widetilde{\mathcal{F}}}{\partial \widetilde{X}^\Lambda}$. Therefore, a prepotential $\widetilde{\mathcal{F}}(\widetilde{X})$ does not exist. Such basis has been studied for a large class of special Kähler manifold $\mathcal{M}_V$ [16,17,46,47,48,49].

2.3 Quaternionic Kähler Geometry

In the hypermultiplet the scalars span a quaternionic Kähler manifold [50,33,42] whose holonomy is a subgroup of $Sp(2)\times Sp(2n_H, \mathrm{I\!R})$. A quaternionic manifold is a $4n_H$-dimensional real manifold endowed with the metric $h_{uv}, (u,v = 1,...,4n_H)$ and three complex structures $J^x, (x=1,2,3)$ that satisfy the quaternionic algebra

$$J^x J^y = -\delta^{xy}\mathbb{1} + \epsilon^{xyz}J^z \quad . \tag{2.7}$$

Associated with the complex structures is a triplet 2-forms

$$K^x_{uv} = h_{uw}(J^x)^w_v \quad , \tag{2.8}$$

which are called the hyperKähler forms which are covariantly closed with respect to an $Sp(2)$-connection ω^x

$$\nabla K^x \equiv dK^x + \varepsilon^{xyz}\omega^y \wedge K^z = 0 \quad . \tag{2.9}$$

Furthermore, the $Sp(2)$-curvature given by

$$-K^x \equiv d\omega^x + \frac{1}{2}\epsilon^{xyz}\omega^y \wedge \omega^z \quad . \tag{2.10}$$

The vielbeins of the quaternionic manifold $\mathcal{M}_H$ will be denoted by $\mathcal{U}^{A\alpha} \equiv \mathcal{U}^{A\alpha}_u dq^u$ where $\alpha = 1,...,2n_H$ is an index labelling the fundamental representation of $Sp(2n_H)$. The vielbein $\mathcal{U}^{A\alpha}$ is covariantly closed with respect to the $SU(2)$ connection ω^x and to some $Sp(2n_H)$ connection $\Delta^{\alpha\beta} = \Delta^{\beta\alpha}$

$$\nabla\mathcal{U}^{A\alpha} = d\mathcal{U}^{A\alpha} + \frac{\mathrm{i}}{2}\omega^x\,\sigma^{xAB}\wedge\mathcal{U}^\alpha_B + \mathbb{C}_{\beta\gamma}\Delta^{\alpha\beta}\wedge\mathcal{U}^{A\gamma} = 0 \quad . \tag{2.11}$$

The metric for a such vielbein is

$$h_{uv} = \mathcal{U}^{A\alpha}_u \mathcal{U}_{A\alpha v} \quad , \tag{2.12}$$

where $\mathcal{U}_{A\alpha} = (\mathcal{U}^{A\alpha})^* = \epsilon_{AB}\mathbb{C}_{\alpha\beta}\mathcal{U}^{B\beta}$, $\epsilon_{AB} = -\epsilon_{BA}$ and $\mathbb{C}_{\alpha\beta} = -\mathbb{C}_{\beta\alpha}$ are the flat $Sp(2n_H)$ and $Sp(2)\cong SU(2)$ invariant metric respectively and the $*$ is the complex conjugate. More specifically one can write a stronger version of eq.(2.12) [33]

$$(\mathcal{U}^{A\alpha}_u\mathcal{U}^{B\beta}_v + \mathcal{U}^{A\alpha}_v\mathcal{U}^{B\beta}_u)\mathbb{C}_{\alpha\beta} = h_{uv}\epsilon^{AB} \quad ,$$

$$(\mathcal{U}_u^{A\alpha}\mathcal{U}_v^{B\beta}+\mathcal{U}_v^{A\alpha}\mathcal{U}_u^{B\beta})\epsilon_{AB}=h_{uv}\frac{1}{n_H}\mathbb{C}^{\alpha\beta} \quad , \tag{2.13}$$

with the additional property

$$h^{uv}\mathcal{U}_u^{A\alpha}\mathcal{U}_v^{B\beta}=\epsilon^{AB}\mathbb{C}^{\alpha\beta} \quad . \tag{2.14}$$

Now consider the equations (2.7), (2.8), and (2.10). We easily deduce that the following relation

$$h^{ts}K_{ut}^{x}\,K_{sv}^{y}=-\delta^{xy}h_{uv}+\epsilon^{xyz}\,K_{uv}^{z} \quad , \tag{2.15}$$

holds in the quaternionic case. The relation (2.15) further implies that the $Sp(2)$-curvature K^x can alternatively be written as

$$\mathrm{i}K_{ts}^{x}(\sigma^x)^{AB}=\mathcal{U}_t^{A\alpha}\,\mathcal{U}_{\alpha s}^{B}-\mathcal{U}_s^{A\alpha}\,\mathcal{U}_{\alpha t}^{B} \quad . \tag{2.16}$$

2.4 Gauging Isometries on Scalar Manifolds

Let us consider the scalar manifold $\mathcal{M}$ with isometries

$$\delta z^i=\epsilon^\Lambda k_\Lambda^i(z)\ , \qquad \delta q^u=\epsilon^\Lambda k_\Lambda^u(q)\ , \tag{2.17}$$

where $k_\Lambda^i(z), k_\Lambda^u(q)$ are Killing vectors of $\mathcal{M}_V$ and $\mathcal{M}_H$, respectively. These isometries can be gauged in that the gauge parameters ϵ^Λ are made spacetime dependent and the following covariant derivatives are introduced

$$\nabla_\mu\, z^i=\partial_\mu z^i+k_\Lambda^i A_\mu^\Lambda\ , \qquad \nabla_\mu\, q^u=\partial_\mu q^u+k_\Lambda^u A_\mu^\Lambda\ , \tag{2.18}$$

In this thesis we are mainly interested in the case where $k_\Lambda^i=0$ and for simplicity we focus on this situation henceforth.

These Killing vectors determine a triplet of prepotentials $P_\Lambda^x, x=1,2,3$ (or momentum maps) via [34]

$$2k_\Lambda^v\,K_{uv}^x=-\nabla_u P_\Lambda^x\equiv-(\partial_u P_\Lambda^x+\epsilon^{xyz}\omega_u^y\,P_\Lambda^z) \quad , \tag{2.19}$$

where K_{uv}^x is defined in (2.10).

As consequence of the above gauging, supersymmetry requires fermionic masses and a scalar potential. These issues will be discussed in the next section.

2.5 Lagrangian of Gauged $N=2$ Supergravity

In this section we record the part of the $N=2$ gauged supergravity Lagrangian which is needed for our analysis. The complete $N=2$ Lagrangian can be found, for example in [51, 42] and also in appendix C. The $N=2$ gauged supergravity up to four-fermion terms can be written as

$$\begin{aligned}\mathcal{L}^{N=2} \;=\;& -\frac{1}{2}R+\frac{1}{2}\mathcal{I}_{\Lambda\Sigma}F_{\mu\nu}^\Lambda F^{\Lambda\mu\nu}+\frac{1}{2}\mathcal{R}_{\Lambda\Sigma}F_{\mu\nu}^\Lambda\tilde{F}^{\Sigma\mu\nu}\\ &+g_{i\bar{j}}\partial_\mu z^i\partial^\mu\bar{z}^{\bar{i}}+h_{uv}\nabla_\mu q^u\nabla^\mu q^v-V^{N=2}(z,\bar{z},q)\\ &+2\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}}\bar{\psi}_\mu^A\gamma_\sigma\nabla_\nu\psi_{A\lambda}-\mathrm{i}g_{i\bar{j}}\bar{\lambda}^{iA}\gamma^\mu\nabla_\mu\lambda_A^{\bar{j}}-2\mathrm{i}\,\bar{\zeta}^\alpha\gamma^\mu\nabla_\mu\zeta_\alpha\end{aligned} \tag{2.20}$$

$$
\begin{aligned}
&- \{g_{i\bar{j}}\partial_\mu \bar{z}^{\bar{i}}(\bar{\psi}^\mu_A \lambda^{iA} - \bar{\lambda}^{iA}\gamma^{\mu\nu}\psi_{A\nu}) - 2\mathcal{U}^{A\alpha}_u \nabla_\mu q^u(\bar{\psi}^\mu_A \zeta_\alpha - \bar{\zeta}_\alpha \gamma^{\mu\nu}\psi_{A\nu}) + \text{h.c.}\} \\
&+ [2S_{AB}\,\bar{\psi}^A_\mu \gamma^{\mu\nu}\psi^B_\nu + \mathrm{i} g_{i\bar{j}} W^{iAB}\bar{\lambda}^{\bar{j}}_A \gamma^\mu \psi_{B\mu} + 2\mathrm{i} N^A_\alpha \bar{\zeta}^\alpha \gamma^\mu \psi^A_\mu \\
&+ \mathcal{M}^{\alpha\beta}\,\bar{\zeta}_\alpha \zeta_\beta + \mathcal{M}^\alpha_{iA}\,\bar{\zeta}_\alpha \lambda^{iA} + \mathcal{M}_{iA|lB}\,\bar{\lambda}^{iA}\lambda^{lB} + \text{h.c.}] + \ \ldots \quad ,
\end{aligned}
$$

where we omitted couplings of the gauge fields to the fermions. The gauge coupling functions $\mathcal{I}_{\Lambda\Sigma}$ and $\mathcal{R}_{\Lambda\Sigma}$ are

$$
\begin{aligned}
\mathcal{I}_{\Lambda\Sigma} &= \mathrm{Im}\,\mathcal{N}_{\Lambda\Sigma} \quad , \\
\mathcal{R}_{\Lambda\Sigma} &= \mathrm{Re}\,\mathcal{N}_{\Lambda\Sigma} \quad ,
\end{aligned}
\tag{2.21}
$$

and $\mathcal{N}_{\Lambda\Sigma}$ are determined in terms of the prepotential by[2]

$$
\mathcal{N}_{\Lambda\Sigma} = \bar{\mathcal{F}}_{\Lambda\Sigma} + 2\mathrm{i}\frac{\mathrm{Im}\mathcal{F}_{\Lambda\Gamma}\mathrm{Im}\mathcal{F}_{\Sigma\Delta}X^\Gamma X^\Delta}{(\mathrm{Im}\,\mathcal{F}\,X\,X)} \quad , \tag{2.22}
$$

where $(\mathrm{Im}\,\mathcal{F}\,X\,X) = \mathrm{Im}\,\mathcal{F}_{\Gamma\Delta}X^\Gamma X^\Delta$. The mass matrix of the gravitinos and their mixing to the spin-$\frac{1}{2}$ fermions are

$$
\begin{aligned}
S_{AB} &= \frac{\mathrm{i}}{2}\,\sigma^x_{AB}P^x_\Lambda L^\Lambda \quad , \\
W^{iAB} &= \mathrm{i}\,\sigma^{xAB}P^x_\Lambda g^{i\bar{j}}\,\bar{f}^\Lambda_{\bar{j}} + \epsilon^{AB}k^i_\Lambda \bar{L}^\Lambda \quad , \\
N^A_\alpha &= 2\mathcal{U}^A_{\alpha u}k^u_\Lambda \bar{L}^\Lambda \quad ,
\end{aligned}
\tag{2.23}
$$

where $L^\Lambda(z,\bar{z}) \equiv e^{\frac{1}{2}\mathcal{K}_V(z,\bar{z})}X^\Lambda(z)$ and $\bar{f}^\Lambda_{\bar{i}}$ is the complex conjugate of $f^\Lambda_i \equiv \nabla_i L^\Lambda = (\partial_i + \frac{1}{2}\mathcal{K}_{V,i})L^\Lambda$. The mass matrices of the spin-$\frac{1}{2}$ fermions are given by

$$
\begin{aligned}
\mathcal{M}_{iA|jB} &= \epsilon_{AB}\,g_{\bar{l}[i}f^\Lambda_{j]}k^{\bar{l}}_\Lambda + \frac{\mathrm{i}}{2}\,\sigma^x_{AB}P^x_\Lambda\,\nabla_i f^\Lambda_j \quad , \\
\mathcal{M}^\alpha_{iA} &= -4\,\mathcal{U}^\alpha_{Au}\,k^u_\Lambda f^\Lambda_i \quad , \\
\mathcal{M}^{\alpha\beta} &= -\mathcal{U}^{A\alpha}_u \mathcal{U}^{B\beta}_v\,\epsilon_{AB}\,\nabla^u k^v_\Lambda L^\Lambda \quad .
\end{aligned}
\tag{2.24}
$$

The scalar potential $V^{N=2}(z,\bar{z},q)$ is given by [51][3]

$$
\delta^A_B\,V^{N=2}(z,\bar{z},q) = -12\bar{S}^{AC}S_{CB} + g_{i\bar{j}}\bar{W}^{\bar{j}}_{BC}W^{iCA} + 2\bar{N}^\alpha_B N^A_\alpha \quad . \tag{2.25}
$$

Now one can derive the first derivative of the scalar potential $V^{N=2}$ with respect to the scalar fields in $N = 2$ supergravity

$$
\begin{aligned}
\delta^A_B\,\frac{\partial V^{N=2}}{\partial z^k} &= -4g_{k\bar{l}}\,\bar{W}^{\bar{l}}_{BC}\,\bar{S}^{CA} + 2\mathcal{M}_{kB|iC}\,W^{iCA} + \mathcal{M}^\alpha_{kB}\,N^A_\alpha \quad , \\
\delta^A_B\,\frac{\partial V^{N=2}}{\partial \bar{z}^{\bar{k}}} &= -4g_{\bar{k}l}W^{lAC}\,S_{CB} + 2\bar{\mathcal{M}}^{A|C}_{\bar{k}|\bar{i}}\,\bar{W}^{\bar{i}}_{CB} + \bar{\mathcal{M}}^A_{\bar{k}\alpha}\,\bar{N}^\alpha_B \quad , \\
\frac{\partial V^{N=2}}{\partial q^u}\mathcal{U}^{B\beta u} &= -4\bar{N}^\beta_A\,\bar{S}^{AB} + \frac{1}{2}\mathcal{M}^\beta_{iA}\,W^{iAB} + \mathcal{M}^{\beta\alpha}\,N^B_\alpha \\
&+ \ \epsilon^{AB}\mathbb{C}^{\alpha\beta}(-4N^C_\alpha\,S_{CA} + \frac{1}{2}\bar{\mathcal{M}}^C_{\alpha\bar{i}}\,\bar{W}^{\bar{i}}_{CA} + \bar{\mathcal{M}}_{\alpha\gamma}\,\bar{N}^\gamma_A) \quad .
\end{aligned}
\tag{2.26}
$$

[2]We assume here that the prepotential $\mathcal{F}$ exists.

[3]See also appendix C.

The potential (2.25) and its first derivative (2.26) are very important equations in order to study the properties of the ground states of $N=2$ theory together with the supersymmetry transformation of the fermions defined below. In addition, (2.26) can be used to show the possibility of the super-Higgs mechanism in the ground states because it relates the gravitino mass matrix S_{AB} with the spin-$\frac{1}{2}$ fermion $\mathcal{M}^{\alpha\beta}, \mathcal{M}^{\alpha}_{iA}, \mathcal{M}_{iA|jB}$ as we will see in section 3.5 and 4.3.

Finally the supersymmetry transformation of the fermions laws up to 3-fermion terms leaving invariant (2.20) are:

$$\begin{aligned} \delta\psi_{A\mu} &= \widehat{\mathcal{D}}_\mu \epsilon_A + \mathrm{i} S_{AB}\gamma_\mu \epsilon^B + \ldots \quad , \\ \delta\lambda^{iA} &= \mathrm{i}\nabla_\mu z^i \gamma^\mu \epsilon^A + W^{iAB}\epsilon_B + \ldots \quad , \\ \delta\zeta_\alpha &= \mathrm{i}\mathcal{U}^{B\beta}_u \nabla_\mu q^u \gamma^\mu \epsilon^A \epsilon_{AB}\, \mathbb{C}_{\alpha\beta} + \sqrt{2} N^A_\alpha \epsilon_A + \ldots \quad , \end{aligned} \tag{2.27}$$

where ϵ^A are the parameters of the two supersymmetry transformations, $\widehat{\mathcal{D}}_\mu \epsilon_A = \partial_\mu \epsilon_A - \frac{1}{4}\gamma_{ab}\,\omega^{ab}_\mu \epsilon_A + \frac{\mathrm{i}}{2}\hat{Q}_\mu \epsilon_A + \hat{\omega}_{\mu|A}{}^B\,\epsilon_B$, and Q_μ is the $U(1)$-connection of special Kähler manifold (see appendix C).

Chapter 3

Spontaneous $N = 2 \rightarrow N = 1$ SUSY Breaking in Minkowski

In this chapter we investigate the spontaneous $N = 2 \rightarrow N = 1$ local supersymmetry breaking in Minkowski backgrounds. Starting with a simplest example, we derive the necessary conditions for spontaneous $N = 2 \rightarrow N = 1$ supersymmetry breaking in such backgrounds. As we are going to see these conditions can be formulated as geometrical conditions on the scalar manifold $\mathcal{M}$. Furthermore we discuss the occurrence of the Higgs and the super-Higgs effect.

3.1 Simplest Example

The simplest realization of the partial $N = 2$ local supersymmetry breaking in Minkowski ground states was discussed by the authors in references [16]. Here we study in detail such example including the Higgs and the super-Higgs mechanism and derive the low energy effective $N = 1$ theory.

The spectrum of the model has six scalars because $n_V = n_H = 1$ and the scalar manifold has form

$$\mathcal{M} = \frac{SU(1,1)}{U(1)} \otimes \frac{SO(4,1)}{SO(4)} \quad . \tag{3.1}$$

The manifold $\frac{SU(1,1)}{U(1)}$ is spanned by the complex scalar. Its Kähler potential and its metric are given by

$$\mathcal{K}_V = -\ln(z + \bar{z}) \ , \quad g_{z\bar{z}} = (z + \bar{z})^{-2} \quad , \tag{3.2}$$

which correspond to a basis where no prepotential $\mathcal{F}$ exists.[1] Instead one has

$$X^0(z) = -\frac{1}{2} \ , \ X^1(z) = \frac{\mathrm{i}}{2} \ , \ \mathcal{F}_0 = \mathrm{i}z \ , \ \mathcal{F}_1 = z \ , \tag{3.3}$$

which can be obtained from $\mathcal{F} = \mathrm{i}X^0X^1$ via the symplectic transformation

$$X^1 \rightarrow -\mathcal{F}_1 \ , \quad \mathcal{F}_1 \rightarrow X^1 \ , \tag{3.4}$$

[1] This choice in turn is a condition in the vector multiplet where the spontaneous breaking can be realized. See section 3.4.

with $z = \frac{X^1}{X^0}$.

The quaternionic Kähler manifold $\frac{SO(4,1)}{SO(4)}$ is parameterized by the four real scalars $b^0, b^x, x = 1,2,3$. The $Sp(2)$ connection ω^x and the $Sp(2)$ curvature Ω^x for this manifold are

$$\omega^x_u = \frac{1}{b^0}\delta^x_u\,, \quad K^x_{0u} = \frac{1}{2(b^0)^2}\delta^x_u\,, \quad K^x_{yz} = -\frac{1}{2(b^0)^2}\epsilon^{xyz}\;, \tag{3.5}$$

then using (2.15) one can get the metric

$$h_{uv} = \frac{1}{2(b^0)^2}\delta_{uv} \quad . \tag{3.6}$$

Additionally we need the vielbein $\mathcal{U}^{A\alpha},\ \alpha, A = 1,2,$. From the metric (3.6) one can read off that the vielbein is

$$\mathcal{U}^{A\alpha} = \frac{1}{2b^0}(-\epsilon^{A\alpha}db^0 + \mathrm{i}\sigma^{xA\alpha}db^x) \qquad , \tag{3.7}$$

where $\sigma^{xA\alpha} = -\epsilon^{\alpha\beta}\sigma^{xA}_{\beta}$ and σ^{xA}_{β} are the standard Pauli matrices (see appendix A).

The metric (3.6) is invariant under three Peccei-Quinn isometries. These isometries are arbitrary constant translation of the coordinates b^1, b^2, b^3. Moreover, it turns out that two of them have to be gauged

$$b^1 \to b^1 + \varepsilon^0 g_0 \qquad , \qquad b^2 \to b^2 + \varepsilon^1 g_1 \quad , \tag{3.8}$$

where $g_0, g_1 \in \mathbb{R}$. The Killing vectors which generate these isometries can be written in the simple form

$$k^u_0 = g_0\,\delta^{u1}\;, \qquad k^u_1 = g_1\delta^{u2}\;, \tag{3.9}$$

and then inserted into (2.19) the corresponding prepotentials for these vectors are

$$P^x_0 = \frac{g_0}{b^0}\delta^{x1}\;, \qquad P^x_1 = \frac{g_1}{b^0}\delta^{x2}\delta_{\Lambda 1} \qquad . \tag{3.10}$$

Inserting all of the above quantities into (2.23) and (2.24), one arrives at the explicit expression of those quantities

$$\begin{aligned}
(S_{AB}) &= \frac{\mathrm{i}}{4b^0}e^{\frac{\mathcal{K}_V}{2}}\begin{pmatrix} g_1 - g_0 & 0 \\ 0 & g_1 + g_0 \end{pmatrix}\;, \\
(W^{zAB}) &= -\frac{\mathrm{i}}{2b^0}e^{\frac{\mathcal{K}_V}{2}}(z+\bar{z})\begin{pmatrix} g_1 - g_0 & 0 \\ 0 & g_1 + g_0 \end{pmatrix}, \\
(N^A_\alpha) &= -\frac{\mathrm{i}}{2b^0}e^{\frac{\mathcal{K}_V}{2}}\begin{pmatrix} 0 & g_0 + g_1 \\ g_0 - g_1 & 0 \end{pmatrix}\;, \\
(\mathcal{M}_{zA|zB}) &= 0\;, \quad (\mathcal{M}^\alpha_{zB}) = \frac{\mathrm{i}}{b^0}e^{\frac{\mathcal{K}_V}{2}}(z+\bar{z})^{-1}\begin{pmatrix} 0 & g_0 - g_1 \\ g_0 + g_1 & 0 \end{pmatrix}, \\
(\mathcal{M}^{\alpha\beta}) &= \frac{\mathrm{i}}{2b^0}e^{\frac{\mathcal{K}_V}{2}}\begin{pmatrix} g_1 + g_0 & 0 \\ 0 & g_1 - g_0 \end{pmatrix}\;.
\end{aligned} \tag{3.11}$$

Note that $\mathcal{M}_{zA|zB} = 0$ because we use the following fact for the complex manifold $\frac{SU(1,1)}{U(1)}$:

$$\nabla_z f^\Lambda_z = \nabla_{\bar{z}} \bar{f}^\Lambda_{\bar{z}} = 0 \qquad . \tag{3.12}$$

Before turning to the properties of this model, let us first write the scalar potential (2.25) and its first derivative (2.26) in the matrix form

$$
\begin{aligned}
(\delta^A_B)V^{N=2}(z,\bar{z},q) &= -12(\bar{S}^{AC})(S_{CB}) + g_{i\bar{j}}(\bar{W}^{\bar{j}}_{BC})(W^{iCA}) + 2(\bar{N}^{\alpha}_{B})(N^A_\alpha) \quad , \\
(\delta^A_B)\frac{\partial V^{N=2}}{\partial z^k} &= -4g_{k\bar{l}}(\bar{W}^{\bar{l}}_{BC})^{\mathrm{T}}(\bar{S}^{CA}) + 2(\mathcal{M}_{kB|iC})(W^{iCA}) + (\mathcal{M}^{\alpha}_{kB})(N^A_\alpha) \quad , \\
\frac{\partial V^{N=2}}{\partial q^u}(\mathcal{U}^{B\beta u}) &= -4(\bar{N}^{\beta}_{A})^{\mathrm{T}}(\bar{S}^{AB}) + \frac{1}{2}(\mathcal{M}^{\beta}_{iA})^{\mathrm{T}}(W^{iAB}) + (\mathcal{M}^{\beta\alpha})\,(N^B_\alpha) \\
&\quad + \epsilon^{AB}\mathbb{C}^{\alpha\beta}\Big(-4(S_{CA})(N^C_\alpha)^{\mathrm{T}} + \frac{1}{2}(\bar{W}^{\bar{i}}_{CA})(\bar{\mathcal{M}}^C_{\alpha\bar{i}})^{\mathrm{T}} + (\bar{N}^{\gamma}_{A})(\bar{\mathcal{M}}_{\alpha\gamma})\Big).
\end{aligned} \tag{3.13}
$$

Subtituting (3.11) into (3.13), one gets

$$
\frac{\partial V^{N=2}}{\partial z^k} = \frac{\partial V^{N=2}}{\partial q^u} = V^{N=2} \equiv 0 \quad , \tag{3.14}
$$

identically, for any value of g_0, g_1 and also of z and b^0, b^x. The equation (3.14) means that this model has Minkowskian ground states.

From the first equation of (3.11), we see that there can be a mass gap between the two eigenvalues of S_{AB}. The interesting case is $g_0 = g_1$ (and nonzero) such that the mass matrix S_{AB} has a zero eigenvalue, *i.e.* $S_{11} = 0$, or in other words one of the two gravitinos becomes massless. Moreover a look at the supersymmetry transformation (2.27) restricted to the parameter ϵ_1, one finds

$$
\begin{aligned}
\langle \delta_{\epsilon_1}\psi_{1\mu}\rangle &= \partial_\mu \epsilon_1 + \mathrm{i}\langle S_{11}\rangle \gamma_\mu \epsilon^1 = \partial_\mu\epsilon_1 = 0 \quad , \\
\langle \delta_{\epsilon_1}\lambda^{iA}\rangle &= \langle W^{iA1}\rangle \epsilon_1 = 0 \quad , \\
\langle \delta_{\epsilon_1}\zeta_\alpha\rangle &= \langle \sqrt{2}N^1_\alpha\rangle\epsilon_1 = 0 \quad ,
\end{aligned} \tag{3.15}
$$

where the bracket $\langle\ \rangle$ means that (2.27) have to be evaluated in the ground states, while for the parameter ϵ_2 (3.15) does not hold. This simply means that the parameter ϵ_1 represents the unbroken direction. Thus, the requirements (3.15) ensure the existence of an unbroken $N = 1$ supersymmetry in the ground states, *i.e.* for $g_0 = g_1$.

The residual $N = 1$ supersymmetry demands the appearing of an $N = 1$ massive spin-$\frac{3}{2}$ multiplet which has spin content $(\frac{3}{2}, 1, 1, \frac{1}{2})$. Both the graviphoton A^0_μ and the matter vector A^1_μ become massive, together with one of the gravitinos. One might wonder how one can simultaneously get the three massive particles with different spin. This can be seen from the $N = 2$ Lagrangian (2.20). Charging the scalar fields b^1, b^2 with the Killing vectors (3.9) implies the appearing of the fermionic mass term in the theory. At $g_0 = g_1$ not all fermions are massive. In fact, besides the massless gravitino ψ^1_μ, there exist a massless spin-$\frac{1}{2}$ fermion. However, in unitary gauge this massless spin-$\frac{1}{2}$ fermion is eaten by the gravitino ψ^2_μ while the scalar fields b^1, b^2 are eaten by the gauge fields $A^{0,1}_\mu$. This leaves only the massive mode in the gravitino multiplet. Thus, we have a super-Higgs and a Higgs mechanisms. In addition to the $N = 1$ massive spin $\frac{3}{2}$ multiplet, there are also two massless chiral multiplets which survive in the ground states. They span a scalar manifold in the low energy effective $N = 1$ theory derived below.

Let us first discuss the occurence of the super-Higgs mechanism. To see this, one has to focus on the fermionic terms in the Lagrangian (2.20). Subtituting all of the quantities in (3.11) into (2.20), one has then [2]

$$\mathcal{L}_{\rm mass}=\mathrm{i}\langle\frac{g_0}{b^0}e^{\frac{\mathcal{K}_V}{2}}\rangle\left(\bar{\psi}^2_\mu\gamma^{\mu\nu}\psi^2_\nu+\mathrm{i}\bar{\eta}\gamma^\mu\psi^2_\mu-\sqrt{2}\langle(z+\bar{z})^{-1}\rangle\bar{\zeta}_1\lambda^{z2}+\frac{1}{2}\bar{\zeta}_1\zeta_1\right)+\mathrm{h.c.}\;, \quad (3.16)$$

where

$$\eta\equiv-\langle(z+\bar{z})^{-1}\rangle\lambda^{z2}+\sqrt{2}\zeta_1 \quad . \quad (3.17)$$

From the supersymmetry transformation of the gaugino and the hyperino (2.27), the supersymmetry transformation of η has form

$$\delta\eta=3\langle\frac{(g_0)^2}{(b^0)^2}e^{\mathcal{K}_V}\rangle\epsilon_2 \quad . \quad (3.18)$$

Such fermion η is called a Goldstone fermion and can be gauged away by a suitable gauge transformation of the gravitino ψ^2_μ. It is easy to see that this gauge transformation is

$$\psi^2_\mu\rightarrow\psi^2_\mu+\frac{\mathrm{i}}{6}\gamma_\mu\eta+\frac{\mathrm{i}}{3}\langle\frac{b^0}{g_0}e^{-\frac{\mathcal{K}_V}{2}}\rangle\partial_\mu\eta \quad (3.19)$$

and then inserting it into (3.16), then the Lagrangian (3.16) takes a form simply like [3]

$$\begin{aligned}\mathcal{L}_{\rm mass}= \;& \mathrm{i}\langle\frac{g_0}{b^0}e^{\frac{\mathcal{K}_V}{2}}\rangle\left(\bar{\psi}^2_\mu\gamma^{\mu\nu}\psi^2_\nu-\frac{1}{3}\big(\langle(z+\bar{z})^{-1}\rangle\bar{\lambda}^{z2}+\frac{1}{\sqrt{2}}\bar{\zeta}_1\big)\big(\langle(z+\bar{z})^{-1}\rangle\lambda^{z2}+\frac{1}{\sqrt{2}}\zeta_1\big)\right)\\ &+\mathrm{h.c.} \quad . \end{aligned} \quad (3.20)$$

Let us define a new massive spin-$\frac{1}{2}$ fermion in (3.20) as

$$\chi\equiv\frac{1}{\sqrt{3}}\big(\langle(z+\bar{z})^{-1}\rangle\lambda^{z2}+\frac{1}{\sqrt{2}}\zeta_1\big) \quad , \quad (3.21)$$

and one can then simplify (3.20) as follows :

$$\mathcal{L}_{\rm mass}=\mathrm{i}\langle\frac{g_0}{b^0}e^{\frac{\mathcal{K}_V}{2}}\rangle\left(\bar{\psi}^2_\mu\gamma^{\mu\nu}\psi^2_\nu-\bar{\chi}\chi\right)+\mathrm{h.c.} \quad . \quad (3.22)$$

We see that the above Lagrangian contains only the physical massive fermions which are degenerate in mass. This also shows that the residual $N=1$ supersymmetry requires the presence of an additional massive spin-$\frac{1}{2}$ fermion in the massive gravitino multiplet. On the other hand, the fermions (λ^{z1},ζ_2) remain massless.

The gauge bosons masses come from the gauge covariant derivative

$$\nabla_\mu b^1=\partial_\mu b^1+g_0A^0_\mu\;,\quad \nabla_\mu b^2=\partial_\mu b^2+g_1A^1_\mu\;, \quad (3.23)$$

in the hypermultiplets (gauged) kinetic term

$$\begin{aligned}\mathcal{L}_{\rm b}&=h_{uv}\nabla^\mu b^u\,\nabla_\mu b^v\\ &=\;....+\frac{1}{2}\left(\frac{g_0}{b^0}\right)^2\left((A^0_\mu)^2+(A^1_\mu)^2\right) \quad ,\end{aligned} \quad (3.24)$$

[2] After rescaling $\sqrt{2}\zeta_1\rightarrow\zeta_1$.

[3] The second term $\sim\partial_\mu\eta$ in the shifting transformation (3.19) is used to cancel the kinetic term of the Goldstone fermion η. We postpone for the moment the discussion until section 3.5.

with $g_0 = g_1$. Furthermore, the massless scalars b^1, b^2 can be eliminated from (3.24) by employing the gauge transformation of $A_\mu^{0,1}$,

$$A_\mu^0 \to A_\mu^0 - \frac{1}{g_0}\partial_\mu b^1 \ , \quad A_\mu^1 \to A_\mu^1 - \frac{1}{g_0}\partial_\mu b^2 \ . \tag{3.25}$$

This is the ordinary Higgs mechanism.

To get the correct mass of the gauge bosons, we must normalize the gauge bosons kinetic term in the Lagrangian (2.20)[4]

$$\frac{1}{2}\mathcal{I}_{\Lambda\Sigma}F^{\Lambda\mu\nu}F^{\Sigma}_{\mu\nu} = -\frac{1}{4}e^{-\mathcal{K}_V}\left(F^{0\mu\nu}F^0_{\mu\nu} + F^{1\mu\nu}F^1_{\mu\nu}\right) \quad . \tag{3.26}$$

Transforming to the canonical normalization $-\frac{1}{4}F^2 + \frac{1}{2}m^2A^2$, we finally obtain the correct squared mass of the gauge bosons

$$m^2_{A^{0,1}} = \langle e^{\mathcal{K}_V}\left(\frac{g_0}{b^0}\right)^2\rangle \quad . \tag{3.27}$$

Comparing the fermion mass in (3.20) and the gauge bosons mass (3.27), we see that the members of the $N = 1$ gravitino multiplet are degenerate in mass

$$m_{\psi^2} = m_{A^0} = m_{A^1} = m_\chi = \langle e^{\frac{\mathcal{K}_V}{2}}\frac{g_0}{b^0}\rangle \quad . \tag{3.28}$$

This property is typical in Minkowski space because the squared mass operator P^aP_a is supersymmetric Casimir operator.

Let us now focus on the low energy effective $N = 1$ theory which is valid well below the scale of the supersymmetry breaking set by $m_{\psi^2} = \langle \frac{g_0}{b^0}e^{\frac{\mathcal{K}_V}{2}}\rangle$. This effective theory can be derived by *integrating out* the massive $\frac{3}{2}$ multiplet [40].[5] At the two derivative level this can be achieved by using the equation of motions of the massive fields to first non-trivial order in p/m_{ψ^2} where in $p \ll m_{\psi^2}$ is a characteristic momentum. For the gravitino and the $\frac{1}{2}$-fermion this is a straightforward procedure because they are simply set to zero. For the spin-1 gauge bosons due to their coupling to the Goldstone bosons (3.12) eliminating $A_\mu^{0,1}$ also eliminates the two Goldstone bosons and furthermore changes the σ-model interactions of the remaining scalar fields. To make it precise, let us consider the hypermultiplets (gauged) kinetic term (3.24). Since we have $p \ll m_{\psi^2}$, then the kinetic terms of $A_\mu^{0,1}$ can be omitted and their equations of motion read[6]

$$\frac{\partial\mathcal{L}_b}{\partial A_\mu^{0,1}} = 0 \quad . \tag{3.29}$$

Then we arrive at

$$A_\mu^0 = -\frac{1}{g_0}\partial_\mu b^1 \ , \quad A_\mu^1 = -\frac{1}{g_0}\partial_\mu b^2 \ . \tag{3.30}$$

Inserting back (3.30) to (3.24), we find that the left over scalar fields b^0, b^3 span a Kähler manifold which is $\frac{SU(1,1)}{U(1)}$. Indeed, we define $u = b^0 + ib^3$ and derive

$$\mathcal{L}'_{\rm b} = g_{u\bar{u}}\,\partial^\mu u\,\partial_\mu\bar{u} \tag{3.31}$$

[4]Since in this basis no prepotential $\mathcal{F}$ exists, one cannot use (2.22) to compute $\mathcal{N}_{\Lambda\Sigma}$ but rather the general formulae (C.1) [46, 48].

[5]We generalize this aspect in [41].

[6]In other words, this is a quotient procedure with respect to the isometries (3.8) [52].

with metric and Kähler potential

$$g_{u\bar{u}} = \partial_u \partial_{\bar{u}} \mathcal{K}_H \ , \quad \mathcal{K}_H = -2 \ln(u + \bar{u}) \ , \tag{3.32}$$

respectively.

Moreover, $N = 1$ supersymmetry demands that the scalar potential can be written in terms of a covariantly holomorphic object the superpotential $\mathcal{W}$, and correspondingly, the inverse gauge couplings are harmonic. Let us check that our simplest model satisfies these requirements. As we have discussed above, this model has Minkowskian ground states in which the $N = 1$ gravitino ψ^1_μ is exactly massless. This implies that $\mathcal{W} = V^{N=1} = 0$, where $V^{N=1}$ is the $N = 1$ scalar potential and hence the first requirement is satisfied. Furthermore, this simplest model contains only one vector multiplet and this immediately implies that the low energy $N = 1$ theory contains no massless $N = 1$ vector multiplets, then the second requirement is trivially satisfied. Thus the low energy effective theory of our simplest model satisfies the properties of $N = 1$ theory.

3.2 General Picture in Minkowski

Using the above example, we are now going to generalize the picture of partial supersymmetry breaking in Minkowskian ground states. It is important to notice that Minkowskian ground states respect the Lorentz invariance which implies that the vacuum expectation values of all fermions and gauge bosons vanish, *i.e.* $\langle \text{fermions} \rangle = \langle A^\Lambda_\mu \rangle = 0$, and in addition, for the scalar fields we only allow constant vacuum expectation values $\langle \partial_\mu z^i \rangle = \langle \partial_\mu q^u \rangle = 0$. The discussion here follows rather closely [40].

The presence of an unbroken $N = 1$ supersymmetry corresponds to the vanishing supersymmetry transformation (2.27) of the fermions evaluated in the ground states for the unbroken supersymmetry generator

$$\langle \delta \psi_{A\mu} \rangle = \langle \delta \lambda^{iA} \rangle = \langle \delta \zeta^\alpha \rangle = 0 \quad , \tag{3.33}$$

while for the broken generator (3.33) should not hold. We see from (2.23) that equations (3.33) can be viewed as geometrical conditions on the scalar manifold $\mathcal{M}$ and its gauged isometries.

Although (3.33) are equivalent to the geometrical condition on the the scalar manifold, still one cannot directly determine the properties of the couplings $S_{AB}, W^{iAB}, N^A_\alpha$ and its gauged isometries from the definitions (2.23). These provide us to use some additional physical input. First of all from the action (2.20) we see that S_{AB} also is the mass matrix for the two gravitinos. As we have learned from the previous example, the necessary condition for the existence of $N = 1$ ground states is that the two eigenvalues m_{ψ^1}, m_{ψ^2} of S_{AB} are non-degenerate and one has, for example, $m_{\psi^1} < m_{\psi^2}$. In Minkowski ground states one further needs $m_{\psi^1} = 0$ which means that one of the gravitinos stays massless while the other one has to become massive.[7] Furthermore, the unbroken $N = 1$ supersymmetry requires the existence of an $N = 1$ massive spin-$\frac{3}{2}$-multiplet which has spin content $s = (\frac{3}{2}, 1, 1, \frac{1}{2})$ and is degenerate in mass.[8] This

[7]In anti-de Sitter ground states setting $m_{\psi^1} = 0$ is not possible but the two eigenvalues should not be degenerate. See chapter 4.

[8]Such situation is different in anti-de Sitter because the mass operator p^2 is no longer Casimir operator. See appendix B.

implies that one also needs two vectors, say A^0_μ, A^1_μ and a spin-$\frac{1}{2}$ fermion χ have to become massive. Thus, the complete spectrum consists of a gravitino ψ^2_μ together with a Goldstone fermion η, two gauge bosons together with two Goldstone bosons, say ϕ^0, ϕ^1 and a massive fermion χ [17]. The Goldstone bosons and the two gauge bosons indicate that the minimal model require at least a hypermultiplet and a vector multiplet. Thus, in order to display spontaneous $N=2 \to N=1$ the minimal $N=2$ spectrum consists of the $N=2$ gravitational multiplet, one vector multipet, and one hypermultiplet.

Furthermore the two Goldstone bosons ϕ^0, ϕ^1 couple to the gauge fields A^0_μ, A^1_μ only via derivative couplings with respect to two constant Killing vectors k_0, k_1 (defined in (2.17)) which generate isometries on $\mathcal{M}_H$. Let us parameterize them as

$$k^u_\Lambda = g_0\, \delta^{u1}\delta_{\Lambda 0} + g_1 \delta^{u2}\delta_{\Lambda 1} \qquad , \tag{3.34}$$

where g_0, g_1 are constant charges. The existence of these Killing vectors implies that the Lagrangian has a Peccei-Quinn symmetry

$$\phi^{0,1} \to \phi^{0,1} + \varepsilon^{0,1}\, g_{0,1} \quad , \tag{3.35}$$

which can be gauged, that is $\varepsilon^{0,1}$ can be made spacetime dependent. As a consequence the covariant derivatives (2.18) read

$$\nabla_\mu \phi^0 = \partial_\mu \phi^0 + g_0\, A^0_\mu \ , \quad \nabla_\mu \phi^1 = \partial_\mu \phi^1 + g_1\, A^1_\mu \ . \tag{3.36}$$

In geometrical terminology this means that $\mathcal{M}_H$ has to admit two commuting translational $\mathbb{R}^2$-isometries and these isometries have to be gauged [17].[9]

Before turning our attention to the necessary conditions of spontaneous $N=2 \to N=1$ breaking, let us discuss further properties of an $N=2$ theory and in particular the quaternionic geometry $\mathcal{M}_H$ which admits $\mathbb{R}^2$-isometries.

3.3 $N=2$ Theories with $\mathbb{R}^2$-Isometries

Besides two linearly independent Killing vectors k^u_0, k^u_1, we also have via (2.19) two Killing prepotentials P^x_0, P^x_1. They carry $SU(2)$ quantum numbers in that they transform as a triplet. In this representation space two triplets (vectors) span a plane and thus, without loss of generality we can always choose an $SU(2)$ basis where $P^3_0 = P^3_1 = 0$ holds. This choice fixes an $SU(2)$ gauge and leaves a $U(1)$ transformation (corresponding to a rotation in the plane) intact. From (2.23) we see that in this basis S_{AB} is diagonal and given by

$$(S_{AB}) = \begin{pmatrix} S_{11} & 0 \\ 0 & S_{22} \end{pmatrix} \quad , \tag{3.37}$$

where

$$S_{11} = \frac{\mathrm{i}}{2} e^{\frac{1}{2}\mathcal{K}_V} (P^1_\Lambda - \mathrm{i} P^2_\Lambda) X^\Lambda \ , \quad S_{22} = -\frac{\mathrm{i}}{2} e^{\frac{1}{2}\mathcal{K}_V} (P^1_\Lambda + \mathrm{i} P^2_\Lambda) X^\Lambda \ , \tag{3.38}$$

and thus the two gravitino masses are given by

$$m_{\psi^1} = 2|\langle S_{11}\rangle| \ , \quad m_{\psi^2} = 2|\langle S_{22}\rangle| \ . \tag{3.39}$$

[9] Strictly speaking we only need these couplings in the neighborhood of the ground states. However, we do not consider the possible generalization for Minkowski backgrounds in this thesis.

Similarly, (2.23) also implies that W^{iAB} is blockdiagonal and one finds

$$W^{i12} = W^{i21} = 0\ , \quad W^{i11} = 2g^{i\bar{j}}\nabla_{\bar{j}}\bar{S}^{11}\ , \quad W^{i22} = 2g^{i\bar{j}}\nabla_{\bar{j}}\bar{S}^{22}\ , \tag{3.40}$$

where $\nabla_{\bar{j}} = \partial_{\bar{j}} + \frac{1}{2}\mathcal{K}_{V,\bar{j}}$. In addition, we also have

$$\mathcal{U}_u^{1\alpha} N_\alpha^1 = \nabla_u \bar{S}^{11}\ , \quad \mathcal{U}_u^{2\alpha} N_\alpha^2 = \nabla_u \bar{S}^{22}\ . \tag{3.41}$$

Inserting (3.37), (3.40) and (3.41) into (2.25), for the block-diagonal components we arrive at

$$\begin{aligned} V^{N=2} &= 4\left(g^{i\bar{j}}\,\nabla_i S_{11}\nabla_{\bar{j}}\bar{S}^{11} + \frac{1}{2}h^{uv}\,\nabla_u S_{11}\nabla_v \bar{S}^{11} - 3\,\bar{S}^{11}S_{11}\right) \qquad , \\ &= 4\left(g_{i\bar{j}}\,\nabla_i S_{22}\nabla_{\bar{j}}\bar{S}^{22} + \frac{1}{2}h^{uv}\,\nabla_u S_{22}\nabla_v \bar{S}^{22} - 3\,\bar{S}^{22}S_{22}\right) \qquad , \end{aligned} \tag{3.42}$$

and for the off-diagonal components, we have

$$\bar{N}_2^\alpha N_\alpha^1 = \bar{N}_1^\alpha N_\alpha^2 = 0 \qquad . \tag{3.43}$$

In addition the covariant derivatives ∇_u with respect to the $Sp(2)$ connection ω_u^x are obtained from (2.19) which for $P_\Lambda^3 = 0$ simplify as

$$\begin{aligned} \nabla_u P_\Lambda^1 &= \partial_u P_\Lambda^1 - \omega_u^3\,P_\Lambda^2 \quad , \\ \nabla_u P_\Lambda^2 &= \partial_u P_\Lambda^2 + \omega_u^3\,P_\Lambda^1 \quad , \\ \nabla_u P_\Lambda^3 &= \omega_u^1\,P_\Lambda^2 - \omega_u^2\,P_\Lambda^1 \quad . \end{aligned} \tag{3.44}$$

The potential cannot depend on the two Goldstone bosons ϕ^0, ϕ^1. This can explicitly be shown by using the identity

$$K_{uv}^x k_\Lambda^u k_\Sigma^v = \frac{1}{2}\epsilon^{xyz} P_\Lambda^y\,P_\Sigma^z \quad , \tag{3.45}$$

which holds in general for Abelian isometries [51]. Inserting (2.19) and (3.44) into (3.45), it results in

$$k_\Lambda^u\,\omega_u^{1,2} = -P_\Lambda^{1,2}\ , \quad k_\Lambda^u\,\partial_u P_\Sigma^{1,2} = 0\ . \tag{3.46}$$

This explicitly establishes that $V^{N=2}$ is independent of the Goldstone bosons.

Furthermore, the first derivative of the scalar potential (2.26) in this $SU(2)$ gauge takes the form

$$\begin{aligned} \frac{\partial V^{N=2}}{\partial z^k} &= -4g_{k\bar{l}}\,\bar{W}_{11}^{\bar{l}}\,\bar{S}^{11} + 2\mathcal{M}_{k1|i1}\,W^{i11} + \mathcal{M}_{k1}^\alpha\,N_\alpha^1 \quad , \\ &= -4g_{k\bar{l}}\,\bar{W}_{22}^{\bar{l}}\,\bar{S}^{22} + 2\mathcal{M}_{k2|i2}\,W^{i22} + \mathcal{M}_{k2}^\alpha\,N_\alpha^2 \quad , \\ 0 &= \mathcal{M}_{k2}^\alpha\,N_\alpha^1 \quad , \\ \frac{\partial V^{N=2}}{\partial q^u}\mathcal{U}^{1\beta u} &= -4\bar{N}_1^\beta\,\bar{S}^{11} + \frac{1}{2}\mathcal{M}_{i1}^\beta\,W^{i11} + \mathcal{M}^{\beta\alpha}\,N_\alpha^1 \\ &+ \ -\mathbb{C}^{\alpha\beta}\big(-4N_\alpha^2\,S_{22} + \frac{1}{2}\bar{\mathcal{M}}_{\alpha\bar{i}}^2\,\bar{W}_{22}^{\bar{i}} + \bar{\mathcal{M}}_{\alpha\gamma}\,\bar{N}_2^\gamma\big) \quad . \end{aligned} \tag{3.47}$$

Finally, from the action (2.20) and using (3.36) one read off the mass matrix for the gauge bosons to be

$$\mathcal{L}_{\rm b} = h_{uv}\nabla_\mu q^u\,\nabla^\mu q^v = \ldots + \frac{1}{2}m_{\Lambda\Sigma}^2 A_\mu^\Lambda\,A^{\Sigma\mu} \quad , \tag{3.48}$$

where

$$\frac{1}{2} m^2_{\Lambda\Sigma} = h_{uv} k^u_\Lambda \, k^v_\Sigma \quad . \tag{3.49}$$

3.4 Necessary Condition for $N = 2 \to N = 1$

Let us now look at the supersymmetry transformation (2.27). Since we are dealing with Minkowski ground states which respect four dimensional Lorentz invariance, the vacuum expectation values of (2.27) can be simplified as

$$\begin{aligned} \langle \delta\psi_{A\mu} \rangle &= \partial_\mu \epsilon_A + \mathrm{i} \langle S_{AB} \rangle \gamma_\mu \, \epsilon^B \quad , \\ \langle \delta\lambda^{iA} \rangle &= \langle W^{iAB} \rangle \, \epsilon_B \quad , \\ \langle \delta\zeta_\alpha \rangle &= \langle \sqrt{2} N^A_\alpha \rangle \, \epsilon_A \quad , \end{aligned} \tag{3.50}$$

where $\langle \omega^{ab}_\mu \rangle = 0$ because the ground states are flat space.

To have an unbroken $N = 1$ supersymmetry, the equations (3.50) vanish for the unbroken supersymmetry generator. Without loss of generality we choose the unbroken direction represented by the supersymmetry transformation parameter ϵ_1 which is associated with the unbroken supersymmetry generator. Thus the equations (3.33) can be obtained by restricting (3.50) to the unbroken direction ϵ_1. Let us first analyze the supersymmetry variation of the spin-$\frac{1}{2}$ fermions. The solution of $\langle \delta_{\epsilon_1} \lambda^{iA} \rangle = 0$ and $\langle \delta_{\epsilon_1} \zeta_\alpha \rangle = 0$ are given by

$$\begin{aligned} \langle W^{i1A} \rangle &= \langle W^{iA1} \rangle = 0 \quad , \\ \langle N^1_\alpha \rangle &= 0 \quad , \end{aligned} \tag{3.51}$$

respectively. These equations together with the fact that $\langle V^{N=2} \rangle = 0$ simplify (3.42) as

$$\begin{aligned} \langle \bar{S}^{11} S_{11} \rangle &= 0 \quad , \\ \langle g_{i\bar{j}} \, \nabla_i S_{22} \nabla_{\bar{j}} \bar{S}^{22} \rangle + \frac{1}{2} \langle h^{uv} \, \nabla_u S_{22} \nabla_v \bar{S}^{22} \rangle - 3 \, \langle \bar{S}^{22} S_{22} \rangle &= 0 \quad , \end{aligned} \tag{3.52}$$

and (3.43) is trivially satisfied. The first equation in (3.52) means that we further need $m_{\psi^1} = 2|\langle S_{11} \rangle| = 0$. Now let us look this requirement in detail. From (3.38), setting $\langle S_{11} \rangle = 0$ means

$$\langle (P^1_\Lambda - \mathrm{i} P^2_\Lambda) X^\Lambda \rangle = 0 \quad , \quad \Lambda = 0, 1 \quad . \tag{3.53}$$

Trivially, one expects that the Killing prepotentials $P^1_\Lambda = P^2_\Lambda = 0$ are the solutions of (3.53). However, such solutions preserve fully $N = 2$ supersymmetry which we do not want. Thus, we need an additional equation in the ground states. From the first equation of (3.51), *i.e.* $\langle \bar{W}^i_{11} \rangle = 0$, we have

$$\langle (P^1_1 - \mathrm{i} P^2_1) \, \partial_i X^1 \rangle = 0 \quad , \tag{3.54}$$

because X^0 is a constant. So we need to find an appropriate basis for X^1. If the basis of X^Λ is linearly independent, this would give $\partial_i X^1 = \delta^1_i$ and further implies $P^1_1 = P^2_1 = 0$. Then it is impossible to have $N = 1$ ground states because one of the Killing vector $k^u_1 = 0$ and moreover, the gauge boson A^1_μ is no longer massive.[10] Therefore, the choice

[10] In other words, the no go theorem cannot be evaded if the basis of X^Λ is linearly independent.

of basis of X^Λ for our purpose cannot be linearly independent. As we know from chapter 2, the pair $(X^\Lambda, \mathcal{F}_\Lambda)$ transforms under symplectic group $Sp(2n_V + 2, \mathbb{R})$. Now, using this symplectic group our problem can be solved by choosing a symplectic basis where $X^1 \to -\mathcal{F}_1,\ \mathcal{F}_1 \to X^1$. Thus, we obtain

$$\langle \mathcal{F}_{i1} \rangle = 0 \quad . \tag{3.55}$$

Let us denote $(\widetilde{X}^\Lambda, \widetilde{\mathcal{F}}_\Lambda)$ as a new basis with the new prepotential $\widetilde{X}^\Lambda \widetilde{\mathcal{F}}_\Lambda = 2\widetilde{\mathcal{F}}$. Consider now the example in section 3.1. One can check using (3.3) that in this new basis $\widetilde{\mathcal{F}} = 0$. This means that one has to choose to a basis where no $\mathcal{F}$ exists in order to have spontaneous $N = 2 \to N = 1$ in Minkowskian ground states. Indeed such basis has been appeared in the literatures which studied spontaneous $N = 2 \to N = 1$ breaking [16, 17, 47, 48].

Next, we analyze the supersymmetry variation of the gravitino fields. The solutions of $\langle \delta_{\epsilon_1} \psi_{A\mu} \rangle = 0$ consist of

$$\begin{aligned} \langle S_{12} \rangle = \langle S_{21} \rangle &= 0 \quad , \\ \partial_\mu \epsilon_1 + \mathrm{i} \langle S_{11} \rangle \gamma_\mu \epsilon^1 &= 0 \quad , \quad . \end{aligned} \tag{3.56}$$

We see that the first equation of (3.56) and also $\langle W^{i12} \rangle = 0$ do not contradict with the particular $SU(2)$ basis where $P_0^3 = P_1^3 = 0$ in the previous section. Hence, such particular $SU(2)$ basis and our choice of the unbroken parameter ϵ_1 are compatible. Using (3.53) the second equation in (3.51) simplify as

$$\partial_\mu \epsilon_1 = 0 \quad , \tag{3.57}$$

which is called Killing spinor equation. The solution of (3.57) is an arbitrary constant four spinor ϵ_0. Thus the existence of the Killing spinor equation (3.57) explicitly shows that the ground states preserve $N = 1$ supersymmetry.

Furthermore, the ground states require that

$$\left\langle \frac{\partial V^{N=2}}{\partial z^i} \right\rangle = \left\langle \frac{\partial V^{N=2}}{\partial q^u} \right\rangle = 0 \quad , \tag{3.58}$$

and inserting (3.51), the first derivative of the scalar potential (3.47) take the form

$$\begin{aligned} -4\, \langle g_{k\bar{l}}\, \bar{W}^{\bar{l}}_{22}\, \bar{S}^{22} \rangle + 2\, \langle \mathcal{M}_{k2|i2}\, W^{i22} \rangle + \langle \mathcal{M}^\alpha_{k2}\, N^2_\alpha \rangle &= 0 \quad , \\ -4\, \langle \bar{N}^\beta_2\, \bar{S}^{22} \rangle + \frac{1}{2} \langle \mathcal{M}^\beta_{i2}\, W^{i22} \rangle + \langle \mathcal{M}^{\beta\alpha}\, N^2_\alpha \rangle &= 0 \quad . \end{aligned} \tag{3.59}$$

To summarize, we have determined the necessary conditions any model has to obey this necessary conditions in order to display spontaneous $N = 2 \to N = 1$ supersymmetry breaking. By comparing (3.37) for $S_{11} = 0$ with the quantities in (3.11) evaluated at $g_0 = g_1$ in the previous simplest model, clearly it satisfies the requirement for spontaneous $N = 2 \to N = 1$ supersymmetry breaking in Minkowski backgrounds.

3.5 The Higgs and Super-Higgs Effects

Having derived the necessary condition of spontaneous $N = 2 \to N = 1$ supersymmetry breaking, we can now investigate the occurence of the Higgs and super-Higgs effects.

Let us first discuss the Higgs effect. Our starting point is the gauged supersymmetric σ-model (3.48) and the coupling (3.36). The Goldstone bosons ϕ^0, ϕ^1 can be eliminated from the theory by employing the gauge transformation of A^0_μ, A^1_μ which has form

$$A^0_\mu \to A^0_\mu - \frac{1}{g_0}\partial_\mu \phi^0 \ , \quad A^1_\mu \to A^1_\mu - \frac{1}{g_1}\partial_\mu \phi^1 \ . \tag{3.60}$$

Then the resulting theory consists only of the remaining physical scalar fields and the massive gauge bosons. Note that since $\mathbb{R}^2$-isometries are our additional input, one can always perform the gauge tranformation (3.60). However, in general such gauge tranformation (3.60) cannot be done because the couplings g_0, g_1 might explicitly depend on q^u. This occurs for example if we consider the situation where it is far enough from the ground states.

Next, we discuss the super-Higgs effect. In order to see this, let us consider the fermionic part of the Lagrangian (2.20), namely the kinetic and the mass-like term which is related to the broken direction in these flat backgrounds. After rescaling $\sqrt{2}\zeta_\alpha \to \zeta_\alpha$ we can write such terms as

$$\begin{aligned} \mathcal{L}_f &= 2\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}}\bar{\psi}^2_\mu \gamma_\sigma \,\partial_\nu \psi_{2\lambda} - \mathrm{i} g_{i\bar{j}} \bar{\lambda}^{i2}\gamma^\mu \partial_\mu \lambda^{\bar{j}}_2 - \mathrm{i}\,\bar{\zeta}^\alpha \gamma^\mu \partial_\mu \zeta_\alpha \\ &\quad + [2\langle S_{22}\rangle \,\bar{\psi}^2_\mu \gamma^{\mu\nu}\psi^2_\nu + \mathrm{i}\langle g_{i\bar{j}} W^{i22}\rangle \bar{\lambda}^{\bar{j}}_2 \gamma^\mu \psi_{2\mu} + \mathrm{i}\sqrt{2}\langle N^2_\alpha\rangle \bar{\zeta}^\alpha \gamma^\mu \psi^2_\mu \\ &\quad + \frac{1}{2}\langle \mathcal{M}^{\alpha\beta}\rangle \,\bar{\zeta}_\alpha \zeta_\beta + \frac{1}{\sqrt{2}}\langle \mathcal{M}^\alpha_{i2}\rangle \,\bar{\zeta}_\alpha \lambda^{i2} + \langle \mathcal{M}_{i2|l2}\rangle \,\bar{\lambda}^{i2}\lambda^{l2} + \mathrm{h.c.}] \quad . \end{aligned} \tag{3.61}$$

The mixing of the spin-$\frac{1}{2}$ fermion $(\lambda^{i2}, \zeta_\alpha)$ to the gravitino field ψ^2_μ plays an important role to identify a massless fermion (called Goldstone fermion)

$$\eta_2 = \langle g_{\bar{i}j} \bar{W}^{\bar{i}}_{22}\rangle \lambda^{j2} + \sqrt{2}\,\langle \bar{N}^\alpha_2\rangle \zeta_\alpha \quad . \tag{3.62}$$

To see that η_2 is indeed a Goldstone fermion, let us first write (3.50) for gaugino λ^{iA} and hyperino ζ_α restricted to the broken parameter ϵ_2,

$$\begin{aligned} \langle \delta\lambda^{i2}\rangle &= \langle W^{i22}\rangle \,\epsilon_2 \quad , \\ \langle \delta\zeta_\alpha\rangle &= \langle \sqrt{2} N^2_\alpha\rangle \,\epsilon_2 \quad , \end{aligned} \tag{3.63}$$

where the right hand side is non-zero. Then using (3.63) and the fact that the cosmological constant is zero, *i.e.* (3.52), the vacuum expectation value of the supersymmetry variation of η_2 is given by

$$\langle \delta\eta_2\rangle = 12\,\langle \bar{S}^{22} S_{22}\rangle \,\epsilon_2 \quad . \tag{3.64}$$

We see that η_2 transform by a shift and furthermore, indicates that η_2 is a Goldstone fermion. A fermion with a supersymmetry transformation like (3.64) can be removed from the theory by a suitable local supersymmetry transformation of the gravitino ψ^2_μ. This is the super-Higgs effect.

Before writing down the suitable local supersymmetry transformation of the gravitino ψ^2_μ, let us first define physical fermions $(\zeta^\perp_\alpha, \lambda^{i2}_\perp)$ as

$$\zeta^\perp_\alpha \;=\; \zeta_\alpha - \frac{\sqrt{2}\langle N^2_\alpha\rangle}{12\langle \bar{S}^{22} S_{22}\rangle}\,\eta_2 \quad ,$$

$$\lambda_\perp^{i2} \;=\; \lambda^{i2} - \frac{\langle W^{i22}\rangle}{12\langle \bar{S}^{22} S_{22}\rangle}\,\eta_2 \quad . \tag{3.65}$$

Then it can be shown by using (3.63) that the physical fermions $(\zeta_\alpha^\perp, \lambda_\perp^{i2})$ cannot be gauged away by any field redefinition of the gravitino ψ_μ^2 because

$$\langle \delta\lambda_\perp^{i2}\rangle = 0 \;, \qquad \langle \delta\zeta_\alpha^\perp\rangle = 0 \;. \tag{3.66}$$

Now, we split off the Goldstone mode η_2 and the physical fermions $(\zeta_\alpha^\perp, \lambda_\perp^{i2})$ in the Lagrangian (3.61) and it can be written down as

$$\begin{aligned}
\mathcal{L}_f \;=\;& 2\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}}\bar{\psi}_\mu^2\gamma_\sigma\,\partial_\nu\psi_{2\lambda} - \mathrm{i}g_{i\bar{j}}\bar{\lambda}_\perp^{i2}\gamma^\mu\partial_\mu\lambda_{2\perp}^{\bar{j}} - \mathrm{i}\,\bar{\zeta}^{\alpha\perp}\gamma^\mu\partial_\mu\zeta_\alpha^\perp \\
&-\frac{\mathrm{i}}{12\langle \bar{S}^{22} S_{22}\rangle}\bar{\eta}_2\gamma^\mu\partial_\mu\eta^2 + [2\langle S_{22}\rangle\,\bar{\psi}_\mu^2\gamma^{\mu\nu}\psi_\nu^2 + \mathrm{i}\bar{\eta}_2\gamma^\mu\psi_\mu^2 \\
&+\frac{1}{6\langle S_{22}\rangle}\bar{\eta}_2\,\eta_2 + \langle \mathcal{M}^{(0)\alpha\beta}\rangle\bar{\zeta}_\alpha^\perp\,\zeta_\beta^\perp + 2\langle \mathcal{M}_{i2}^{(0)\alpha}\rangle\,\bar{\zeta}_\alpha^\perp\lambda_\perp^{i2} \\
&+\langle \mathcal{M}_{i2|l2}^{(0)}\rangle\,\bar{\lambda}_\perp^{i2}\lambda_\perp^{l2} + \text{h.c.}] \quad .
\end{aligned} \tag{3.67}$$

where[11]

$$\begin{aligned}
\langle \mathcal{M}_{i2|l2}^{(0)}\rangle \;&=\; \langle \mathcal{M}_{i2|l2}\rangle - \frac{\langle g_{i\bar{j}}\,g_{l\bar{k}}\,\bar{W}_{22}^{\bar{j}}\,\bar{W}_{22}^{\bar{k}}\rangle}{6\langle S_{22}\rangle} \quad , \\
\langle \mathcal{M}_{i2}^{(0)\alpha}\rangle \;&=\; \frac{1}{2\sqrt{2}}\langle \mathcal{M}_{i2}^\alpha\rangle - \frac{\sqrt{2}\,\langle g_{i\bar{j}}\,\bar{W}_{22}^{\bar{j}}\,\bar{N}_2^\alpha\rangle}{6\langle S_{22}\rangle} \quad , \\
\langle \mathcal{M}^{(0)\alpha\beta}\rangle \;&=\; \frac{1}{2}\langle \mathcal{M}^{\alpha\beta}\rangle - \frac{\langle \bar{N}_2^\alpha\,\bar{N}_2^\beta\rangle}{3\langle S_{22}\rangle}
\end{aligned} \tag{3.68}$$

are the physical masses of the fermions $(\zeta_\alpha^\perp, \lambda_\perp^{i2})$ for vanishing cosmological constant denoted by superscript (0).

To find a suitable redifinition of the gravitino ψ_μ^2, we need the form of its supersymmetry variation in the ground states. From the first equation in (3.50) restricted to the broken parameter ϵ_2, one finds

$$\langle \delta\psi_{2\mu}\rangle = \partial_\mu\epsilon_2 + \mathrm{i}\langle S_{22}\rangle\gamma_\mu\epsilon^2 \quad . \tag{3.69}$$

In addition, (3.64) means that the Goldstone fermion η_2 can be regarded as a supersymmetry transformation parameter and thus one can introduce a replacement

$$\epsilon_2 \to \frac{1}{12\langle \bar{S}^{22} S_{22}\rangle}\,\eta_2 \quad . \tag{3.70}$$

Hence, from (3.69) and (3.70) one concludes that the local supersymmetry transformation of the gravitino ψ_μ^2 which eliminates the Goldstone fermion η_2 from the Lagrangian (3.67) should have form

$$\psi_\mu^2 \to \psi_\mu^2 + \frac{1}{12\langle \bar{S}^{22} S_{22}\rangle}\left(\partial_\mu\eta^2 + \mathrm{i}\langle \bar{S}^{22}\rangle\,\gamma_\mu\,\eta_2\right) \quad . \tag{3.71}$$

[11] See (3.79) for the explicit form of $\langle \mathcal{M}_{i2|l2}^{(0)}\rangle$ and $\langle \mathcal{M}_{i2}^\alpha\rangle$.

We want to notice that the transformation (3.71) could not be defined if $\langle S_{22}\rangle = 0$, which is precisely the condition for unbroken supersymmetry in the Minkowski backgrounds. The supersymmetry transformation required to remove the Goldstone fermion would then be singular and in fact the splitting off in the Lagrangian (3.67) would not be possible as the physical fermions would diverge.

Inserting the trasformation (3.71) into the Lagrangian (3.67), the Goldstone fermion does vanish from the theory. The Lagrangian (3.67) takes the form

$$\begin{aligned}\mathcal{L}_f &= 2\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}}\bar{\psi}^2_\mu\gamma_\sigma\,\partial_\nu\psi_{2\lambda} - \mathrm{i}g_{i\bar{j}}\bar{\lambda}^{i2}_\perp\gamma^\mu\partial_\mu\lambda^{\bar{j}}_{2\perp} - \mathrm{i}\,\bar{\zeta}^{\alpha\perp}\gamma^\mu\partial_\mu\zeta^\perp_\alpha \\ &\quad +[2\langle S_{22}\rangle\,\bar{\psi}^2_\mu\gamma^{\mu\nu}\psi^2_\nu + \langle\mathcal{M}^{(0)\alpha\beta}\rangle\bar{\zeta}^\perp_\alpha\,\zeta^\perp_\beta + 2\langle\mathcal{M}^{(0)\alpha}_{i2}\rangle\,\bar{\zeta}^\perp_\alpha\lambda^{i2}_\perp \\ &\quad +\langle\mathcal{M}^{(0)}_{i2|l2}\rangle\,\bar{\lambda}^{i2}_\perp\lambda^{l2}_\perp + \mathrm{h.c.}]\quad .\end{aligned} \tag{3.72}$$

where we have used the property of the flat space, i.e. the Riemannian curvature is zero or $[\partial_\mu,\partial_\nu] = 0$. We see that the Lagrangian (3.72) only contains the physical massive fields. The massive gravitino ψ^2_μ has four instead of two degrees of freedom as a consequence of spontaneous local supersymmetry breaking.

To give an explicit expression of the physical mass matrix (3.68) we need some additional equations in the ground states. First, let us consider the first equation in (C.2) for $\Lambda,\Sigma = 0,1$. Since $\langle\partial_i X^\Lambda\rangle = 0$ for $\Lambda = 0,1$ in the ground states, the first equation in (C.2) becomes

$$\langle g^{i\bar{j}}\mathcal{K}_{V,i}\,\mathcal{K}_{V,\bar{j}}\,L^\Lambda\bar{L}^\Sigma\rangle = -\frac{1}{2}\langle(\mathcal{I}^{-1})^{\Lambda\Sigma}\rangle - \langle\bar{L}^\Lambda L^\Sigma\rangle \quad . \tag{3.73}$$

Antisymmetrizing (3.73), one obtains

$$\langle g^{i\bar{j}}\mathcal{K}_{V,i}\mathcal{K}_{V,\bar{j}}\rangle = 1 \quad , \tag{3.74}$$

while the symmetric part of (3.73) gives

$$\langle(\mathcal{I}^{-1})^{\Lambda\Sigma}\rangle = -2\langle e^{\mathcal{K}_V}\,(X^\Lambda\bar{X}^\Sigma + \bar{X}^\Lambda X^\Sigma)\rangle \quad . \tag{3.75}$$

Furthermore, evaluated the second equation of (C.2) in the ground states, we have

$$\langle\nabla_i\,f^\Lambda_j\rangle = \mathrm{i}\langle C_{ijk}\,g^{k\bar{l}}\mathcal{K}_{V,\bar{l}}\,\bar{L}^\Lambda\rangle \quad . \tag{3.76}$$

The equation (3.76) together with (3.53) can be used to show

$$\langle\mathcal{M}_{k2|i2}\rangle = 0 \quad . \tag{3.77}$$

In the ground states the quantities W^{i22} and $\mathcal{M}^{(0)\alpha}_{k2}$ simplify as

$$\begin{aligned}\langle W^{i22}\rangle &= 2\langle\bar{S}^{22}g^{i\bar{j}}\mathcal{K}_{V,\bar{j}}\rangle \quad , \\ \langle\mathcal{M}^\alpha_{k2}\rangle &= 2\langle\mathcal{K}_{V,k}\,\bar{N}^\alpha_2\rangle \quad .\end{aligned} \tag{3.78}$$

Now, using (3.74), (3.77) and (3.78) the physical mass (3.68) have the form

$$\langle\mathcal{M}^{(0)}_{i2|l2}\rangle = -\frac{2}{3}\,\langle\mathcal{K}_{V,i}\,\mathcal{K}_{V,l}\,S_{22}\rangle \quad ,$$

$$\langle \mathcal{M}^{(0)\alpha}_{i2} \rangle = \frac{\sqrt{2}}{6} \langle \mathcal{K}_{V,i} \bar{N}^\alpha_2 \rangle \quad , \tag{3.79}$$

where $\langle \mathcal{M}^{(0)\alpha\beta} \rangle$ have the same form as in (3.68).

Finally, we can discuss the properties of the physical mass (3.68), namely these must have a zero eigenvalue and are degenerate with the gravitino mass $\langle S_{22} \rangle$. Let us first check the previous property. Our starting point is to check that the following equations

$$\begin{aligned} &\langle \mathcal{M}^{(0)}_{k2|i2} W^{i22} \rangle + \sqrt{2} \langle \mathcal{M}^{(0)\alpha}_{k2} N^2_\alpha \rangle \quad , \\ &\langle \mathcal{M}^{(0)\beta}_{i2} W^{i22} \rangle + \sqrt{2} \langle \mathcal{M}^{(0)\beta\alpha} N^2_\alpha \rangle \quad , \end{aligned} \tag{3.80}$$

vanish. Using (3.79) together with (3.52), we can show that

$$\begin{aligned} \langle \mathcal{M}^{(0)}_{k2|i2} W^{i22} \rangle + \sqrt{2} \langle \mathcal{M}^{(0)\alpha}_{k2} N^2_\alpha \rangle &= \frac{1}{3} \mathcal{K}_{V,k} \left(-4 \langle \bar{S}^{22} S_{22} \rangle + \langle \bar{N}^\alpha_2 N^2_\alpha \rangle \right) \\ &= 0 \quad , \end{aligned} \tag{3.81}$$

while for the last equation in (3.80), we use (3.52) and the second equations in (3.59) to show

$$\begin{aligned} \langle \mathcal{M}^{(0)\beta}_{i2} W^{i22} \rangle + \sqrt{2} \langle \mathcal{M}^{(0)\beta\alpha} N^2_\alpha \rangle &= \frac{1}{\sqrt{2}} \left(\langle \mathcal{M}^{(0)\beta\alpha} N^2_\alpha \rangle - 2 \langle \bar{S}^{22} \bar{N}^\beta_2 \rangle \right) \\ &= 0 \quad . \end{aligned} \tag{3.82}$$

This proves that there exist a vector $(W^{i22}, \sqrt{2} N^2_\alpha)$ such that the physical masses (3.68) has a zero eigenvalue which means the physical mode is already separated from the Goldstone mode.

The second property can be shown by using the second equation in (3.59) and (3.79), the first equation of (3.78) together with (3.52) that

$$\begin{aligned} \langle \mathcal{M}^{(0)}_{k2|i2} W^{i22} \rangle - \frac{1}{\sqrt{2}} \langle \mathcal{M}^{(0)\alpha}_{k2} N^2_\alpha \rangle &= -\langle \bar{S}^{22} g_{k\bar{l}} \bar{W}^{\bar{l}}_{22} \rangle \quad , \\ \langle \mathcal{M}^{(0)\beta}_{i2} W^{i22} \rangle - \frac{1}{\sqrt{2}} \langle \mathcal{M}^{(0)\beta\alpha} N^2_\alpha \rangle &= \frac{1}{\sqrt{2}} \langle \bar{S}^{22} \bar{N}^\beta_2 \rangle \quad . \end{aligned} \tag{3.83}$$

This proves the existence of a vector $(W^{i22}, \frac{1}{\sqrt{2}} N^2_\alpha)$ such that the physical fermion mass (3.68) has an eigenvalue $\langle \bar{S}^{22} \rangle$, or in other words the gravitino ψ^2_μ and the physical fermion $(\zeta^\perp_\alpha, \lambda^{i2}_\perp)$ are degenerate in mass.

Chapter 4

Spontaneous $N = 2 \rightarrow N = 1$ SUSY Breaking in Anti-de Sitter

In this chapter we extend the previous analysis to curved backgrounds with negative cosmological constant, namely anti-de Sitter spaces. Our analysis here uses appendix B where the aspects of anti-de Sitter supersymmetry are mainly reviewed. Furthermore, the general setting in this chapter is the same as in section 3.3. First, we start by discussing the $N = 1$ massive gravitino multiplet with its mass relation and then describe general picture of spontaneous $N = 2 \rightarrow N = 1$ breaking. Next, we derive the necessary conditions of such breaking and finally, discuss the super-Higgs effect.

4.1 General Picture in Anti-de Sitter

First of all let us illustrate the general description of the spontaneous $N = 2 \rightarrow N = 1$ supersymmetry breaking in curved backgrounds with negative cosmological constant. These backgrounds respect the local Lorentz invariance and naturally appear as solution of the supersymmetry transformation (2.27) which maintain a residual $N = 1$ supersymmetry in the ground states. However, so far no simple model with spontaneous $N = 2 \rightarrow N = 1$ supersymmetry breaking in these backgrounds has been found in the literature.

In this section we first discuss the definition of physical mass for the $N = 1$ massive gravitino multiplet because this problem is rather delicate. For this purpose, we review several facts about anti-de Sitter representation of this massive multiplet and then construct its Lagrangian. At the end we discuss a particular property of spontaneous $N = 2 \rightarrow N = 1$ supersymmetry breaking in anti-de Sitter space.

Now we turn to anti-de Sitter ground states which preserve $N = 1$ supersymmetry. It follows that the unbroken $N = 1$ supersymmetry ensures the existence of the $N = 1$ massive gravitino multiplet which has spin content $s = (\frac{3}{2}, 1, 1, \frac{1}{2})$. To make it clear, let us consider anti-de Sitter representation of the $N = 1$ massive gravitino multiplet. Using the massive multiplet (B.33) for $s = 1$, we get

$$D(E_0 + \frac{1}{2}, \frac{3}{2}) \oplus D(E_0, 1) \oplus D(E_0 + 1, 1) \oplus D(E_0 + \frac{1}{2}, \frac{1}{2}) \quad , \tag{4.1}$$

where $D(E_0, s)$ denote the unitary irreducible representation of the non-compact group $SO(2,3)$ and E_0, s denote energy and spin, respectively which are the eigenvalues of the diagonal operators of the maximal compact subgroup $SO(2) \times SO(3) \subset SO(2,3)$.[1] To see the mass relation between the particles in (4.1), we use the mass formula (B.24). Replacing the energy label E_0 in (B.24) by $E_0 + \frac{1}{2}$ for the gravitino, $E_0 + 1, E_0$ for the two gauge bosons, and $E_0 + \frac{1}{2}$ for spin-$\frac{1}{2}$ fermion, we obtain the physical mass of each particle in (4.1) in the following:

$$\begin{aligned}
(m_{\frac{3}{2}} - \ell)^2 &= \ell^2 (E_0 - 1)^2 \quad , \\
m_1^2 &= \ell^2 (E_0 - 2)(E_0 - 1) \quad , \\
(m_1')^2 &= \ell^2 E_0 \, (E_0 - 1) \quad , \\
m_{\frac{1}{2}}^2 &= \ell^2 (E_0 - 1)^2 \quad ,
\end{aligned} \tag{4.2}$$

for $E_0 > 2$, where ℓ is a constant related to the cosmological constant Λ_0 via $\Lambda_0 = -3\ell^2$, m_1' is the mass of the gauge bosons with the energy label $E_0 + 1$, while m_1 is the mass for the gauge bosons with the energy label E_0. Furthermore, the physical mass in (4.2) satisfy the mass relation (B.37)

$$-4(m_{\frac{3}{2}} - \ell)^2 + 3m_1^2 + 3(m_1')^2 - 2m_{\frac{1}{2}}^2 = 0 \quad . \tag{4.3}$$

We see from (4.1) that the three fields, *e.g.* the gravitino and the two gauge bosons, have different energy labels. This means that these fields are not degenerate in mass. Such situation occur in anti-de Sitter backgrounds because the mass operator $P^a P_a$ is no longer Casimir operator.

Next, we construct the Lagrangian of the $N = 1$ massive gravitino multiplet. Due to its spin content, the Lagrangian should have form[2]

$$\begin{aligned}
\mathcal{L}_{\text{gravitino}} = \quad & \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}} \bar{\psi}^2_\mu \gamma_\sigma \mathcal{D}_\nu \psi_{2,\lambda} - \mathrm{i}\, \bar{\chi} \gamma^\mu \mathcal{D}_\mu \chi - \frac{1}{4} F^0_{\mu\nu} F^{0\mu\nu} - \frac{1}{4} F^1_{\mu\nu} F^{1\mu\nu} \\
& + \frac{1}{2} m^2_{A^0}\, A^{0\mu} A^0_\mu + \frac{1}{2} m^2_{A^1}\, A^{1\mu} A^1_\mu \\
& + \left(m_{\psi^2}\, \bar{\psi}^2_\mu \gamma^{\mu\nu} \psi^2_\nu + m_\chi\, \bar{\chi}\chi + \text{h.c.} \right) \quad ,
\end{aligned} \tag{4.4}$$

and the mass parameters in (4.4) are not the physical masses given in (4.2) but instead obey the relation

$$\begin{aligned}
m_{\psi^2} &= m + 2\ell \quad , \\
m^2_{A^0} &= m(m - \ell) \quad , \\
m^2_{A^1} &= m(m + \ell) \quad , \\
m_\chi &= m \quad ,
\end{aligned} \tag{4.5}$$

where $m = \ell(E_0 - 1)$ and $\mathcal{D}_\mu$ is the anti-de Sitter covariant derivative. For gravitino ψ_μ, the physical mass is defined to be $m_{\frac{3}{2}} \equiv m_{\psi^2} - \ell = m + \ell$ [53,54]. An interesting feature of this definition is the case where $m_{\psi^2} = \ell$. It then follows that $m_{\frac{3}{2}} = 0$. In view of the Rarita-Schwinger field equation this simply means that we are dealing with a 'massless'

[1] For an extended discussion about anti-de Sitter supersymmetry see appendix B and the references given there.

[2] For a review of the Lagrangian of the $N = 1$ massive gravitino multiplet in anti-de Sitter space, see [14].

gravitino fields in anti-de Sitter space. On the other hand, the physical mass of the two gauge bosons A^0_μ, A^1_μ are $m_{A^0} = m_1 = \sqrt{m(m-\ell)}$ and $m_{A^1} = m_1' = \sqrt{m(m+\ell)}$ respectively, while the physical mass of the spin-$\frac{1}{2}$-fermion χ is $m_{\frac{1}{2}} = m$. Thus, the members of the $N = 1$ gravitino multiplet are not degenerate in mass.

Finally, let us turning our attention to the necessary condition for the existence of $N = 1$ ground states that the two eigenvalues of the mass parameter S_{AB} satisfy, for example, $m_{\psi^1} < m_{\psi^2}$. From the discussion in the previous paragraph, we see that a massless gravitino in anti-de Sitter space corresponds to the case where the mass parameter in the Lagrangian is simply $m_{\psi^1} = \ell$ and non-zero. This shows that one cannot set $m_{\psi^1} = 0$ in anti-de Sitter space.

4.2 Necessary Condition for $N = 2 \to N = 1$

Let us now consider the supersymmetry transformation (2.27). Again Lorentz invariance simplifies the vacuum expectation values of (2.27) as

$$\begin{aligned} \langle \delta\psi_{A\mu} \rangle &= \mathcal{D}_\mu \epsilon_A + \mathrm{i}\langle S_{AB} \rangle \gamma_\mu \, \epsilon^B \quad , \\ \langle \delta\lambda^{iA} \rangle &= \langle W^{iAB} \rangle \, \epsilon_B \quad , \\ \langle \delta\zeta_\alpha \rangle &= \langle \sqrt{2} N^A_\alpha \rangle \, \epsilon_A \quad , \end{aligned} \tag{4.6}$$

where $\mathcal{D}_\mu = \partial_\mu - \frac{1}{4}\langle \omega^{ab}_\mu \rangle$ is an anti-de Sitter covariant derivative.

In order to maintain an unbroken $N = 1$ supersymmetry in the ground states, (4.6) has to vanish for the unbroken supersymmetry generator. In this case, we also choose the supersymmetry transformation parameter ϵ_1 to be the unbroken direction associated with the unbroken supersymmetry generator. Let us first analyze the supersymmetry variation of the gaugino λ^{iA} and the hyperino ζ_α. The solutions of the variations $\langle \delta_{\epsilon_1} \lambda^{iA} \rangle = 0$ and $\langle \delta_{\epsilon_1} \zeta_\alpha \rangle = 0$ are given by

$$\begin{aligned} \langle W^{i1A} \rangle &= \langle W^{iA1} \rangle = 0 \quad , \\ \langle N^1_\alpha \rangle &= 0 \quad . \end{aligned} \tag{4.7}$$

We see that compared with (3.51) no modification arises at the level of spin-$\frac{1}{2}$ fermion. The anti-de Sitter ground states further require $\langle V^{N=2} \rangle = \Lambda_0$. Then the potential (3.42) in these ground states can be simplified as

$$\begin{aligned} -\,12\langle \bar{S}^{11} S_{11} \rangle &= \Lambda_0 \quad , \\ 4\,\langle g_{i\bar{j}}\, \nabla_i S_{22} \nabla_{\bar{j}} \bar{S}^{22} \rangle + 2\,\langle h^{uv}\, \nabla_u S_{22} \nabla_v \bar{S}^{22} \rangle - 12\,\langle \bar{S}^{22} S_{22} \rangle &= \Lambda_0 \quad , \end{aligned} \tag{4.8}$$

and the off-diagonal components of (3.43)

$$\langle \bar{N}^\alpha_2 N^1_\alpha \rangle = \langle \bar{N}^\alpha_1 N^2_\alpha \rangle \equiv 0 \quad . \tag{4.9}$$

are trivially satisfied. As we see from the first equation in (4.8), $m_{\psi^1} = 2|\langle S_{11} \rangle|$ cannot be set to zero but rather generates the cosmological constant Λ_0. Using (3.38), this implies

$$\langle \mathrm{i} e^{\frac{1}{2}\mathcal{K}_V} (P^1_\Lambda - \mathrm{i} P^2_\Lambda) X^\Lambda \rangle = \sqrt{-\frac{\Lambda_0}{3}} \quad , \quad \Lambda = 0, 1 \quad . \tag{4.10}$$

Furthermore, $\langle W^{i11} \rangle = 0$ give an additional condition

$$\langle \mathrm{i} e^{\frac{1}{2}\mathcal{K}_V} (P^1_\Lambda - \mathrm{i} P^2_\Lambda) g^{\bar{j}i} \partial_i \, X^\Lambda \rangle = -\langle g^{\bar{j}i} \mathcal{K}_{V,i} \rangle \sqrt{-\frac{\Lambda_0}{3}} \quad . \tag{4.11}$$

A look at (4.11), it is not obvious to find an approriate basis for X^Λ where the no go theorem can be avoided. However, as the cosmological constant $\Lambda_0 \to 0$, we should retrieve (3.55). Therefore, the basis of X^Λ cannot be linearly independent.

Now let us consider the supersymmetry variation of the gravitino fields. The solutions of $\langle \delta_{\epsilon_1} \psi_{A\mu} \rangle = 0$ are given by

$$\begin{aligned} \langle S_{12} \rangle = \langle S_{21} \rangle &= 0 \quad , \\ \mathcal{D}_\mu \epsilon_1 + \mathrm{i} \langle S_{11} \rangle \gamma_\mu \, \epsilon^1 &= 0 \quad , \end{aligned} \tag{4.12}$$

We see that $\langle S_{21} \rangle = 0$ and $\langle W^{i21} \rangle = 0$ are compatible with our choice of the $SU(2)$ basis where $P_0^3 = P_1^3 = 0$ holds. Thus, we have a set of consistent equation. The second equation in (4.12) is a Killing spinor equation in anti-de Sitter space. To solve this let us take an ansatz of metric [55, 56]

$$ds^2 = e^{2m_{\psi^1} z} \, \eta_{\underline{ab}} \, dx^{\underline{a}} \, dx^{\underline{b}} - dz^2 \quad , \tag{4.13}$$

where $\underline{a}, \underline{b} = 0, 1, 2$ and $m_{\psi^1} = 2\langle S_{11} \rangle = \sqrt{-\Lambda_0/3}$. With this choice of metric, the Ricci curvature is $R_{\mu\nu} = \Lambda_0 \, g_{\mu\nu}$ which is indeed anti-de Sitter space. The solution of the second equation in (4.12) is then given by

$$\epsilon = e^{\frac{\mathrm{i}}{2} m_{\psi^1} z \gamma_3} \left(1 + \frac{\mathrm{i}}{2} m_{\psi^1} \, x^{\underline{a}} \, \gamma_{\underline{a}} (1 - \mathrm{i}\gamma_3) \right) \epsilon_0 \quad , \tag{4.14}$$

where ϵ_0 is an arbitrary constant four spinor. Thus the existence of Killing spinor ensure the presence of residual $N = 1$ supersymmetry in the ground states.

Finally we want to mention that the ground states require the vanishing of the first derivative of the scalar potential (3.47). Again the resulting equations are described by (3.59).

To summarize, we have derived the necessary condition of spontaneous $N = 2 \to N = 1$ supersymmetry breaking in anti-de Sitter backgrounds. As we see from the first equation of (4.8), the cosmological constant Λ_0 is negative and furthermore, the second equation of (4.12) shows the existence of a Killing spinor in the ground states. Thus, anti-de Sitter backgrounds appear naturally as a solution in order to have a residual $N = 1$ supersymmetry in the ground states.

4.3 The Super-Higgs Effects

In this section we mainly discuss the super-Higgs effect because the Higgs effect in anti-de Sitter backgrounds is similar as in Minkowski backgrounds. The difference is that the two gauge bosons are not degenerate in mass. For the super-Higgs effect, the situation is slightly modified from Minkowski backgrounds but the philosophy is the same: one needs to find a unitary gauge in which a massless spin-$\frac{1}{2}$ fermion is eaten by a gravitino which acquires a mass.

To discuss the super-Higgs effect, let us consider the fermionic part of the Lagrangian (2.20), namely the kinetic and the mass-like term which is related to the broken direction in these curved backgrounds. After rescaling $\sqrt{2}\zeta_\alpha \to \zeta_\alpha$ such terms can be written down as[3]

$$\mathcal{L}_f(\Lambda_0) \quad = \quad 2 \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}} \bar{\psi}^2_\mu \gamma_\sigma \, \mathcal{D}_\nu \psi_{2\lambda} - \mathrm{i} g_{i\bar{j}} \bar{\lambda}^{i2} \gamma^\mu \mathcal{D}_\mu \lambda_2^{\bar{j}} - \mathrm{i} \, \bar{\zeta}^\alpha \gamma^\mu \mathcal{D}_\mu \zeta_\alpha$$

[3] As $\Lambda_0 \to 0$, then $\mathcal{L}_f(\Lambda_0) \to \mathcal{L}_f$, see (3.61).

$$+\Bigg(2\langle S_{22}\rangle\,\bar\psi^2_\mu\gamma^{\mu\nu}\psi^2_\nu+\mathrm{i}\langle g_{i\bar j}W^{i22}\rangle\bar\lambda^{\bar j}_2\gamma^\mu\psi_{2\mu}+\mathrm{i}\sqrt{2}\langle N^2_\alpha\rangle\bar\zeta^\alpha\gamma^\mu\psi^2_\mu \qquad (4.15)$$
$$+\frac{1}{2}\langle\mathcal{M}^{\alpha\beta}\rangle\,\bar\zeta_\alpha\,\zeta_\beta+\frac{1}{\sqrt{2}}\langle\mathcal{M}^\alpha_{i2}\rangle\,\bar\zeta_\alpha\lambda^{i2}+\langle\mathcal{M}_{i2|l2}\rangle\,\bar\lambda^{i2}\lambda^{l2}+\mathrm{h.c.}\Bigg)\quad .$$

Focusing on the mixing spin-$\frac{1}{2}$ fermion $(\lambda^{i2},\zeta_\alpha)$ and the gravitino field ψ^2_μ term, we arrive at the identification of a fermion

$$\eta_2=\langle g_{\bar i j}\bar W^{\bar i}_{22}\rangle\lambda^{j2}+\sqrt{2}\,\langle\bar N^\alpha_2\rangle\zeta_\alpha\quad . \qquad (4.16)$$

To see its supersymmetry transformation in the ground states, let us first consider (4.6) for gaugino λ^{iA} and hyperino ζ_α restricted to the broken parameter ϵ_2,

$$\begin{aligned}\langle\delta\lambda^{i2}\rangle &= \langle W^{i22}\rangle\,\epsilon_2\quad ,\\ \langle\delta\zeta_\alpha\rangle &= \langle\sqrt{2}N^2_\alpha\rangle\,\epsilon_2\quad ,\end{aligned} \qquad (4.17)$$

where the right hand side is non-zero. Using (4.17) together with the fact that $\langle V^{N=2}\rangle=\Lambda_0$, we find that the vacuum expectation values of the supersymmetry transformation of η_2 is given by

$$\langle\delta\eta_2\rangle=(12\,\langle\bar S^{22}S_{22}\rangle+\Lambda_0)\,\epsilon_2\quad . \qquad (4.18)$$

Thus η_2 transforms non-trivially which shows that η_2 is a Goldstone fermion. Thus, η_2 can be gauged away from the theory by a suitable local supersymmetry transformation of the gravitino ψ^2_μ.

Next we define define physical fermions $(\zeta^\perp_\alpha,\lambda^{i2}_\perp)$ as

$$\begin{aligned}\zeta^\perp_\alpha &= \zeta_\alpha-\frac{\sqrt{2}\langle N^2_\alpha\rangle}{12\langle\bar S^{22}S_{22}\rangle+\Lambda_0}\,\eta_2\quad ,\\ \lambda^{i2}_\perp &= \lambda^{i2}-\frac{\langle W^{i22}\rangle}{12\langle\bar S^{22}S_{22}\rangle+\Lambda_0}\,\eta_2\quad .\end{aligned} \qquad (4.19)$$

Then using (4.17), the physical fermions $(\zeta^\perp_\alpha,\lambda^{i2}_\perp)$ cannot be gauged away by any gauge transformation of the gravitino ψ^2_μ since

$$\langle\delta\lambda^{i2}_\perp\rangle=0\ ,\qquad\langle\delta\zeta^\perp_\alpha\rangle=0\ . \qquad (4.20)$$

Now, we split off the Goldstone mode η_2 and the physical fermions $(\zeta^\perp_\alpha,\lambda^{i2}_\perp)$ in the Lagrangian (4.15) which can be written down as

$$\begin{aligned}\mathcal{L}_f(\Lambda_0) &= 2\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}}\bar\psi^2_\mu\gamma_\sigma\,\mathcal{D}_\nu\psi_{2\lambda}-\mathrm{i}\,g_{i\bar j}\bar\lambda^{i2}_\perp\gamma^\mu\mathcal{D}_\mu\lambda^{\bar j}_{2\perp}-\mathrm{i}\,\bar\zeta^{\alpha\perp}\gamma^\mu\mathcal{D}_\mu\zeta^\perp_\alpha\\ &\quad -\frac{\mathrm{i}}{12\langle\bar S^{22}S_{22}\rangle+\Lambda_0}\,\bar\eta_2\gamma^\mu\mathcal{D}_\mu\eta^2+\Big\{2\langle S_{22}\rangle\,\bar\psi^2_\mu\gamma^{\mu\nu}\psi^2_\nu+\mathrm{i}\bar\eta_2\gamma^\mu\psi^2_\mu\\ &\quad +\frac{2\langle\bar S^{22}\rangle}{12\langle\bar S^{22}S_{22}\rangle+\Lambda_0}\,\bar\eta_2\,\eta_2+\,\langle\mathcal{M}^{(\Lambda_0)\alpha\beta}\rangle\,\bar\zeta^\perp_\alpha\,\zeta^\perp_\beta+2\langle\mathcal{M}^{(\Lambda_0)\alpha}_{i2}\rangle\,\bar\zeta^\perp_\alpha\lambda^{i2}_\perp\\ &\quad +\langle\mathcal{M}^{(\Lambda_0)}_{i2|l2}\rangle\,\bar\lambda^{i2}_\perp\lambda^{l2}_\perp+\mathrm{h.c.}\Big\}\quad .\end{aligned} \qquad (4.21)$$

where

$$\langle\mathcal{M}^{(\Lambda_0)}_{i2|l2}\rangle\;=\;\langle\mathcal{M}_{i2|l2}\rangle-\frac{2\,\langle g_{i\bar j}\,g_{l\bar k}\,\bar W^{\bar j}_{22}\bar S^{22}\bar W^{\bar k}_{22}\rangle}{12\langle\bar S^{22}S_{22}\rangle+\Lambda_0}\qquad ,$$

$$\begin{aligned} \langle \mathcal{M}_{i2}^{(\Lambda_0)\alpha} \rangle &= \frac{1}{2\sqrt{2}} \langle \mathcal{M}_{i2}^{\alpha} \rangle - \frac{2\sqrt{2}\, \langle g_{i\bar{j}}\, \bar{W}_{22}^{\bar{j}} \bar{S}^{22} \bar{N}_2^{\alpha} \rangle}{12 \langle \bar{S}^{22} S_{22} \rangle + \Lambda_0} \quad , \qquad (4.22) \\ \langle \mathcal{M}^{(\Lambda_0)\alpha\beta} \rangle &= \frac{1}{2} \langle \mathcal{M}^{\alpha\beta} \rangle - \frac{4\, \langle \bar{N}_2^{\alpha}\, \bar{S}^{22}\, \bar{N}_2^{\beta} \rangle}{12 \langle \bar{S}^{22} S_{22} \rangle + \Lambda_0} \quad , \end{aligned}$$

are the physical masses of the fermions $(\zeta_\alpha^{\perp}, \lambda_{\perp}^{i2})$ for non-zero cosmological constant denoted by superscript (Λ_0). The requirement of the ground states (3.59) together with (4.8) can be used to show that

$$\begin{aligned} \langle \mathcal{M}_{k2|i2}^{(\Lambda_0)}\, W^{i22} \rangle + \sqrt{2} \langle \mathcal{M}_{k2}^{(\Lambda_0)\alpha}\, N_\alpha^2 \rangle &= \langle \mathcal{M}_{k2|i2}\, W^{i22} \rangle + \frac{1}{2} \mathcal{M}_{k2}^{\alpha}\, N_\alpha^2 \bar{S}^{22} \rangle \\ &\quad -2 \langle g_{k\bar{l}}\, \bar{W}_{22}^{\bar{l}} \rangle \frac{\langle g_{i\bar{j}}\, W^{i22} \bar{W}_{22}^{\bar{j}} \rangle + 2 \langle \bar{N}_2^{\alpha} N_\alpha^2 \rangle}{12 \langle \bar{S}^{22} S_{22} \rangle + \Lambda_0} \\ &= 0 \quad , \qquad (4.23) \end{aligned}$$

and also,

$$\begin{aligned} \langle \mathcal{M}_{i2}^{(\Lambda_0)\beta}\, W^{i22} \rangle + \sqrt{2} \langle \mathcal{M}^{(\Lambda_0)\beta\alpha}\, N_\alpha^2 \rangle &= \frac{1}{\sqrt{2}} \left(\frac{1}{2} \langle \mathcal{M}_{i2}^{\beta}\, W^{i22} \rangle + \langle \mathcal{M}^{\beta\alpha}\, N_\alpha^2 \rangle \right) \\ &\quad -2\sqrt{2} \langle \bar{N}_2^{\beta} \bar{S}^{22} \rangle \frac{\langle g_{i\bar{j}}\, W^{i22} \bar{W}_{22}^{\bar{j}} \rangle + 2 \langle \bar{N}_2^{\alpha} N_\alpha^2 \rangle}{12 \langle \bar{S}^{22} S_{22} \rangle + \Lambda_0} \\ &= 0 \quad . \qquad (4.24) \end{aligned}$$

The equations (4.23) and (4.24) mean that the physical masses (4.22) has a zero eigenvalue which means that the physical modes are already separated from the Goldstone mode.

Next, we discuss the gauge transformation of the gravitino ψ_μ^2 which can be used to eliminate η_2 from the theory. Let us consider the first equation in (4.6) restricted to the broken parameter ϵ_2. Then we arrive at

$$\langle \delta \psi_{2\mu} \rangle = \mathcal{D}_\mu \epsilon_2 + \mathrm{i} \langle S_{22} \rangle \gamma_\mu\, \epsilon^2 \quad . \qquad (4.25)$$

Moreover, (4.18) means that the Goldstone fermions η_2 acts as a supersymmetry transformation parameter and we can perform the replacement

$$\epsilon_2 \to \frac{1}{12 \langle \bar{S}^{22} S_{22} \rangle + \Lambda_0}\, \eta_2 \quad . \qquad (4.26)$$

Thus, (4.25) and (4.26) tell that the local supersymmetry transformation of the gravitino ψ_μ^2 which eliminates the Goldstone fermion η_2 from the Lagrangian (4.21) should take form

$$\psi_\mu^2 \to \psi_\mu^2 + \frac{1}{12 \langle \bar{S}^{22} S_{22} \rangle + \Lambda_0} \left(\mathcal{D}_\mu \eta^2 + \mathrm{i} \langle \bar{S}^{22} \rangle\, \gamma_\mu\, \eta_2 \right) \quad . \qquad (4.27)$$

Note that the transformation (4.27) could not be defined if $12 \bar{S}^{22} S_{22} = -\Lambda_0$, which is precisely the condition for ψ_μ^2 to be invariant in anti-de Sitter backgrounds. The supersymmetry transformation needed to remove the Goldstone fermion would then be singular and in fact the splitting off in the Lagrangian (4.21) would not be possible as the physical fermions would diverge.

Inserting the trasformation (4.27) into the Lagrangian (4.21), we have then

$$\begin{aligned}
\mathcal{L}_f(\Lambda_0) \;=\;& 2\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}}\bar{\psi}^2_\mu\gamma_\sigma\,\mathcal{D}_\nu\psi_{2\lambda} - \mathrm{i}\,g_{i\bar{\jmath}}\bar{\lambda}^{i2}_\perp\gamma^\mu\mathcal{D}_\mu\lambda^{\bar{\jmath}}_{2\perp} - \mathrm{i}\,\bar{\zeta}^{\alpha\perp}\gamma^\mu\mathcal{D}_\mu\zeta^\perp_\alpha \\
&+\frac{1}{(12\langle\bar{S}^{22}S_{22}\rangle+\Lambda_0)^2}\left(\mathrm{i}\Lambda_0\,\bar{\eta}_2\gamma^\mu\mathcal{D}_\mu\eta^2+\epsilon^{\mu\nu\lambda\sigma}\bar{\mathcal{D}}_\mu\bar{\eta}^2\gamma_\sigma[\mathcal{D}_\nu,\mathcal{D}_\lambda]\eta_2+\mathrm{h.c.}\right) \\
&+\frac{1}{(12\langle\bar{S}^{22}S_{22}\rangle+\Lambda_0)}\left(\mathrm{i}\Lambda_0\,\bar{\eta}_2\gamma^\mu\psi^2_\mu+\epsilon^{\mu\nu\lambda\sigma}\bar{\psi}^2_\mu\gamma_\sigma[\mathcal{D}_\nu,\mathcal{D}_\lambda]\eta_2+\mathrm{h.c.}\right) \qquad (4.28)\\
&-\frac{1}{(12\langle\bar{S}^{22}S_{22}\rangle+\Lambda_0)^2}\left(2\Lambda_0\langle\bar{S^{22}}\rangle\bar{\eta}_2\,\eta_2+\frac{\mathrm{i}}{2}\langle\bar{S}^{22}\rangle\,\epsilon^{\mu\nu\lambda\sigma}\bar{\eta}_2\gamma_{\mu\nu}[\mathcal{D}_\lambda,\mathcal{D}_\sigma]\eta_2+\mathrm{h.c.}\right) \\
&+\Big\{2\langle S_{22}\rangle\,\bar{\psi}^2_\mu\gamma^{\mu\nu}\psi^2_\nu+\langle\mathcal{M}^{(\Lambda_0)\alpha\beta}\rangle\bar{\zeta}^\perp_\alpha\,\zeta^\perp_\beta+2\langle\mathcal{M}^{(\Lambda_0)\alpha}_{i2}\rangle\,\bar{\zeta}^\perp_\alpha\lambda^{i2}_\perp \\
&+\langle\mathcal{M}^{(\Lambda_0)}_{i2|l2}\rangle\,\bar{\lambda}^{i2}_\perp\lambda^{l2}_\perp+\mathrm{h.c.}\Big\} \quad .
\end{aligned}$$

Now using the property of this curved spacetime with non zero cosmological constant,

$$\begin{aligned}
[\mathcal{D}_\mu,\mathcal{D}_\nu]\eta_2 &= -\frac{\Lambda_0}{6}\,\gamma_{\mu\nu}\,\eta_2 \quad , \\
[\bar{\mathcal{D}}_\mu,\bar{\mathcal{D}}_\nu]\bar{\eta}_2 &= \frac{\Lambda_0}{6}\,\bar{\eta}_2\gamma_{\mu\nu} \quad , \qquad (4.29)
\end{aligned}$$

with $\bar{\mathcal{D}}_\mu\bar{\eta}_2=\partial_\mu\,\bar{\eta}_2+\frac{1}{4}\bar{\eta}_2\gamma_{ab}\,\langle\omega^{ab}_\mu\rangle$, the Lagrangian (4.28) can be simplified as

$$\begin{aligned}
\mathcal{L}_f(\Lambda_0) \;=\;& 2\frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}}\bar{\psi}^2_\mu\gamma_\sigma\,\mathcal{D}_\nu\psi_{2\lambda} - \mathrm{i}\,g_{i\bar{\jmath}}\bar{\lambda}^{i2}_\perp\gamma^\mu\mathcal{D}_\mu\lambda^{\bar{\jmath}}_{2\perp} - \mathrm{i}\,\bar{\zeta}^{\alpha\perp}\gamma^\mu\mathcal{D}_\mu\zeta^\perp_\alpha \\
&+\Big\{2\langle S_{22}\rangle\,\bar{\psi}^2_\mu\gamma^{\mu\nu}\psi^2_\nu+\langle\mathcal{M}^{(\Lambda_0)\alpha\beta}\rangle\bar{\zeta}^\perp_\alpha\,\zeta^\perp_\beta+2\langle\mathcal{M}^{(\Lambda_0)\alpha}_{i2}\rangle\,\bar{\zeta}^\perp_\alpha\lambda^{i2}_\perp \qquad (4.30)\\
&+\langle\mathcal{M}^{(\Lambda_0)}_{i2|l2}\rangle\,\bar{\lambda}^{i2}_\perp\lambda^{l2}_\perp+\mathrm{h.c.}\Big\} \quad .
\end{aligned}$$

We see that the Lagrangian (4.30) only contains the physical massive fields. The massive gravitino ψ^2_μ has four instead of two degrees of freedom as a consequence of spontaneous local supersymmetry breaking.

Chapter 5

Conclusion

Let us finally summarize our main results in chapter 3 and 4 and give an outlook for future investigations.

Some aspects of spontaneous $N = 2 \to N = 1$ supersymmetry breaking in Minkowskian ground states have been discussed in chapter 3. First, the simplest example in [16] has been studied in detail including the Higgs and the super-Higgs effects, showing the mass degeneracy of the $N = 1$ gravitino multiplet, and then deriving the low energy effective $N = 1$ theory described by the left over massless scalar fields in section 3.1. Also we have desribed the general description of spontaneous $N = 2 \to N = 1$ supersymmetry breaking and also discussed our additional input to solve the fermionic supersymmetry transformation in ground states, see (3.33). These are in section 3.2 and 3.3. Furthermore, we have derived the necessary conditions in section 3.4. An interesting aspect is that the necessary conditions constrain the symplectic vectors X^Λ of the special Kähler manifold that they have to choose a basis where no prepotential $\mathcal{F}$ exists in order to avoid the no go theorem. In section 3.5 we have discussed the Higgs and super-Higgs mechanisms and then, showed the mass degeneracy of the $N = 1$ massive gravitino multiplet using some properties of Minkowsian ground states.

In chapter 4 we discussed the possibility of spontaneous $N = 2 \to N = 1$ supersymmetry breaking in anti-de Sitter spaces by applying the same procedure as in chapter 3. We have illustrated the general picture of such breaking and discussed the properties of the $N = 1$ massive gravitino multiplet in section 4.1. Then, the necessary conditions have been discussed in section 4.2 and finally, the super-Higgs effect in anti-de Sitter backgrounds has been showed in section 4.3.

A lot of open problems remain. Firstly, we do not address here the general derivation of the low energy effective $N = 1$ theory which is described by the left over massless scalar fields. It is worth to mention that in Minkowski ground states the theory choose a basis of X^Λ where no $\mathcal{F}$ exists in order the inverse gauge couplings of the gauge bosons to be harmonic in the low energy effective $N = 1$ theory. However, this feature is still unclear for anti-de Sitter backgrounds, see (4.11). Another important condition is to show that the left-over massless scalar fields span a manifold which is Kähler as $N = 1$ supersymmetry requires. This manifold is a quotient $\mathcal{M}_H/\mathbb{R}^2$ [40]. These aspects will be discussed in our publication [41].

Secondly, the situation changes if one allows the possibility of a reduced Lorentz invariance or in other words considers domain wall solutions of $N = 2$ supergravity. In this case quite generically BPS-type solutions exist which do preserve half supercharges. At this level, it is still not obvious that our framework can consistently be applied to domain wall case. We hope that in future investigation this might explain the hierarchical spontaneous $N = 2 \to N = 1 \to N = 0$ supersymmetry breaking.

Appendix A

Convention and Notation

The purpose of this appendix is to assemble our conventions in this thesis. The spacetime metric is taken to have the signature $(+,-,-,-)$ while the Riemann tensor is defined to be $-R^{\mu}_{\nu\lambda\rho} = \partial_\lambda \Gamma^{\mu}_{\nu\rho} - \partial_\nu \Gamma^{\mu}_{\lambda\rho} + \Gamma^{\mu}_{\lambda\sigma}\Gamma^{\sigma}_{\nu\rho} - \Gamma^{\mu}_{\nu\sigma}\Gamma^{\sigma}_{\lambda\rho}$. The Christoffel symbol is given by $\Gamma^{\mu}_{\nu\rho} = \frac{1}{2} g^{\mu\sigma}(\partial_\nu g_{\rho\sigma} + \partial_\rho g_{\nu\sigma} - \partial_\sigma g_{\nu\rho})$ where $g_{\mu\nu}$ is the spacetime metric.

Indices

$\mu, \nu = 0, ..., 3$	label curved four dimensional spacetime indices
$a, b = 0, ..., 3$	label flat four dimensional spacetime indices
$\hat{a}, \hat{b} = -, 0, ..., 3$	label flat five dimensional space indices with two time-like directions
$\underline{a}, \underline{b} = 0, 1, 2$	label flat three dimensional spacetime indices
$\widehat{A}, \widehat{B} = 1, ..., N$	label the number of supercharges
$A, B = 1, 2$	label the fundamental representation of the R-symmetry group $SU(2) \otimes U(1)$
$x, y, z = 1, 2, 3$	label adjoint representation of $SU(2)$ operator
$i, j, k = 1, .., n_V$ or n_c	label the $N = 2$ vector multiplet or the $N = 1$ chiral multiplet respectively
$\bar{i}, \bar{j}, \bar{k} = 1, .., n_V$ or n_h	label conjugate indices of i, j, k
$u, v, w = 1, ..., 4n_H$	label the real scalars of the $N = 2$ hypermultiplet
$\alpha, \beta = 1, ..., 2n_H$	label the fundamental representation of $Sp(2n_H)$
$\Lambda, \Sigma = 0, ..., n_V$	label holomorphic and antiholomorphic section of Special Kähler manifold and also all gauge fields

$g_{i\bar{j}}$ denotes the metric of the special Kähler manifold whose Levi-Civita connection is defined as $\Gamma^{l}_{ij} = g^{l\bar{k}}\partial_i g_{j\bar{k}}$ and its conjugate $\Gamma^{\bar{l}}_{\bar{i}\bar{j}} = g^{\bar{l}k}\partial_{\bar{i}} g_{\bar{j}k}$, while h_{uv} denotes the metric of the quaternionic Kähler manifold.

$SU(2)$ and $Sp(2n_H)$ metrics

$$\epsilon^{AB}\epsilon_{BC} = -\delta^{A}_{C}, \qquad \epsilon^{AB} = -\epsilon^{BA}, \tag{A.1}$$

$$\mathbb{C}^{\alpha\beta}\mathbb{C}_{\beta\gamma} = -\delta^{\alpha}_{\gamma}, \qquad \mathbb{C}^{\alpha\beta} = -\mathbb{C}^{\beta\alpha} \tag{A.2}$$

For any $SU(2)$ vector (and Lorentz scalar) V_A we have:

$$\epsilon_{AB}V^B = V_A, \qquad \epsilon^{AB}V_B = -V^A, \tag{A.3}$$

and also for $Sp(2n_H)$ vectors (and Lorentz scalar) V_α:

$$\mathbb{C}_{\alpha\beta}V^\beta = V_\alpha, \qquad \mathbb{C}^{\alpha\beta}V_\beta = -V^\alpha. \tag{A.4}$$

The above formulae are useful to lower and raise indices for quantity such as the vielbein $\mathcal{U}^{A\alpha}$ in the quaternionic Kähler manifold and the Pauli matrices. Note that the lower and upper index of any $SU(2)$ Weyl spinor ϵ_A is related by hermitian conjugate (see spinor convention)

Pauli matrices

The standard Pauli matrices used in this thesis are

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma^2 = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}, \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{A.5}$$

The Pauli matrices with two lower indices can be defined as

$$(\sigma^x)_{AB} \equiv (\sigma^x)_A^{\ C}\,\epsilon_{BC} \tag{A.6}$$

where $x = 1, 2, 3$ and $(\sigma^x)_A^{\ C}$ are the standard Pauli matrices. The equation(A.6) can be read as

$$(\sigma^1)_{AB} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, (\sigma^2)_{AB} = \begin{pmatrix} -\mathrm{i} & 0 \\ 0 & -\mathrm{i} \end{pmatrix}, (\sigma^3)_{AB} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \tag{A.7}$$

and their complex cojugate are

$$(\sigma^x_{AB})^* = -\sigma^{xAB}. \tag{A.8}$$

Symmetric and antisymmetric indices

For any indices I, J, we define symmetric indices $\{I, J\}$ and antisymmetric indices $[I, J]$ as

$$\{I, J\} \equiv \frac{1}{2}(IJ + JI), \qquad [I, J] \equiv \frac{1}{2}(IJ - JI) \tag{A.9}$$

respectively.

Clifford algebra

$$\begin{aligned}
\{\gamma_a,\gamma_b\} &\equiv \gamma_a\gamma_b+\gamma_b\gamma_a = 2g_{ab} \\
[\gamma_a,\gamma_b] &\equiv \gamma_a\gamma_b-\gamma_b\gamma_a = 2\gamma_{ab} \\
\gamma_5 &\equiv -\mathrm{i}\gamma_0\gamma_1\gamma_2\gamma_3 \\
\{\gamma_a,\gamma_5\} &= 0 \\
\gamma_0^\dagger &= \gamma_0, \qquad \gamma_0\gamma_i^\dagger\gamma_0 = \gamma_i \quad (i=1,2,3), \qquad \gamma_5^\dagger = \gamma_5 \\
\epsilon_{abcd}\gamma^{cd} &= 2\mathrm{i}\gamma_{ab}\gamma_5 \\
\gamma_a\gamma_b\gamma_c &= g_{ac}\,\gamma_b - g_{bc}\,\gamma_a - g_{ab}\,\gamma_c - \mathrm{i}\epsilon_{abcd}\gamma^d\gamma_5
\end{aligned} \tag{A.10}$$

Spinor convention
For any fermion Ψ :

$$\bar{\Psi} \equiv \Psi^\dagger\,\gamma^0 = \Psi^{\mathrm{T}}\,C \quad . \tag{A.11}$$

Let Ψ_A be a Majorana spinor in four dimensions which can be decomposed into Weyl spinors ψ_A and ψ^A as

$$\begin{aligned}
\psi_A &\equiv \frac{1}{2}(1+\gamma_5)\Psi_A \quad , \\
\psi^A &\equiv \frac{1}{2}(1-\gamma_5)\Psi_A \quad ,
\end{aligned} \tag{A.12}$$

and ψ_A, ψ^A have right or left chirality respectively. In this paper the right chiral spinors are

$$\begin{aligned}
\psi_{A\mu} &\equiv \frac{1}{2}(1+\gamma_5)\Psi_{A\mu} \quad , \\
\lambda^{iA} &\equiv \frac{1}{2}(1+\gamma_5)\Lambda^I_A \quad , \\
\zeta_\alpha &\equiv \frac{1}{2}(1+\gamma_5)\Upsilon_\alpha \quad , \\
\epsilon_A &\equiv \frac{1}{2}(1+\gamma_5)\Xi_A \quad ,
\end{aligned} \tag{A.13}$$

while the left chiral spinors are

$$\begin{aligned}
\psi^A_\mu &\equiv \frac{1}{2}(1-\gamma_5)\Psi_{A\mu} \quad , \\
\lambda^{\bar{i}}_A &\equiv \frac{1}{2}(1-\gamma_5)\Lambda^I_A \quad , \\
\zeta^\alpha &\equiv \frac{1}{2}(1-\gamma_5)\Upsilon_\alpha \quad , \\
\epsilon^A &\equiv \frac{1}{2}(1-\gamma_5)\Xi_A \quad ,
\end{aligned} \tag{A.14}$$

where $\Psi_{A\mu}, \Lambda^I_A, \Upsilon_\alpha, \Xi_A$ are the Majorana spinors in four dimensions. As an example (for ϵ_A) the eq.(A.11) becomes

$$\bar{\epsilon}^A = \epsilon_A^\dagger\gamma^0 = \epsilon_A^\dagger \,, \tag{A.15}$$

where $\bar{\epsilon}^A$ has the same chirality as ϵ^A and they are 'inverse' into each other in the representation with respect to the $SL(2,\mathbb{C})$ [57].

Let χ_A, η^B are (0-form) spinors which have right or left chirality respectively, then Hermiticity of currents for these spinors are

$$\begin{aligned}
(\bar{\chi}_A \eta_B)^\dagger &= \bar{\eta}^B \chi^A = \bar{\chi}^A \eta^B \quad , \\
(\bar{\chi}_A \gamma^a \eta^B)^\dagger &= \bar{\eta}_B \gamma^a \chi^A = -\bar{\chi}^A \gamma^a \eta_B \quad , \\
(\bar{\chi}_A \gamma^{ab} \eta_B)^\dagger &= -\bar{\eta}^B \gamma^{ab} \chi^A = \bar{\chi}^A \gamma^{ab} \eta^B \quad .
\end{aligned} \tag{A.16}$$

Appendix B

Anti-de Sitter Supersymmetry

In this appendix we give a discussion about the structure of supersymmetry in a curved spacetime with negative cosmological constant, namely anti-de Sitter spacetime. The interested reader can consult the literature for more details, see for example [21, 58].

B.1 Anti-de Sitter Spacetime

A four dimensional anti-de Sitter spacetime can be viewed as a hypersurface (or hyperboloid) embedded in a five dimensional space with two time-like directions. Denoting the extra coordinate of the five dimensional space by y^-, so that we have coordinates $y^{\hat{a}}$ with $\hat{a} = -, 0, .., 3$, this hypersurface is defined by [58]

$$(y^-)^2 + \eta_{ab}\, y^a y^b = \eta_{\hat{a}\hat{b}}\, y^{\hat{a}} y^{\hat{b}} = -\frac{3}{\Lambda_0} \quad , \tag{B.1}$$

where $\Lambda_0 < 0$. Obviously, the hypersurface is invariant under linear transformations that leave the metric $\eta_{\hat{a}\hat{b}} = \mathrm{diag}(+, +, -, -, -)$ invariant. These transformations constitute the group $SO(3, 2)$ whose 10 generators are denoted by $\hat{J}_{\hat{a}\hat{b}}$ satisfy the $SO(3, 2)$ Lie algebra in its standard form:

$$[\hat{J}_{\hat{a}\hat{b}}, \hat{J}_{\hat{c}\hat{d}}] = -\mathrm{i}\big(\eta_{\hat{b}\hat{c}}\, \hat{J}_{\hat{a}\hat{d}} + \eta_{\hat{a}\hat{d}}\, \hat{J}_{\hat{b}\hat{c}} - \eta_{\hat{b}\hat{d}}\, \hat{J}_{\hat{a}\hat{c}} - \eta_{\hat{a}\hat{c}}\, \hat{J}_{\hat{b}\hat{d}}\big) \quad . \tag{B.2}$$

In other words, anti de-Sitter space has 10 Killing vectors which generate the isometries corresponding to the group $SO(3, 2)$. Furthermore, by these isometries any two points on anti de-Sitter space can be related into each other. This means that anti de Sitter space is a homogeneous space.

Using a redefinition

$$\hat{J}_{-a} = \frac{1}{2\ell} P_a \ , \qquad \hat{J}_{ab} = J_{ab} \ , \tag{B.3}$$

where ℓ is a real quantity related to the cosmological constant Λ_0 via $\Lambda_0 = -3\ell^2$, then the algebra (B.2) takes form

$$\begin{aligned} [J_{ab}, J_{cd}] &= -\mathrm{i}\big(\eta_{bc}\, J_{ad} + \eta_{ad}\, J_{bc} - \eta_{bd}\, J_{ac} - \eta_{ac}\, J_{bd}\big) \quad , \\ [J_{ab}, P_c] &= \mathrm{i}\big(\eta_{ac}\, P_b - \eta_{bc}, P_a\big) \quad , \\ [P_a, P_b] &= 4\mathrm{i}\ell^2 J_{ab} \quad . \end{aligned} \tag{B.4}$$

In perfect analogy with the Poincaré case, a unitary irreducible representation of $SO(3, 2)$ is what one calls a particle in anti-de Sitter space. However, the mass squared operator $P^a P_a$ is no longer invariant under the $SO(3,2)$ subalgebra (B.2). Hence in anti-de Sitter a particle is not characterized by the eigenvalue of $P^a P_a$, rather by the eigenvalue of the true second order Casimir of $SO(3,2)$ which in our normalization has the following expression [21]:

$$\mathrm{C}_2 = \frac{1}{2} \hat{J}^{\hat{a}\hat{b}} \hat{J}_{\hat{a}\hat{b}} = \frac{1}{2} J^{ab} J_{ab} + \frac{1}{4\ell^2} P^a P_a \quad , \tag{B.5}$$

where we lower and raise indices by contracting with $\eta_{\hat{a}\hat{b}}$ and its inverse $\eta^{\hat{a}\hat{b}}$.

Now, let us consider the unitary irreducible representation of $SO(3,2)$. Our starting point is its compact subgroup $SO(2) \times SO(3)$ corresponding to rotations of the compact anti-de Sitter time and spatial rotations. It is convenient to decompose the 10 generators as follows. First, the generator J_{-0} is related to the energy operator when the radius of the anti-de Sitter space is taken to infinity. The eigenvalues of this operator, which is associated with motions along the circle, are quantized in integer units in order to have single-valued functions. So we define the energy operator H by

$$H \equiv J_{-0} = \frac{1}{2\ell} P_0 \quad . \tag{B.6}$$

Obviously the generators of the spatial rotations are J_{xy} with $x, y = 1, 2, 3$. The remaining 6 generators J_{-x} and J_{0x} are combined into pairs of mutually conjugate operators,

$$\begin{aligned} J_x^{\pm} &\equiv J_{0x} \pm \mathrm{i} J_{-x} \quad , \\ (J_x^{+})^{\dagger} &= J_x^{-} \quad . \end{aligned} \tag{B.7}$$

From the $SO(3,2)$ algebra (B.4), one can see that the $J_x^{\pm}$ play the role of raising and lowering operators: when applied to an eigenstate of H with eigenvalue E_0, application of $J_x^{\pm}$ yields a state with eigenvalue $E_0 \pm 1$.

The representation we are interested in must have an energy spectrum bounded from below, $H \geq E_0$, and the lowest eigenvalue E_0 is realized on states that we denote $|E_0, s\rangle$, where E_0 is the eigenvalue of H and s indicates the value of the total angular momentum operator. Since states with $E < E_0$ should not appear, vacuum states are given by the condition, $J_x^{-} |E_0, s\rangle = 0$. Evaluating the Casimir operator (B.5) on the vacuum states $|E_0, s\rangle$ we get

$$\mathrm{C}_2 = E_0 \, (E_0 - 3) + s(s+1) \quad . \tag{B.8}$$

The other states with energy $E_0 + n$ are constructed by an n-fold product of creation operators J_x^{+}. In this way we obtain states of higher eigenvalues E with higher spin.

The states constructed above might have three possible norms: negative, zero, and positive. To obtain a physical Hilbert space, the states must have a positive norm. Therefore, if there are zero-norm states, the Hilbert space is composed of the equivalence classes of all states modulo the zero-norm states. Such a situation is typical of all massless theories. However, to avoid the negative-norm states we must impose conditions which are expressed as the lower bounds on the energy E_0 relative to the spin s. We give the results in the following [21]:

1. For $s \geq 1$, then $E_0 \geq s + 1$.

When $E_0 = s+1$, the corresponding representation is massless and the states are indeed zero-norm, while for $E_0 > s+1$ no zero-norm states occur, so the representation is massive.

2. For $s = 1/2$, then $E_0 \geq 1$.
In this case the zero-norm states occur for $E_0 = 3/2$ and $E_0 = 1$. The first value is a massless representation, while the latter corresponds to a so called *Dirac singleton* for which no field-theoretic interpretation has been found. It is called singleton because we have only one state for a given value of spin. In addition, such representation has no counterpart in the Poincaré case.

3. For $s = 0$, then $E_0 \geq 1/2$.
The zero-norm states are found for the special values $E_0 = 1, 2$ which yield the standard massless representation, and $E_0 = 1/2$ is again a Dirac singleton where again we are left with just one state for every spin value, with no counterpart in the Poincaré case and no field theory interpretation.

B.2 Mass in Anti-de Sitter Spacetime

To make contact between the properties of the unitary irreducble representation of $SO(3,2)$ and the physical mass of a field, it is important to establish a relation between the D'Alembertian in anti-de Sitter space, *i.e.* $\Box_{\text{adS}} \equiv D^a D_a$, and the Casimir operator C_2 defined in (B.5) of the isometry group. For that purpose, let us first look at a field equation described by the following wave equation:

$$(-\Box_{\text{adS}} + b_s)\varphi_{(s)}(x) = m_s^2\, \varphi_{(s)}(x) \quad , \tag{B.9}$$

where b_s is a constant and m_s is the physical mass for a particle with spin s. In the following we derive the equation (B.9) for $s = 0, \frac{1}{2}, 1, \frac{3}{2}$ [21]:

i) $s = 0$.
Let us consider the Klein-Gordon equation in general curved spacetime

$$(-\Box + \frac{R}{6})\varphi_{(0)}(x) = m_0^2\, \varphi_{(0)}(x) \quad . \tag{B.10}$$

Since we are dealing with anti-de Sitter spacetime with constant scalar curvature $R = 12\ell^2$, (B.10) takes the form

$$(-\Box_{\text{adS}} + 2\ell^2)\phi(x) = m_0^2\, \phi(x) \quad . \tag{B.11}$$

ii) $s = \frac{1}{2}$.
For spin-$\frac{1}{2}$ fermion, (B.9) can be obtained as follows. We start with the Dirac equation $(-\mathrm{i}\gamma^a D_a + m_{\frac{1}{2}})\chi = 0$. Then, we evaluate $(-\mathrm{i}\gamma^a D_a + m_{\frac{1}{2}})(-\mathrm{i}\gamma^a D_a - m_{\frac{1}{2}})\chi = 0$, which gives rise to the wave equation

$$(-\Box_{\text{adS}} - \frac{1}{2}\gamma^{ab}[D_a, D_b] - m_{\frac{1}{2}}^2)\chi(x) = 0 \quad . \tag{B.12}$$

Inserted $[D_a, D_b]\chi = \frac{1}{2}\ell^2\gamma_{ab}\chi$ into (B.12), we obtain

$$(-\Box_{\text{adS}} + 3\ell^2)\chi(x) = m_{\frac{1}{2}}^2\chi(x) \quad . \tag{B.13}$$

iii) $s = 1$.
The equation of motion of a gauge boson A_a is given by

$$D^a(D_b\, A_a - D_a\, A_b) = m_1^2 A_b \quad . \tag{B.14}$$

Imposing the Lorentz condition $D^a A_a = 0$, we can write (B.14) as

$$(-\Box_{\text{adS}} A_b - R_{ab}^{ac}\, A_c) = m_1^2 A_b \quad . \tag{B.15}$$

In anti-de Sitter spacetime we have $R_{cd}^{ab} = -\ell^2(\delta_c^a\,\delta_d^b - \delta_d^a\,\delta_c^b)$, so (B.15) can be simplified as

$$(-\Box_{\text{adS}} + 3\ell^2)A_b = m_1^2 A_b \quad . \tag{B.16}$$

iv) $s = \frac{3}{2}$.
The dynamical of a spin-$\frac{3}{2}$ fermion is described by the Rarita-Schwinger equation

$$\epsilon^{abcd}\gamma_5\gamma_b(D_c + \frac{\mathrm{i}}{2}\ell\,\gamma_c)\psi_d = m_{\frac{3}{2}}\gamma^{ad}\,\psi_d \quad . \tag{B.17}$$

Imposing the conditions $D^a\psi_a = \gamma^a\psi_a = 0$, then (B.17) reduces to the Dirac equation

$$-\mathrm{i}\gamma^a D_a\psi_d = (m_{\frac{3}{2}} - \ell)\psi_d \quad . \tag{B.18}$$

Using the same procedure as spin-$\frac{1}{2}$ fermion, one obtains

$$(-\Box_{\text{adS}} - \frac{1}{4}\gamma^{ce}R_{ce}^{ab}\gamma_{ab})\psi_d + \gamma^{ab}R_{dab}^{e}\,\psi_e = (m_{\frac{3}{2}} - \ell)^2\psi_d \quad , \tag{B.19}$$

which can be simplified as

$$(-\Box_{\text{adS}} + 4\ell^2)\psi_d = (m_{\frac{3}{2}} - \ell)^2\psi_d \quad . \tag{B.20}$$

To summarize, below we collect b_s for $s = 0, \frac{1}{2}, 1, \frac{3}{2}$:

$$b_0 = 2\ell\ , \quad b_{\frac{1}{2}} = 3\ell^2\ , \quad b_1 = 3\ell^2\ , \quad b_{\frac{3}{2}} = 4\ell^2\ . \tag{B.21}$$

In addition, the operator $\Box_{\text{adS}}$ is a second order invariant differential operator, *i.e.* it is invariant along the isometries generated by the $SO(3,2)$ Killing vectors. Therefore, since we have only one Casimir operator, it must be $C_2(\Box_{\text{adS}}) \sim \Box_{\text{adS}}$. Using the characteristic of the massless multiplet described in the previous section, we find that the physical mass m_s^2 can be written as [21][1]

$$\frac{m_s^2}{\ell^2} = C_2 - 2(s^2 - 1) + \delta_{s,\frac{3}{2}} = E_0\,(E_0 - 3) - s^2 + s + 2 + \delta_{s,\frac{3}{2}} \quad , \tag{B.22}$$

[1] Note that the additional term $\delta_{s,\frac{3}{2}}$ is introduced in order to get the correct (B.9) for Rarita-Schwinger field.

where

$$\delta_{s,\frac{3}{2}} = \begin{cases} 1 & \text{for} \quad s = \frac{3}{2} \\ \\ 0 & \text{for} \quad s \neq \frac{3}{2} \end{cases} \quad . \tag{B.23}$$

Below, we list the physical mass for $s = 0, \frac{1}{2}, 1, \frac{3}{2}$:

$$\begin{aligned} \frac{m_0^2}{\ell^2} &= (E_0 - 2)(E_0 - 1) \quad , \\ \frac{m_{\frac{1}{2}}^2}{\ell^2} &= E_0\,(E_0 - 3) + \frac{9}{4} = (E_0 - \frac{3}{2})^2 \quad , \\ \frac{m_1^2}{\ell^2} &= (E_0 - 2)(E_0 - 1) \quad , \\ \frac{(m_{\frac{3}{2}} - \ell)^2}{\ell^2} &= E_0\,(E_0 - 3) + \frac{5}{4} + 1 = (E_0 - \frac{3}{2})^2 \quad . \end{aligned} \tag{B.24}$$

B.3 Anti-de Sitter Superalgebra

As in Minkowski case, the supersymmetry algebra in anti de-Sitter space also admits a $\mathbb{Z}_2$ graded structure. The $SO(3,2)$ algebra (B.4) together with the following algebra

$$\begin{aligned} \{Q^{\widehat{A}}, Q^{\widehat{B}}\} &= -2\big(\,(\gamma C)^a P_a + \ell(\gamma C)^{ab}\,J_{ab}\big)\delta^{\widehat{A}\widehat{B}} - 4C\,Z^{\widehat{A}\widehat{B}} \quad , \\ [P_a, Q^{\widehat{A}}] &= -\ell\,\gamma_a Q^{\widehat{A}} \quad , \\ [J_{ab}, Q^{\widehat{A}}] &= -\frac{\mathrm{i}}{2}\gamma_{ab}\,Q^{\widehat{A}} \quad , \\ [\mathcal{Z}^{\widehat{A}\widehat{B}}, Q^{\widehat{C}}] &= \frac{\ell}{4}\big(\delta^{\widehat{A}\widehat{C}}Q^{\widehat{B}} - \delta^{\widehat{B}\widehat{C}}Q^{\widehat{A}}\big) \,, \\ [\mathcal{Z}^{\widehat{A}\widehat{B}}, \mathcal{Z}^{\widehat{C}\widehat{D}}] &= \ell\big(\delta^{\widehat{B}\widehat{C}}\mathcal{Z}^{\widehat{A}\widehat{D}} + \delta^{\widehat{A}\widehat{D}}\mathcal{Z}^{\widehat{B}\widehat{C}} - \delta^{\widehat{B}\widehat{D}}\mathcal{Z}^{\widehat{A}\widehat{C}} - \delta^{\widehat{A}\widehat{C}}\mathcal{Z}^{\widehat{B}\widehat{D}}\big) \quad , \end{aligned} \tag{B.25}$$

forms an anti-de Sitter superalgebra which is usually called $OSp(4/N)$ superalgebra [21]. Its Lie subalgebra is $SO(2,3) \times SO(N)$. Additionally the supercharges $Q^{\widehat{A}}$ transform under $SO(3,2)$ as a four dimensinal spinor and under $SO(N)$ as a vector. From the last equation in (B.25) we see that the antisymmetric $\mathcal{Z}^{\widehat{A}\widehat{B}}$ form an $SO(N)$ algebra rather than being supersymmetrically invariant. Unlike in the Minkowski case, one cannot set the $SO(N)$ generators $\mathcal{Z}^{\widehat{A}\widehat{B}}$ to zero for N-extended supersymmetry since the $SO(N)$ group belongs to the Lie subalgebra of the superalgebra $OSp(4/N)$. However, setting $\mathcal{Z}^{\widehat{A}\widehat{B}}$ to zero would reduce the superalgebra $OSp(4/N)$ to $OSp(4/1)$ which is the minimal supersymmetry, *i.e.* $N = 1$ supersymmetry in anti-de Sitter space. Furthermore, as the cosmological constant $\Lambda_0 \to 0$, then the first generator in (B.3) becomes singular. In this case we regain the supersymmetry algebra in the Minkowski case (1.4) and the $SO(N)$ generators $\mathcal{Z}^{\widehat{A}\widehat{B}}$ become the central charges $Z^{\widehat{A}\widehat{B}}$ which are supersymmetric invariants.

Furthermore, the squared mass operator $P^a P_a$ is no longer a supersymmetric Casimir operator with respect to the $OSp(4/N)$ superalgebra. This implies that the particle spectrum in a supermultiplet is not degenerate in mass. We discuss this feature in detail by giving example for Wess-Zumino multiplets (see next section).

Next, we construct the supermultiplets in anti-de Sitter space. To derive the structure of the supermultiplets we begin by choosing a gamma matrix basis. For reasons of convenience we do not take the same basis in (1.9), rather we adopt the following convention

$$\begin{aligned} \gamma^0 &= \begin{pmatrix} -\mathbb{1} & 0 \\ 0 & \mathbb{1} \end{pmatrix} , \quad \gamma^x = \begin{pmatrix} 0 & \sigma^x \\ -\sigma^x & 0 \end{pmatrix} , \\ \gamma_5 &= \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} , \end{aligned} \tag{B.26}$$

which corresponds to an exchange of γ^0 with $-\gamma_5$. In this basis the charge conjugation matrix is given by

$$C = \begin{pmatrix} 0 & \mathrm{i}\sigma^2 \\ \mathrm{i}\sigma^2 & 0 \end{pmatrix} , \tag{B.27}$$

and we write the decomposition of the supercharges $Q^{\widehat{A}}$ as

$$Q^{\widehat{A}} = \begin{pmatrix} \tilde{Q}_+^{\widehat{A}} \\ \tilde{Q}_-^{\widehat{A}} \end{pmatrix} \quad . \tag{B.28}$$

Also the commutation relation between the supercharges $Q^{\widehat{A}}$ and the Hamiltonian H in the supersymmetry algebra (B.25) can be written down as

$$[H, \tilde{Q}_\pm^{\widehat{A}}] = \pm\frac{1}{2}\tilde{Q}_\pm^{\widehat{A}} \quad . \tag{B.29}$$

It appears that $\tilde{Q}_+^{\widehat{A}}$ like J_x^+ is a raising operator for the energy eigenvalue in addition to the spin. Differently from J_x^+, however, $\tilde{Q}_+^{\widehat{A}}$ raises the energy eigenvalue by a half unit rather than a unit. Moreover $\tilde{Q}_+^{\widehat{A}}$ is an $SO(N)$ vector.

To build up a unitary irreducible representation of $OSp(4/N)$ we introduce a multiplet of vacuum states $|E_0, s, \mathcal{Z}\rangle$ which are annihilated not only by J_x^- but also by all the $\tilde{Q}_-^{\widehat{A}}$, *i.e.* $\tilde{Q}_-^{\widehat{A}}|E_0, s, \mathcal{Z}\rangle = 0$, where E_0 is, as usual, the eigenvalue of H, s is the spin, and $\mathcal{Z}$ is the quantum numbers of the $SO(N)$ representation. Furthermore, from the anticommutation relation in (B.25) it follows that all even symmetric combination of the operators $\tilde{Q}_+^{\widehat{A}}$ can be expressed through the even elements of the $OSp(4/N)$ superalgebra, namely the generators of $SO(3,2) \times SO(N)$. Therefore applying such combinations to the vacuum state we simply build up the the unitary irreducible representation $D(E_0, s) \times \mathcal{Z}$ of $SO(3,2) \times SO(N)$. Such a representation is one of the finite number of particles into which the $OSp(4/N)$ irreducible representation breaks up. Henceforth we suppress $SO(N)$ labels $\mathcal{Z}$ for simplicity. To find the other particles $D(E_0', s')$ which sit with $D(E_0, s)$ in the same multiplet, we just need to consider the action on the vacuum state of the antisymmetric combinations of $\tilde{Q}_+^{\widehat{A}}$ operators. Then there are C_n^{2N} at level n and the dimension of the representation is given by 2^{2N}.

The resulting $OSp(4/N)$ supermultiplets are of the form

$$D(E_0^{(1)}, s^{(1)}) \oplus .... \oplus D(E_0^{(r)}, s^{(r)}) , \quad r < \infty , \tag{B.30}$$

and additionally, in general, we shall have $E_0^{(1)} \neq E_0^{(2)} \neq ... \neq E_0^{(r)}$ so that the particles within the same supermultiplet having different energy labels also have different mass. This is because the mass squared operator $P^a P_a$ is not supersymmetric invariant. Furthermore the supermultiplets in (B.30) are also constrained by the energy label E_0 in order for no negative-norm states to be present in the spectrum. This problem can be solved by expressing the energy eigenvalue E_0 relative to the spin s and the $SO(N)$ quantum number $\mathcal{Z}$. Also there are zero-norm states related to the critical value of E_0 which can be decoupled producing a shortening of the multiplet. For $N \geq 2$, these are the *short massive representations* where E_0 is related in a convenient way to the $SO(N)$ quantum numbers. These short representation are somehow the counterpart of the Poincaré massive multiplet with central charge.

Here, we just give the results for the minimal $N = 1$ supermultiplet in anti-de Sitter space which is the case relevant for our analysis in chapter four. In $N = 1$ case, there is no $SO(N)$ group, so the vacuum state is labeled only by E_0 and by its spin s. Thus the $OSp(4/1)$ supermultiplets are [59]

1. Wess-Zumino multiplets ($E_0 > \frac{1}{2}$)

$$D(E_0, 0) \oplus D(E_0 + \frac{1}{2}, \frac{1}{2}) \oplus D(E_0 + 1, 0) \quad , \tag{B.31}$$

2. Massless higher spin multiplets ($E_0 = s + 1, s \geq \frac{1}{2}$)

$$D(s + 1, s) \oplus D(s + \frac{3}{2}, s + \frac{1}{2}) \quad , \tag{B.32}$$

3. Massive higher spin multiplets ($E_0 > s + 1, s \geq \frac{1}{2}$)

$$D(E_0, s) \oplus D(E_0 + \frac{1}{2}, s + \frac{1}{2}) \oplus D(E_0 + \frac{1}{2}, s - \frac{1}{2}) \oplus D(E_0 + 1, s) \quad , \tag{B.33}$$

4. The Dirac singleton

$$D(\frac{1}{2}, 0) \oplus D(1, \frac{1}{2}) \quad . \tag{B.34}$$

Apart from the Dirac singleton which has no counterpart in the Poincaré case, we see that the particle content is perfectly the same between the $OSp(4/1)$ and the $N = 1$ Poincaré multiplets. However, the latter case the spectrum is degenerate in mass. For the massless multiplet, *i.e.* (B.32) and massless Wess-Zumino multiplet (see next section), we have the same picture as in the Minkowski space.

B.4 Example: Wess-Zumino Multiplet

To make the above supermultiplet clear, we discuss here in detail the Wess-Zumino multiplet including its mass relation and construct a simple Lagrangian in order to see the difference between rigid supersymmetry in anti-de Sitter and Minkowski spacetimes.

Let us first discuss about the mass relation for Wess-Zumino multiplet (B.31). Using the mass formula (B.24) and replacing the energy labels E_0 by, $E_0 + 1$ for real scalar 0^+, E_0 for real pseudoscalar 0^-, and $E_0 + \frac{1}{2}$ for spin-$\frac{1}{2}$ fermion, we have then

$$\begin{aligned} m^2_{0^+} &= \ell^2 E_0 \,(E_0 - 1) \\ m^2_{0^-} &= \ell^2 (E_0 - 2)(E_0 - 1) \\ m^2_{\frac{1}{2}} &= \ell^2 (E_0 - 1)^2 \end{aligned} \tag{B.35}$$

for $E_0 > \frac{1}{2}$. We see that for $E_0 = 1$ we obtain a massless Wess-Zumino multiplet in anti-de Sitter space which is the same as in Minkowski space.

Furthermore, one can check that the masses in (B.35) satisfy the following sum rule

$$m_{0^+}^2 + m_{0^-}^2 - 2m_{\frac{1}{2}}^2 = 0 \quad . \tag{B.36}$$

This is an example of a more general relation satisfied by the mass of any supersymmetric theory in an unbroken phase [54]

$$\mathrm{Str}M^2 = \sum_s (-1)^{2s}(2s+1)m_s^2 = 0 \quad , \tag{B.37}$$

where Str denotes the supertrace of the squared mass matrix.

Now we construct the Lagragian for the Wess-Zumino multiplet in anti-de Sitter space. Let us first consider the massless case. Due to its spin content, the free Lagrangian of the on-shell massless Wess-Zumino multiplet in anti-de Sitter space should have form [60, 61]

$$L = \frac{1}{2}\mathcal{D}_a A\, \mathcal{D}^a A + \frac{1}{2}\mathcal{D}_a B\, \mathcal{D}^a B + \frac{\mathrm{i}}{2}\bar{\zeta}\gamma^a \mathcal{D}_a \zeta + \ell^2 \left(A^2 + B^2\right) \quad , \tag{B.38}$$

where (A, B, ζ) are a scalar, a pseudoscalar, and a single Majorana fermion respectively. The derivative $\mathcal{D}_a$ is the anti-de Sitter covariant derivative and acts on scalar as $(\mathcal{D}_a A, \mathcal{D}_a B) = (\partial_a A, \partial_a B)$ and $D_a\zeta = \partial_a\zeta - \frac{1}{4}\langle\omega_a^{bc}\rangle\gamma_{bc}\zeta$. The supersymmetry transformations of the fields leaving invariant (B.38) are

$$\begin{aligned} \delta A &= \bar{\epsilon}\zeta \ , \quad \delta B = -\mathrm{i}\bar{\epsilon}\gamma_5\zeta \quad , \\ \delta\zeta &= -\mathrm{i}\gamma^a \mathcal{D}_a(A + \mathrm{i}\gamma_5 B)\epsilon - \ell(A + \mathrm{i}\gamma_5 B)\epsilon \quad , \end{aligned} \tag{B.39}$$

where ϵ is a Killing spinor satisfy the Killing spinor equation

$$(\mathcal{D}_a + \frac{\mathrm{i}}{2}\,\ell\,\gamma_a)\epsilon = 0 \quad . \tag{B.40}$$

Unlike in the Minkowski case, the appearing new term in the free model (B.38) is caused in order for maintaining invariance under the supersymmetry transformation (B.39) and correspondingly, by the fact that the spinor parameter ϵ is a Killing spinor.

Indeed, as the cosmological constant $\Lambda_0 \to 0$, we retrieve the Lagrangian and the supersymmetry transformation of the Wess-Zumino multiplet constructed in the previous section. Furthermore, the Killing spinor equation (B.40) reduces to $\partial_\mu\epsilon = 0$ whose solution is a constant spinor. Thus rigid supersymmetry in general is supersymmetry which admits Killing spinor [60, 58].

With the above example, one can further generalize to the massive case which can be written down as [60]

$$\begin{aligned} L_1 &= \frac{1}{2}\mathcal{D}_a A\, \mathcal{D}^a A + \frac{1}{2}\mathcal{D}_a B\, \mathcal{D}^a B + \frac{\mathrm{i}}{2}\bar{\zeta}\gamma^a \mathcal{D}_a \zeta \\ &\quad -\frac{1}{2}(m^2 + m\ell - 2\ell^2)A^2 - \frac{1}{2}(m^2 - m\ell - 2\ell^2)B^2 - \frac{1}{2}m\bar{\zeta}\zeta \quad , \end{aligned} \tag{B.41}$$

together with its supersymmetry transformation

$$\begin{aligned} \delta A &= \bar{\epsilon}\zeta \ , \quad \delta B = \mathrm{i}\bar{\epsilon}\gamma_5\zeta \quad , \\ \delta\zeta &= -\mathrm{i}\gamma^a \mathcal{D}_a(A + \mathrm{i}\gamma_5 B)\epsilon - \ell(A + \mathrm{i}\gamma_5 B) - m(A - \mathrm{i}\gamma_5 B)\epsilon \quad . \end{aligned} \tag{B.42}$$

Comparing (B.38) and (B.41), we see that the parameter m in the Lagrangian (B.41) is related to the mass $m^2_{0^+}, m^2_{0^-}, m^2_{\frac{1}{2}}$ by the formulae

$$\begin{aligned} m^2_{0^+} &= m(m+\ell) \quad , \\ m^2_{0^-} &= m(m-\ell) \quad , \\ m^2_{\frac{1}{2}} &= m^2 \quad . \end{aligned} \tag{B.43}$$

Appendix C

More on $N = 2$ Supergravity

In this section we give some additional facts about $N = 2$ supergravity. The homogeneous symmetric special and quaternionic Kähler manifolds are listed. In addition, the gauged $N = 2$ Lagragian up to four-fermion terms and the supersymmetry transformation of the field content are written down.

C.1 Special Kähler Geometry

n_V	Coset Manifold
1	$\frac{SU(1,1)}{U(1)}$
n	$\frac{SU(n,1)}{SU(n)\times U(1)}$
$n+1$	$\frac{SU(1,1)}{U(1)} \otimes \frac{SO(n,2)}{SO(n)\times SO(2)}$
6	$\frac{Sp(6,\mathbb{R})}{U(3)}$
9	$\frac{U(3,3)}{U(3)\times U(3)}$
15	$\frac{SO^*(12)}{U(6)}$
27	$\frac{E_{7(-26)}}{E_6\times SO(2)}$

Table C.1: Homogeneous Symmetric Special Kähler Manifold

Useful formulae in special Kähler geometry

$$\begin{aligned} \mathcal{F}_\Lambda &= \mathcal{N}_{\Lambda\Sigma}\, X^\Sigma \quad , \\ D_{\bar{j}}\bar{\mathcal{F}}_\Lambda &= \mathcal{N}_{\Lambda\Sigma}\, D_{\bar{j}}\bar{X}^\Sigma \, , \end{aligned} \tag{C.1}$$

$$
\begin{aligned}
g^{i\bar{j}} f_i^\Lambda \bar{f}_{\bar{j}}^\Sigma &= -\frac{1}{2}(\mathcal{I}^{-1})^{\Lambda\Sigma} - \bar{L}^\Lambda L^\Sigma \quad , \\
\nabla_i f_j^\Lambda &= \mathrm{i} C_{ijk} g^{k\bar{l}} \bar{f}_{\bar{l}}^\Lambda \quad ,
\end{aligned}
\tag{C.2}
$$

where

$$
\begin{aligned}
D_i X^\Lambda &= \partial_i X^\Lambda + \mathcal{K}_{V,i} X^\Lambda \quad , \\
D_i \mathcal{F}_\Lambda &= \partial_i \mathcal{F}_\Lambda + \mathcal{K}_{V,i} \mathcal{F}_\Lambda \quad , \\
\mathcal{I}_{\Lambda\Sigma} &= \mathrm{Im} \mathcal{N}_{\Lambda\Sigma} \quad , \\
C_{ijk} &= f_i^\Lambda f_j^\Sigma f_k^\Gamma \mathcal{F}_{\Lambda\Sigma\Gamma} \quad .
\end{aligned}
$$

C.2 Quaternionic Kähler Geometry

n_H	Coset Manifold
n	$\mathbb{H}P(n) = \frac{Sp(2n,2)}{Sp(2n)\times Sp(2)}$
n	$X(n) = \frac{SU(n,2)}{SU(n)\times SU(2)\times U(1)}$
n	$Y(n) = \frac{SO(n,4)}{SO(n)\times SO(4)}$
2	$\frac{G_2}{SO(4)}$
7	$\frac{F_4}{Sp(6)\times Sp(2)}$
10	$\frac{E_6}{SU(6)\times SU(2)}$
16	$\frac{E_7}{SO(12)\times SU(2)}$
28	$\frac{E_8}{E_7\times SU(2)}$

Table C.2: Homogeneous Symmetric Quaternionic Kähler Manifold

Note that $X(2) \cong Y(2)$.

C.3 Gauged $N=2$ Lagrangian

The gauged $N=2$ Lagrangian up to 4-fermion terms

$$
\mathcal{L}^{N=2} = -\frac{1}{2}R + g_{i\bar{j}} \nabla_\mu z^i \nabla^\mu \bar{z}^{\bar{i}} + h_{uv} \nabla_\mu q^u \nabla^\mu q^v + \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}} \left(\bar{\psi}_\mu^A \gamma_\sigma \nabla_\nu \psi_{A\lambda} - \bar{\psi}_{A\mu} \gamma_\sigma \nabla_\nu \psi_\lambda^A \right)
$$

$$
\begin{aligned}
& -\frac{\mathrm{i}}{2} g_{i\bar{j}} \left(\bar{\lambda}^{iA}\gamma^\mu\nabla_\mu\lambda^{\bar{j}}_A + \bar{\lambda}^{\bar{j}}_A\gamma^\mu\nabla_\mu\lambda^{iA}\right) - \mathrm{i}\left(\bar{\zeta}^\alpha\gamma^\mu\nabla_\mu\zeta_\alpha + \bar{\zeta}_\alpha\gamma^\mu\nabla_\mu\zeta^\alpha\right)\\
& + \mathrm{i}\left(\bar{\mathcal{N}}_{\Lambda\Sigma}F^{-\Lambda}_{\mu\nu}F^{-\Lambda\mu\nu} - \mathcal{N}_{\Lambda\Sigma}F^{+\Lambda}_{\mu\nu}F^{+\Lambda\mu\nu}\right) + \{-g_{i\bar{j}}\nabla_\mu\bar{z}^{\bar{i}}\bar{\psi}^\mu_A\lambda^{iA}\\
& - 2\mathcal{U}^{A\alpha}_u\nabla_\mu q^u\bar{\psi}^\mu_A\zeta_\alpha + g_{i\bar{j}}\nabla_\mu\bar{z}^{\bar{j}}\bar{\lambda}^{iA}\gamma^{\mu\nu}\psi_{A\nu} + 2\mathcal{U}^{A\alpha}_u\nabla_\mu q^u\bar{\zeta}_\alpha\gamma^{\mu\nu}\psi_{A\nu} + \text{h.c.}\}\\
& + \{F^{-\Lambda}_{\mu\nu}\mathcal{I}_{\Lambda\Sigma}(4L^\Sigma\bar{\psi}^{A\mu}\psi^{B\nu}\epsilon_{AB} - 4\mathrm{i}\bar{f}^\Sigma_{\bar{i}}\bar{\lambda}^{\bar{i}}_A\gamma^\nu\psi^\mu_B\epsilon^{AB}\\
& + \frac{1}{2}\nabla_i f^\Sigma_j\bar{\lambda}^{iA}\gamma^{\mu\nu}\lambda^{jB}\epsilon_{AB} - L^\Sigma\bar{\zeta}_\alpha\gamma^{\mu\nu}\zeta_\beta\mathbb{C}^{\alpha\beta}) + \text{h.c.}\}\\
& + [2S_{AB}\bar{\psi}^A_\mu\gamma^{\mu\nu}\psi^B_\nu + \mathrm{i}g_{i\bar{j}}W^{iAB}\bar{\lambda}^{\bar{j}}_A\gamma^\mu\psi_{\mu B} + 2\mathrm{i}N^A_\alpha\bar{\zeta}^\alpha\gamma^\mu\psi^A_\mu\\
& + \mathcal{M}^{\alpha\beta}\bar{\zeta}_\alpha\zeta_\beta + \mathcal{M}^\alpha_{iA}\bar{\zeta}_\alpha\lambda^{iA} + \mathcal{M}_{iA|lB}\bar{\lambda}^{iA}\lambda^{lB} + \text{h.c.}] - V^{N=2}(z,\bar{z},q) \quad , \qquad \text{(C.3)}
\end{aligned}
$$

The $N=2$ Scalar Potential

$$
V^{N=2}(z,\bar{z},q) = \left(g_{i\bar{j}}k^i_\Lambda k^{\bar{j}}_\Sigma + 4h_{uv}k^u_\Lambda k^v_\Sigma\right)\bar{L}^\Lambda L^\Sigma + \left(g^{i\bar{j}}f^\Lambda_i\bar{f}^\Sigma_{\bar{j}} - 3\bar{L}^\Lambda L^\Sigma\right)P^x_\Lambda P^x_\Sigma \,. \qquad \text{(C.4)}
$$

Supergravity transformation rules of the Fermi fields

$$
\begin{aligned}
\delta\psi_{A\mu} &= \widehat{\mathcal{D}}_\mu\epsilon_A + \mathrm{i}S_{AB}\gamma_\mu\epsilon^B + \epsilon_{AB}T^-_{\mu\nu}\gamma_\nu\epsilon^B \quad ,\\
\delta\lambda^{iA} &= \mathrm{i}\nabla_\mu z^i\gamma^\mu\epsilon^A + G^{-i}_{\mu\nu}\gamma^{\mu\nu}\epsilon^{AB}\epsilon_B + W^{iAB}\epsilon_B \quad , \qquad \text{(C.5)}\\
\delta\zeta_\alpha &= \mathrm{i}\mathcal{U}^{B\beta}_u\nabla_\mu q^u\gamma^\mu\epsilon^A\epsilon_{AB}\,\mathbb{C}_{\alpha\beta} + \sqrt{2}N^A_\alpha\epsilon_A \quad ,
\end{aligned}
$$

where $\widehat{\mathcal{D}}_\mu\epsilon_A = \partial_\mu\epsilon_A - \frac{1}{4}\gamma_{ab}\,\omega^{ab}_\mu\epsilon_A + \frac{\mathrm{i}}{2}\hat{Q}_\mu\epsilon_A + \hat{\omega}_{\mu|A}{}^B\,\epsilon_B$, and also $T^-_{\mu\nu} = 2\mathrm{i}\,\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}L^\Sigma F^{-\Lambda}_{\mu\nu}$ and $G^{-i}_{\mu\nu} = -g^{i\bar{j}}\bar{f}^\Lambda_{\bar{j}}\mathrm{Im}\mathcal{N}_{\Lambda\Sigma}F^{-\Sigma}_{\mu\nu}$.

Supergravity transformation rules of the Bose fields

$$
\begin{aligned}
\delta e^a_\mu &= -\mathrm{i}\bar{\psi}_{A\mu}\gamma^a\epsilon^A - \mathrm{i}\bar{\psi}^{A\mu}\gamma^a\epsilon_A \quad ,\\
\delta A^\Lambda_\mu &= 2\bar{L}^\Lambda\bar{\psi}_{A\mu}\epsilon_B\,\epsilon^{AB} + 2\bar{L}^\Lambda\bar{\psi}^A_\mu\epsilon^B\epsilon_{AB}\\
&+ \mathrm{i}f^\Lambda_i\bar{\lambda}^{iA}\gamma_\mu\epsilon^B\epsilon_{AB} + \mathrm{i}\bar{f}^\Lambda_{\bar{i}}\bar{\lambda}^{\bar{i}}_A\gamma_\mu\epsilon_B\,\epsilon^{AB} \qquad \text{(C.6)}\\
\delta z^i &= \bar{\lambda}^{iA}\epsilon_A \quad ,\\
\delta\bar{z}^{\bar{i}} &= \bar{\lambda}^{\bar{i}}_A\epsilon^A \quad ,\\
\delta q^u &= \mathcal{U}^u_{A\alpha}(\bar{\zeta}^\alpha\epsilon^A + \mathbb{C}^{\alpha\beta}\epsilon^{AB}\bar{\zeta}_\beta\epsilon_B) \quad .
\end{aligned}
$$

Appendix D

Spontaneous $N = 1 \rightarrow N = 0$ SUSY Breaking and the Super-Higgs Effect

In this appendix we discuss spontaneous $N = 1 \rightarrow N = 0$ supersymmetry breaking for arbitrary cosmological constant in the absence of vector multiplets. We start by reviewing $N = 1$ supergravity and also introduce the notation used in our analysis. At the end, we discuss the super-Higgs effect for spontaneous $N = 1 \rightarrow N = 0$ supersymmetry breaking.

D.1 Short Review of $N = 1$ Supergravity

In four dimensions the spectrum of a generic $N = 1$ theory consists of a gravitational multipet, n_v vector multiplets and n_c chiral multiplets. These multiplets are decomposed of the following component fields:

- *a gravitational multiplet*

$$(g_{\mu\nu}, \psi_\mu^1) \ , \qquad \mu = 0, ..., 3 \ . \tag{D.1}$$

  This multiplet consist of the graviton $g_{\mu\nu}$ and a gravitino ψ_μ^1. For the gravitino $\psi_\mu^1, \psi_{1\mu}$ and the upper or lower index denotes left or right chirality respectively, see appendix A.

- n_v *vector multiplets*

$$(A_\mu^{\Lambda_0}, \lambda^{\Lambda_0}) \ , \qquad \Lambda_0 = 1, ..., n_v \ . \tag{D.2}$$

  Each vector multiplet contains a gauge boson $A_\mu^{\Lambda_0}$ and a gaugino λ^{Λ_0}.

- n_c *chiral multiplets*

$$(\chi^i, z^i) \ , \qquad i = 1, ..., n_c \, . \tag{D.3}$$

  Each chiral multiplet consist of a spin-$\frac{1}{2}$ fermion χ^i and a complex scalar z^i.

The complex scalars $(\bar{z}^{\bar{i}}, z^i)$ span a Hodge-Kähler manifold with a metric $g_{i\bar{j}} = \partial_i \partial_{\bar{j}} \mathcal{K}_h$ where the Kähler potential $\mathcal{K}_h$ is an arbitrary real function [22, 62, 63].

The $N = 1$ supergravity Lagrangian which can be written, up to four-fermions terms and in the absence of the vector multiplets [62, 63][1]:

$$
\begin{aligned}
\mathcal{L}^{N=1} \;=\; & -\frac{1}{2}R + g_{i\bar{j}}\partial_\mu z^i \partial^\mu \bar{z}^{\bar{i}} + \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}}\left(\bar{\psi}^1_\mu \gamma_\sigma \widetilde{\nabla}_\nu \psi_{1\lambda} - \bar{\psi}_{1\mu}\gamma_\sigma \widetilde{\nabla}_\nu \psi^1_\lambda\right) \\
& - \frac{\mathrm{i}}{2} g_{i\bar{j}}\left(\bar{\chi}^i \gamma^\mu \nabla_\mu \chi^{\bar{j}} + \bar{\chi}^{\bar{j}}\gamma^\mu \nabla_\mu \chi^i\right) - g_{i\bar{j}}\left(\bar{\psi}_{1\nu}\gamma^\mu\gamma^\nu \chi^i \partial_\mu \bar{z}^{\bar{j}} + \bar{\psi}^1_\nu \gamma^\mu \gamma^\nu \chi^{\bar{j}}\partial_\mu z^i\right) \\
& + \mathcal{W}(z,\bar{z})\,\bar{\psi}^1_\mu \gamma^{\mu\nu}\psi^1_\nu + \bar{\mathcal{W}}(z,\bar{z})\,\bar{\psi}_{1\mu}\gamma^{\mu\nu}\psi_{1\nu} + \mathrm{i} g_{i\bar{j}}\left(\bar{N}^{\bar{j}}\bar{\chi}^i \gamma^\mu \psi^1_\mu + N^i \bar{\chi}^{\bar{j}}\gamma^\mu \psi_{1\mu}\right) \\
& + \mathcal{M}_{ij}\bar{\chi}^i \chi^j + \bar{\mathcal{M}}_{\bar{i}\bar{j}}\;\bar{\chi}^{\bar{i}}\chi^{\bar{j}} - V^{N=1}(z,\bar{z}) \quad ,
\end{aligned} \tag{D.4}
$$

where $\mathcal{W}(z,\bar{z})(\bar{\mathcal{W}}(z,\bar{z}))$ can be written in terms of an (anti)-holomorphic superpotential function of $W(z)(\bar{W}(\bar{z}))$,

$$
\begin{aligned}
\mathcal{W}(z,\bar{z}) &= e^{\frac{1}{2}\mathcal{K}_h(z,\bar{z})} W(z) \quad , \\
\bar{\mathcal{W}}(z,\bar{z}) &= e^{\frac{1}{2}\mathcal{K}_h(z,\bar{z})} \bar{W}(\bar{z}) \quad ,
\end{aligned} \tag{D.5}
$$

with $\mathcal{K}_h(z,\bar{z})$ is a Kähler potential of the chiral multiplets, and the quantities $N^i, \mathcal{M}_{ij}$ are given by:

$$
\begin{aligned}
N^i &= g^{i\bar{j}}\bar{\nabla}_{\bar{j}}\bar{\mathcal{W}}(z,\bar{z}) \quad , \\
\mathcal{M}_{ij} &= \frac{1}{2}\nabla_i \nabla_j \mathcal{W}(z,\bar{z}) \quad .
\end{aligned} \tag{D.6}
$$

On the other hand, the $N = 1$ scalar potential can be expressed in terms of $\mathcal{W}(\bar{\mathcal{W}})$ as

$$
V^{N=1}(z,\bar{z}) = g^{i\bar{j}}\nabla_i \mathcal{W}\,\bar{\nabla}_{\bar{j}}\bar{\mathcal{W}} - 3\mathcal{W}\bar{\mathcal{W}} \quad , \tag{D.7}
$$

where $\nabla_i \mathcal{W} = \partial_i \mathcal{W} + \frac{1}{2}\mathcal{K}_{h,i}\,\mathcal{W}$. The supersymmetry transformation laws up to 3-fermion terms leaving invariant (D.4) are:

$$
\begin{aligned}
\delta\psi_{1\mu} &= \widetilde{\mathcal{D}}_\mu \epsilon_1 + \frac{\mathrm{i}}{2}\mathcal{W}(z,\bar{z})\,\gamma_\mu\,\epsilon^1 \quad , \\
\delta\chi^i &= \mathrm{i}\partial_\mu z^i \gamma^\mu \epsilon_1 + N^i \epsilon_1 \quad , \\
\delta e^a_\mu &= -\mathrm{i}\bar{\psi}_{1\mu}\gamma^a \epsilon^1 - \mathrm{i}\bar{\psi}^{1\mu}\gamma^a \epsilon_1 \quad , \\
\delta z^i &= \bar{\chi}^i \epsilon_1 \quad ,
\end{aligned} \tag{D.8}
$$

where $\widetilde{\mathcal{D}}_\mu \epsilon_1 = \partial_\mu \epsilon_1 - \frac{1}{4}\gamma_{ab}\,\omega^{ab}_\mu \epsilon_1 + \frac{\mathrm{i}}{2}Q_{h,\mu}\epsilon_1$, and $Q_{h,\mu}$ is a $U(1)$-connection with respect to Kähler potential $\mathcal{K}_h$.

D.2 $N = 1 \to N = 0$ Supersymmetry Breaking

In this section we demand that the ground states respect fully Lorentz invariance which implies that the vacuum expectation value of all fermions and $\partial_\mu z^i$ are set to zero. Thus the supersymmetry variation (D.8) becomes

$$
\langle \delta\psi_{1\mu}\rangle = \mathcal{D}_\mu \epsilon_1 + \frac{\mathrm{i}}{2}\langle \mathcal{W}(z,\bar{z})\rangle\,\gamma_\mu\,\epsilon^1 \quad ,
$$

[1] Many articles and books review $N = 1$ supergravity see, for example [22], but the convention used in this thesis follows rather closely references [62, 63](see Appendix A).

$$\begin{aligned} \langle\delta\chi^i\rangle &= \langle N^i\rangle\,\epsilon_1 \quad , \\ \langle\delta e^a_\mu\rangle &= \langle\delta z^i\rangle = 0 \quad , \end{aligned} \tag{D.9}$$

The ground states require

$$\left\langle \frac{\partial V^{(N=1)}}{\partial z^i} \right\rangle = \langle N^i \nabla_j N_i\rangle - 2\langle\bar{\mathcal{W}} N_j\rangle = 0 \quad , \tag{D.10}$$

where $N_i = \nabla_i \mathcal{W}(z, \bar{z})$ and in addition, assuming that the potential (D.7) evaluated at the minimum is Λ_0, one gets

$$\langle N^i N_i\rangle = 3\langle\mathcal{W}\bar{\mathcal{W}}\rangle + \Lambda_0 \qquad , \tag{D.11}$$

where Λ_0 is the cosmological constant. Thus, in this case we have three possible ground states, namely, Minkowski ground states ($\Lambda_0 = 0$), anti-de Sitter ground states ($\Lambda_0 < 0$), and de Sitter ground states ($\Lambda_0 > 0$).[2]

Let us now consider the fermionic part of the Lagrangian (D.4) around the ground states

$$\begin{aligned} \mathcal{L}'_f \;\; = \;\; & \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}} \left(\bar{\psi}^1_\mu \gamma_\sigma \mathcal{D}_\nu \psi_{1\lambda} - \bar{\psi}_{1\mu}\gamma_\sigma \mathcal{D}_\nu \psi^1_\lambda\right) - \frac{\mathrm{i}}{2} g_{i\bar{j}} \left(\bar{\chi}^i \gamma^\mu \mathcal{D}_\mu \chi^{\bar{j}} + \bar{\chi}^{\bar{j}}\gamma^\mu \mathcal{D}_\mu \chi^i\right) \\ & + \langle\mathcal{W}(z,\bar{z})\rangle\, \bar{\psi}^1_\mu \gamma^{\mu\nu}\psi^1_\nu + \langle\bar{\mathcal{W}}(z,\bar{z})\rangle\, \bar{\psi}_{1\mu}\gamma^{\mu\nu}\psi_{1\nu} \\ & + \mathrm{i}\left(\langle N_i\rangle\, \bar{\chi}^i \gamma^\mu \psi^1_\mu + \langle\bar{N}_{\bar{i}}\rangle\, \bar{\chi}^{\bar{i}}\gamma^\mu \psi_{1\mu}\right) + \langle\mathcal{M}_{ij}\rangle\, \bar{\chi}^i \chi^j + \langle\bar{\mathcal{M}}_{\bar{i}\bar{j}}\rangle\, \bar{\chi}^{\bar{i}}\chi^{\bar{j}} \quad , \end{aligned} \tag{D.12}$$

where $\mathcal{D}_\mu = \partial_\mu - \frac{1}{4}\gamma_{ab}\,\langle\omega^{ab}_\mu\rangle$ is the anti-de Sitter or de Sitter covariant derivative and reduced to ordinary partial derivative if the ground states are Minkowski. As it has been discussed in the sections 3.5 and 4.3, the mixing spin-$\frac{1}{2}$ fermion χ^i and the gravitino field ψ^1_μ term in the mass-like term (D.12) plays an important role to define a Goldstone fermion

$$\eta_1 = \langle N_i\rangle\,\chi^i \quad . \tag{D.13}$$

Using the second equation in (D.9), the supersymmetry transformation of η_1 evaluated at the ground states has form

$$\langle\delta\eta_1\rangle = (3\langle\mathcal{W}\bar{\mathcal{W}}\rangle + \Lambda_0)\epsilon_1 \quad . \tag{D.14}$$

which indicates that supersymmetry is spontaneously broken. As the Goldstone fermion η_1 transform by a shift, it has a consequence that it can be eliminated from the theory by a suitable local supersymmetry transformation of the gravitino ψ^1_μ [22].

We define then the physical fermions $\chi^i_\perp$ as

$$\chi^i_\perp = \chi^i - \frac{\langle N^i\rangle}{3\langle\mathcal{W}\bar{\mathcal{W}}\rangle + \Lambda_0}\,\eta_1 \qquad . \tag{D.15}$$

which cannot be removed from the theory by any local supersymmetry transformation of the gravitino ψ^1_μ, because

$$\langle\delta\chi^i_\perp\rangle = 0 \quad . \tag{D.16}$$

[2] The stability of these ground states have been studied in reference [54].

Let us write the Lagrangian (D.12) in terms of the Goldstone mode η_1 and the physical fermions

$$\begin{aligned}
\mathcal{L}'_f &= \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}} \left(\bar{\psi}^1_\mu \gamma_\sigma \mathcal{D}_\nu \psi_{1\lambda} - \bar{\psi}_{1\mu}\gamma_\sigma \mathcal{D}_\nu \psi^1_\lambda\right) - \frac{\mathrm{i}}{2} g_{i\bar{j}} \left(\bar{\chi}^i_\perp \gamma^\mu \mathcal{D}_\mu \chi^{\bar{j}}_\perp + \bar{\chi}^{\bar{j}}_\perp \gamma^\mu \mathcal{D}_\mu \chi^i_\perp\right) \\
&\quad - \frac{\mathrm{i}}{2(3\langle \mathcal{W}\bar{\mathcal{W}}\rangle + \Lambda_0)} \left(\bar{\eta}_1 \gamma^\mu \mathcal{D}_\mu \eta^1 + \bar{\eta}^1 \gamma^\mu \mathcal{D}_\mu \eta_1\right) + \langle \mathcal{W}\rangle\, \bar{\psi}^1_\mu \gamma^{\mu\nu} \psi^1_\nu \\
&\quad + \langle \bar{\mathcal{W}}\rangle\, \bar{\psi}_{1\mu}\gamma^{\mu\nu}\psi_{1\nu} + \mathrm{i}\bar{\eta}_1 \gamma^\mu \psi^1_\mu + \mathrm{i}\bar{\eta}^1 \gamma^\mu \psi_{1\mu} \\
&\quad + \frac{1}{(3\langle \mathcal{W}\bar{\mathcal{W}}\rangle + \Lambda_0)} \left(\langle \bar{\mathcal{W}}\rangle\, \bar{\eta}_1 \eta_1 + \langle \mathcal{W}\rangle\, \bar{\eta}^1 \eta^1\right) + \langle \mathcal{M}^{(\Lambda_0)}_{ij}\rangle\, \bar{\chi}^i_\perp \chi^j_\perp \\
&\quad + \langle \bar{\mathcal{M}}^{(\Lambda_0)}_{\bar{i}\bar{j}}\rangle\, \bar{\chi}^{\bar{i}}_\perp \chi^{\bar{j}}_\perp \quad ,
\end{aligned} \tag{D.17}$$

where

$$\langle \mathcal{M}^{(\Lambda_0)}_{ij}\rangle = \langle \mathcal{M}_{ij}\rangle - \frac{\langle N_i\, N_j\rangle}{3\langle \mathcal{W}\bar{\mathcal{W}}\rangle + \Lambda_0}\langle \bar{\mathcal{W}}\rangle \qquad , \tag{D.18}$$

is the physical mass of the fermions $\chi^i_\perp$ for arbitrary cosmological constant (denoted by superscript (Λ_0)). Using (D.10) one can shows that the physical mass (D.18) satisfies

$$\langle \mathcal{M}^{(\Lambda_0)}_{ij}\, N^j\rangle = 0 \qquad , \tag{D.19}$$

which means that the Goldstone fermion is already split off from the physical mode.

Now, let us consider the supersymmetry variation of ψ^1_μ in (D.9) and of η_1 (D.14). The equation (D.14) means that the Goldstone fermion η_1 can be viewed as a supersymmetry transformation parameter and further, we have a replacement

$$\epsilon_1 \to \frac{1}{(3\langle \mathcal{W}\bar{\mathcal{W}}\rangle + \Lambda_0)}\, \eta_1 \quad . \tag{D.20}$$

Thus, the local supersymmetry transformation of the gravitino ψ^1_μ which removes the Goldstone fermion η_1 is given by

$$\psi^1_\mu \to \psi^1_\mu + \frac{1}{3\langle \mathcal{W}\bar{\mathcal{W}}\rangle + \Lambda_0}\left(\mathcal{D}_\mu \eta^1 + \frac{\mathrm{i}}{2}\langle \bar{\mathcal{W}}\rangle\, \gamma_\mu\, \eta_1\right) \qquad . \tag{D.21}$$

This transformation would not be defined if $3\langle \mathcal{W}\bar{\mathcal{W}}\rangle = -\Lambda_0$, which is precisely the condition for ψ^1_μ to be invariant under supersymmetry in these backgrounds, i.e. Minkowski or anti-de Sitter backgrounds [53,54]. Then the supersymmetry transformation needed to eliminate the Goldstone fermion would be singular and in fact the definition of the physical fermions (D.15) would not be possible as it would diverge. However, such condition cannot occur in de Sitter ground states. Our argument is as follows. From (D.11) we see that in order to get the unbroken supersymmetry one should have $\langle N^i\rangle = 0$. But since we have $\Lambda_0 > 0$, so it is impossible to have $\langle N^i\rangle = 0$. Thus, in de Sitter ground states supersymmetry is always broken.

Finally, to see the super-Higgs we insert the above gauge transformation to the Lagrangian (D.17), then we arrive at

$$\begin{aligned}
\mathcal{L}'_f &= \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}} \left(\bar{\psi}^1_\mu \gamma_\sigma \mathcal{D}_\nu \psi_{1\lambda} - \bar{\psi}_{1\mu}\gamma_\sigma \mathcal{D}_\nu \psi^1_\lambda\right) - \frac{\mathrm{i}}{2} g_{i\bar{j}} \left(\bar{\chi}^i_\perp \gamma^\mu \mathcal{D}_\mu \chi^{\bar{j}}_\perp + \bar{\chi}^{\bar{j}}_\perp \gamma^\mu \mathcal{D}_\mu \chi^i_\perp\right) \\
&\quad + \frac{\mathrm{i}\Lambda_0}{(3\langle \mathcal{W}\bar{\mathcal{W}}\rangle + \Lambda_0)^2} \left(\bar{\eta}_1 \gamma^\mu \mathcal{D}_\mu \eta^1 + \bar{\eta}^1 \gamma^\mu \mathcal{D}_\mu \eta_1\right)
\end{aligned}$$

$$
\begin{aligned}
&+\frac{1}{(3\langle\mathcal{W}\bar{\mathcal{W}}\rangle+\Lambda_0)^2}\,\epsilon^{\mu\nu\lambda\sigma}\left(\bar{\mathcal{D}}_\mu\bar{\eta}_1\gamma_\sigma[\mathcal{D}_\nu,\mathcal{D}_\lambda]\eta^1+[\bar{\mathcal{D}}_\mu,\bar{\mathcal{D}}_\nu]\bar{\eta}^1\gamma_\lambda\mathcal{D}_\sigma\eta_1\right)\\
&+\frac{\Lambda_0}{(3\langle\mathcal{W}\bar{\mathcal{W}}\rangle+\Lambda_0)}\left(\mathrm{i}\bar{\eta}_1\gamma^\mu\psi^1_\mu+\mathrm{i}\bar{\eta}^1\gamma^\mu\psi_{1\mu}\right) \qquad \text{(D.22)}\\
&+\frac{1}{(3\langle\mathcal{W}\bar{\mathcal{W}}\rangle+\Lambda_0)}\,\epsilon^{\mu\nu\lambda\sigma}\left(\bar{\psi}^1_\mu\gamma_\sigma[\mathcal{D}_\nu,\mathcal{D}_\lambda]\eta_1+[\bar{\mathcal{D}}_\mu,\bar{\mathcal{D}}_\nu]\bar{\eta}^1\gamma_\lambda\,\psi_{1\sigma}\right)\\
&+\frac{1}{(3\langle\mathcal{W}\bar{\mathcal{W}}\rangle+\Lambda_0)^2}\Big\{-\Lambda_0\left(\langle\bar{\mathcal{W}}\rangle\,\bar{\eta}_1\eta_1+\langle\mathcal{W}\rangle\,\bar{\eta}^1\eta^1\right)\\
&+\frac{\mathrm{i}}{4}\,\epsilon^{\mu\nu\lambda\sigma}(-\langle\bar{\mathcal{W}}\rangle\,\bar{\eta}_1\gamma_{\mu\nu}[\mathcal{D}_\lambda,\mathcal{D}_\sigma]\eta_1+\langle\mathcal{W}\rangle\,[\bar{\mathcal{D}}_\mu,\bar{\mathcal{D}}_\nu]\bar{\eta}^1\,\gamma_{\lambda\sigma}\eta^1)\Big\}\\
&+\langle\mathcal{W}\rangle\,\bar{\psi}^1_\mu\gamma^{\mu\nu}\psi^1_\nu+\langle\bar{\mathcal{W}}\rangle\,\bar{\psi}_{1\mu}\gamma^{\mu\nu}\psi_{1\nu}+\langle\mathcal{M}^{(\Lambda_0)}_{ij}\rangle\,\bar{\chi}^i_\perp\chi^j_\perp+\langle\bar{\mathcal{M}}^{(\Lambda_0)}_{\bar{i}\bar{j}}\rangle\,\bar{\chi}^{\bar{i}}_\perp\chi^{\bar{j}}_\perp \quad .
\end{aligned}
$$

Now using the following equations

$$
\begin{aligned}
[\mathcal{D}_\mu,\mathcal{D}_\nu]\eta_1 &= -\frac{\Lambda_0}{6}\,\gamma_{\mu\nu}\,\eta_1 \quad ,\\
[\bar{\mathcal{D}}_\mu,\bar{\mathcal{D}}_\nu]\bar{\eta}_1 &= \frac{\Lambda_0}{6}\,\bar{\eta}_1\gamma_{\mu\nu} \quad , \qquad \text{(D.23)}
\end{aligned}
$$

where $\bar{\mathcal{D}}_\mu\bar{\eta}_1=\partial_\mu\,\bar{\eta}_1+\frac{1}{4}\bar{\eta}_1\gamma_{ab}\,\langle\omega^{ab}_\mu\rangle$, the Lagrangian (D.22) takes then simply form as

$$
\begin{aligned}
\mathcal{L}'_f &= \frac{\epsilon^{\mu\nu\lambda\sigma}}{\sqrt{-g}}\left(\bar{\psi}^1_\mu\gamma_\sigma\mathcal{D}_\nu\psi_{1\lambda}-\bar{\psi}_{1\mu}\gamma_\sigma\mathcal{D}_\nu\psi^1_\lambda\right)-\frac{\mathrm{i}}{2}g_{i\bar{j}}\left(\bar{\chi}^i_\perp\gamma^\mu\mathcal{D}_\mu\chi^{\bar{j}}_\perp+\bar{\chi}^{\bar{j}}_\perp\gamma^\mu\mathcal{D}_\mu\chi^i_\perp\right)\\
&\quad+\mathcal{W}\,\bar{\psi}^1_\mu\gamma^{\mu\nu}\psi^1_\nu+\bar{\mathcal{W}}\,\bar{\psi}_{1\mu}\gamma^{\mu\nu}\psi_{1\nu}+\mathcal{M}^{(\Lambda_0)}_{ij}\bar{\chi}^i_\perp\chi^j_\perp+\bar{\mathcal{M}}^{(\Lambda_0)}_{\bar{i}\bar{j}}\,\bar{\chi}^{\bar{i}}_\perp\chi^{\bar{j}}_\perp \quad . \qquad \text{(D.24)}
\end{aligned}
$$

From the resulting Lagrangian (D.24) one can see that the gravitino $\psi_{1\lambda}$ eats the Goldstone fermion η_1 and then becomes massive. It has four instead of two degrees of freedom. Thus we have seen that the super-Higgs effect occurs if the $N = 1$ local supersymmetry is spontaneously broken.

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Erklärung

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