

Type IIA orientifolds on $SU(2)$ -structure Manifolds

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Abstract

We investigate the possible supersymmetry-preserving orientifold projections of type IIA string theory on a six-dimensional background with $SU(2)$ -structure. We find two categories of projections which preserve half of the low-energy supersymmetry, reducing the effective theory from an $\mathcal{N} = 4$ supergravity theory, to an $\mathcal{N} = 2$ supergravity. For these two cases, we impose the projection on the low-energy spectrum and reduce the effective $\mathcal{N} = 4$ supergravity action accordingly. We can identify the resulting gauged $\mathcal{N} = 2$ supergravity theory and bring the action into canonical form. We compute the scalar moduli spaces and characterize the gauged symmetries in terms of the geometry of these moduli spaces. Due to their origin in $\mathcal{N} = 4$ supergravity, which is a highly constrained theory, the moduli spaces are of a very simple form. We find that, for suitable background manifolds, isometries in all scalar sectors can become gauged. The obtained gaugings share many features with those of $\mathcal{N} = 2$ supergravities obtained previously from other G -structure compactifications.

Zusammenfassung

Das Thema dieser Arbeit sind Orientifold-Projektionen der Typ IIA Stringtheorie auf Mannigfaltigkeiten mit $SU(2)$ Struktur. Wir finden zwei Klassen von Projektionen, welche die Hälfte der Supersymmetrie der niederenergetischen effektiven Theorie erhalten, und damit die $\mathcal{N} = 4$ Supergravitation zu einer $\mathcal{N} = 2$ Supergravitation reduzieren. Für die beiden Projektionen wird das resultierende niederenergetische Spektrum berechnet, und in die Wirkung der effektiven $\mathcal{N} = 4$ Theorie eingesetzt. Wir zeigen, dass das Ergebnis jeweils einer geeichten $\mathcal{N} = 2$ Supergravitation entspricht, und bringen die Wirkung in die kanonische Form. Wir beschreiben die Moduli-Räume, welche von den skalaren Feldern parametrisiert werden, und charakterisieren die geeichten Symmetrien der Theorie anhand der Geometrie dieser Räume. Für geeignete Mannigfaltigkeiten können Isometrien aller skalaren Sektoren geeicht werden. Die Eichalgebren, die sich ergeben, weisen viele Ähnlichkeiten auf mit den Algebren aus $\mathcal{N}=2$ Supergravitationstheorien, die aus früheren Kompaktifizierungen auf Mannigfaltigkeiten mit reduzierter Strukturgruppe erhalten wurden. Da die resultierenden $\mathcal{N} = 2$ Supergravitationstheorien ihren Ursprung in einer $\mathcal{N} = 4$ Supergravitation haben, deren Kopplungen aufgrund der hohen Symmetrie stark eingeschränkt sind, bilden die erhaltenen effektiven Wirkungen eine sehr begrenzte Unterklasse der $\mathcal{N} = 2$ Supergravitationstheorien.

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Chapter 1

Introduction

The Standard Model has been a tremendous success in explaining the physics of elementary particles. Ever since its inception in the 1970's, experiments have confirmed the predictions of the model to high accuracy, with only the Higgs boson still remaining unobserved. From a theoretical perspective, it is satisfying that the relatively constrained framework of renormalizable quantum field theories can describe the complex behavior of all known fundamental particles.

Nevertheless, there are reasons to look for a more fundamental description, for which string theory is considered a promising candidate. One cause of dissatisfaction with the Standard Model is its relatively large number of remaining free parameters, some of which have to be finely tuned in order for the model to be consistent. Since it promises a description of all particle physics in terms of a single fundamental object, string theory could in principle be a vast improvement in this area. However, the large number of possible vacua in string theory could also mean that we have traded off one set of parameters for a new, far bigger one. It remains to be seen if the combination of observational input and string theoretical consistency constraints can remove some of the arbitrariness in the choice of a vacuum state.

A more fundamental issue is the lack of a quantum mechanical description of gravitation, where quantum field theory breaks down. The fact that perturbative string theory is a perturbatively finite quantum theory, with a spectrum which naturally contains the graviton, is probably the strongest argument in favor of string theory as a fundamental theory of nature.

1.1 String Compactification

If string theory is to be a fundamental theory of nature, it should be able to reproduce known physics. Therefore, a lot of effort is put into the construction of models that can explain the observations in particle physics and cosmology. This implies that the low-energy limit of the model should feature some extension of the standard

model with spontaneously broken supersymmetry, and that it should reproduce the known features of cosmology, such as a de Sitter vacuum and a correct inflationary mechanism. Though a model satisfying all these constraints has not yet been found, there is still a lot of progress in this area, and promising partial results have been achieved using a variety of techniques.

Perturbative string theory [1–3] describes one-dimensional objects, strings, moving through space-time. It is commonly formulated in terms of a two-dimensional conformal field theory. Classically, this theory describes the time-dependent embedding of the string, described as a two-dimensional space called the world-sheet, into space-time, which is called the target-space. A first difficulty lies in the fact that, in the perturbative regime, a consistent string theory is naturally formulated with a ten-dimensional target space.¹ In order to reconcile this with the four-dimensional observed universe, the most common solution is to assume that six dimensions form a compact space, whose size is below the scale probed by current experiments. Thus we have a ten-dimensional background space-time which is a product $\mathcal{M} = \mathcal{M}_{1,3} \times \mathcal{Y}_6$. Originally, the most attention was given to compactifications on internal manifolds \mathcal{Y}_6 which are Calabi-Yau [1, 4], since such backgrounds, being Ricci-flat, fulfill the vacuum Einstein equations. Using an effective field theory approximation to string theory, which is given by a ten-dimensional supergravity theory, one may then obtain a four-dimensional low-energy theory via so-called Kaluza-Klein reduction. This essentially exploits the fact that momentum along the compact directions gets a discrete spectrum, with a separation between the different mass levels of the order of the inverse length scale of the compact space \mathcal{Y}_6 . Still under the assumption that the compact space is sufficiently small, one can neglect all but the lightest modes, which are massless modes in the Calabi-Yau case, to obtain an effective theory valid at low energies. This effectively reduces the degrees of freedom of each ten-dimensional field to those of a finite number of four-dimensional fields.

With the compactification ansatz comes the practical problem of moduli stabilization. After the Kaluza-Klein reduction, the remaining four-dimensional theory generically contains a large number of massless scalar fields, or moduli. These moduli include the effective four-dimensional fields originating from the metric on the internal manifold, as well as the string coupling. The metric moduli describe the “shape” of the internal manifold, including, more specifically, its overall volume. In order to remain within the regime of validity of the effective theory, the volume of the internal manifold should be fixed at a value which is large compared to the string length, and the string coupling should stay small. In a realistic model, furthermore, all scalar fields should obtain a sufficiently high mass, since no scalar field has been observed in the real world yet, let alone the large numbers of scalar fields which typically arise

¹The requirement of a ten-dimensional space-time arises from the mathematical consistency conditions of the conformal field theory (CFT). These consistency conditions can alternatively be satisfied by considering more complicated two-dimensional CFT’s on the world-sheet. However, explicit low-energy actions for these theories are hard to obtain, and one is restricted to CFT techniques in their study.

from string theory compactifications. To address this problem of moduli stabilization, more sophisticated background configurations must be considered. The first string models where all moduli could be stabilized used contributions from non-perturbative effects, as well as background fluxes to generate a suitable potential [5].

1.2 Fluxes and generalized Calabi-Yau manifolds

So-called “flux backgrounds” are vacua where, in addition to the background metric, non-zero vacuum expectation values are specified for some of the p -form field strengths (reviews and references can be found in [6–10]). When a p -form field has flux through a non-trivial cycle in the internal manifold, there is no continuous transformation that can reduce the flux to zero. Thus, the space of different flux configurations is discrete, protecting these backgrounds against quantum corrections. The fact that these fluxes contribute to the potential for the moduli can be understood intuitively as follows: the presence of flux through a cycle adds an energetic cost to deformations of the cycle, which then leads to a potential for the moduli describing these deformations [7].

When studying string theory compactifications, one is usually interested in background configurations that preserve some amount of low-energy supersymmetry, since supersymmetry ensures that quantum corrections will not distort the (classical) effective theory obtained from compactification too heavily. In a compactification scenario, the four-dimensional supersymmetry will be inherited from the supersymmetry of the original ten-dimensional theory [1]. Explicitly, this can be seen by decomposing the parameter of the ten-dimensional supersymmetry transformations into a tensor product of a four-dimensional spinor $\varepsilon_4(x)$ on $\mathcal{M}_{1,3}$ and a six-dimensional spinor $\eta(y)$ on \mathcal{Y}_6 as follows:

$$\varepsilon_{10} = \varepsilon_4(x) \otimes \eta(y) + \text{c.c.} . \quad (1.1)$$

ε_4 will appear as a low-energy supersymmetry transformation, as long as the corresponding ten-dimensional supersymmetry ε_{10} leaves the background field configuration invariant. This way, a ten-dimensional supersymmetry transformation gives rise to one four-dimensional supersymmetry transformation for every η satisfying the conditions imposed by the preservation of the background.

The background configuration consists of a set of vacuum expectation values for the bosonic fields, such as the metric on the internal manifold and the dilaton, or, in the presence of background fluxes, some of the p -form field strengths. Supersymmetry of the background then amounts to the condition that the supersymmetry variations of the fermionic fields vanish. In the absence of fluxes, the condition that the supersymmetry variation of the gravitino vanishes, tells us that the supersymmetry parameter describing the preserved supersymmetry transformation must be covariantly constant with respect to the Levi-Civita connection. This implies that the holonomy group of the background manifold \mathcal{Y}_6 is reduced, such that it leaves in-

variant at least one internal spinor η , and therefore the background manifold satisfies the Calabi-Yau condition.

It turns out that supersymmetric vacua with background fluxes require an internal manifold \mathcal{Y}_6 which is no longer necessarily Calabi Yau [6]. The field strengths of the various p -form fields appear in the gravitino variation, and thus non-zero expectation values for these field strengths modify the supersymmetry condition. In the presence of fluxes, the supersymmetry parameter which leaves the background invariant must be covariantly constant, not with respect to the Levi-Civita connection, but with respect to a torsionful connection [11–13], where the fluxes appear as torsion components. In this sense, supersymmetry imposes a careful balancing between the geometry and the background flux. In the approach taken here, the internal geometry or the chosen set of fluxes will not be specified in detail.² Instead we work with a whole class of internal manifolds and derive the general form of the effective action, independent of the way the various consistency conditions are met.

Despite the fact that we no longer impose the Calabi-Yau condition, and do not specify the internal geometry in detail, we can still find meaningful results, due to the strong constraints imposed by low-energy supersymmetry. In order for the low-energy theory to have well-defined supersymmetry transformations, as discussed in the context of equation (1.1), a globally defined spinor η must still exist on the internal manifold \mathcal{Y}_6 . In mathematical terms, this means that \mathcal{Y}_6 must have a reduced structure group, which is a strictly weaker requirement than that of a covariantly constant spinor, or reduced holonomy group, but still imposes strong restrictions on the background geometry.

So far, compactifications on SU(3)-structure manifolds have received the most attention in the literature, since they possess only one globally defined spinor and hence lead to effective theories with the minimal amount of low-energy supersymmetry, which is $\mathcal{N} = 2$ in the case of type II compactifications. In this sense, SU(3)-structure manifolds are the direct generalization of conventional compactifications on Calabi-Yau threefolds. SU(2)-structure manifolds are more restricted, and possess two globally defined spinors, leading to $\mathcal{N} = 4$ effective theories [14–20]. The amount of low-energy supersymmetry can be reduced by including orientifold planes, leading to $\mathcal{N} = 1$ theories in the case SU(3)-structure compactifications [21–33], and to $\mathcal{N} = 2$ or $\mathcal{N} = 1$ in the case of SU(2)-structure compactifications [15, 34, 31, 35, 36].

1.3 D-branes and O-planes

Since space-time symmetry in the four non-compact dimensions forbids the escape of flux along the non-compact directions, the total charge in the compact space \mathcal{Y}_6 must be zero, and therefore charges induced by the fluxes must be canceled on the compactification manifold. D-branes and O-planes are higher-dimensional objects

²In fact, background flux is not explicitly considered in our calculations.

which are charged with respect to the p -form fields and as such can balance the total charge on \mathcal{Y}_6 . Furthermore, the perturbative spectrum of type II string theories does not contain the non-Abelian gauge fields necessary for any realistic model of particle physics. This also can be resolved by incorporating stacks of D-branes into the model, since intersecting stacks of D-branes naturally contain the vector fields and matter fields that make up a unitary gauge field theory.

Therefore, a fully consistent flux background generally also requires the presence of D-branes and O-planes. In model-building scenarios, in order not to break any space-time symmetry, these higher-dimensional objects extend uniformly along all of the four-dimensional space-time, and wrap a number cycles in the compact space \mathcal{Y}_6 . In the perturbative string theory, Dp -branes can be described as surfaces to which the endpoints of open strings are attached. The O-planes arise from the orientifold projection, which restricts the space of perturbative string to those states that remain invariant under a specific discrete symmetry of the theory. This symmetry combines orientation reversal of the two-dimensional world-sheet on which the string perturbation theory is defined, with a discrete symmetry of the target space in which the strings live. O-planes arise as the fixed points of the target space symmetry. At those fixed points, only modes which are invariant under the world-sheet orientation reversal survive the projection, leading to a truncation of the spectrum and a corresponding reduction of supersymmetry. At low energies, information about the localization of modes in the compact space \mathcal{Y}_6 is lost, and one finds that the low-energy effective action has a reduced amount of supersymmetry as well.

1.4 Structure of this thesis

In this thesis, which contains results from the publications [36, 20], we study orientifold projections of type IIA string theory compactified on manifolds with $SU(2)$ -structure. Due to the reduction of the structure group to $SU(2)$ there exist not one but two globally defined, nowhere vanishing spinors on the internal space \mathcal{Y}_6 . As a simple example, we will also consider the case where the torsion vanishes, or in other words, where also the holonomy reduces to $SU(2)$. This is the compactification on the Calabi-Yau manifold $K3 \times T^2$, which is the unique six-dimensional compact manifold with $SU(2)$ holonomy. In the absence of the orientifold projection, this leads to an effective low-energy theory which is an $\mathcal{N} = 4$ supergravity. Our goal is to perform an orientifold projection which halves the amount of supersymmetry, leading to an $\mathcal{N} = 2$ supergravity theory.

The starting point of this work are the results of [37], where the four-dimensional gauged $\mathcal{N} = 4$ supergravity theory corresponding to the effective action of type IIA string theory compactified on $SU(2)$ -structure backgrounds was computed. We then look for orientifold projections which preserve $\mathcal{N} = 2$ supersymmetry in four dimensions. There are two possible types of projection, one of which leads to seven-

dimensional, or O6, orientifold planes, whereas the other gives rise to O4 and O8 planes, which are five-, respectively nine-dimensional.

In both cases, the resulting theory after projection is a gauged $\mathcal{N} = 2$ supergravity. The scalar fields determine a σ -model with a target space of the form

$$\mathcal{M} = \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(2,n)}{\mathrm{SO}(2) \times \mathrm{SO}(n)} \times \frac{\mathrm{SO}(4,m)}{\mathrm{SO}(4) \times \mathrm{SO}(m)}, \quad (1.2)$$

which descends from the scalar field space

$$\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(6,n+m)}{\mathrm{SO}(6) \times \mathrm{SO}(n+m)} \quad (1.3)$$

of $\mathcal{N} = 4$ supergravity. The first two factors in (1.2) are spanned by the scalars in the vector multiplets and form a special Kähler manifold. The last factor is quaternion-Kähler and is spanned by the scalars in the hypermultiplets. Furthermore we find that isometries of all three components can be simultaneously gauged when appropriate torsion components are present. To our knowledge, this situation has not been encountered previously in any $\mathcal{N} = 2$ compactification of type II string theory.³

We then identify suitable variables which bring the action into the canonical form of an $\mathcal{N} = 2$ supergravity given in [41]. We calculate all the Killing prepotentials for the gauged symmetries, and verify that the theory obtained from compactification satisfies all constraints imposed by $\mathcal{N} = 2$ supergravity.

The thesis is organized as follows: chapter 2 gives a short review of the results obtained in [37, 19, 38, 42]. We describe the main properties of SU(2)-structure manifolds, and the derivation of the effective action of type IIA string theory on these backgrounds in section 2.2. We describe the essential steps leading to the results in a pedagogical way, explaining technical details only when they are needed for the subsequent chapters. The main input from this introductory chapter is the effective gauged $\mathcal{N} = 4$ supergravity theory obtained in [37], and we give the bosonic Lagrangian in 2.3.

The orientifold projections are discussed in chapter 3. In section 3.1 we first derive the action of the two different orientifold projections on SU(2)-structure manifolds. We then discuss how these affect the Kaluza-Klein procedure used to obtain the effective action from chapter 2. This allows us to implement the projection at the level of the effective action. This is carried through in detail for the O6 projection in section 3.2. We then check the consistency of the result with the various constraints imposed by local $\mathcal{N} = 2$ supersymmetry. The case of O4/O8 orientifold projections is treated in section 3.3. This calculation proceeds in a similar way, and therefore we do not repeat all of the arguments in the same level of detail, but focus on the results instead. We present our conclusions in chapter 4.

³It does occur in certain heterotic SU(2)-structure compactification [38] and can probably also be arranged in appropriate generalizations of M-theory compactifications on SU(3)-structure manifolds considered in [39, 40].

The appendices contain some further background information, as well as calculational details that we omitted from the main text. Appendix A contains details on the spinor conventions we used, and their relation to the orientifold projections from chapter 3. Appendix B contains the calculation of the Killing prepotentials of the effective $\mathcal{N} = 2$ supergravity theories obtained from compactification, and some calculations verifying the consistency of the couplings of the effective theories obtained in chapter 3 with the restrictions imposed by $\mathcal{N} = 2$ supergravity. Finally, appendix C contains some information on the various coset spaces that appear throughout the thesis. It provides an explicit picture of the effect of the projection on the scalar field sector, complementing chapter 3 which discusses this in terms of the special and quaternionic geometry of the $\mathcal{N} = 2$ moduli spaces.

Chapter 2

Compactification on SU(2)-structure Manifolds

In this introductory chapter, we recall some results on the compactification of type IIA supergravity theories on a background $\mathcal{M}_{1,3} \times \mathcal{Y}_6$, where \mathcal{Y}_6 is an SU(2)-structure manifold. We will be using results from [19], where the moduli space for such compactifications was determined, as well as [37, 38, 42, 20], where the effective action was computed. The main goal of this section is to obtain the four-dimensional low-energy effective action, given in equations (2.47)-(2.49), which is that of a specific $\mathcal{N} = 4$ supergravity theory. We also review some of the geometric properties of SU(2)-structure manifolds, which will play a role in the compactification.

2.1 Type IIA supergravity

Type IIA supergravity is an effective field theory which describes the low-energy dynamics of type IIA string theory in the perturbative regime. This means that the effects of massive string modes and string loop corrections are ignored. Its field content is thus the massless string spectrum, where the bosonic fields are given by the NS sector, which consists of the metric \hat{g} , the dilaton $\hat{\varphi}$ and the two-form \hat{B} , and the RR-sector, which consists of a one-form \hat{A} as well as a three-form \hat{C} . The bosonic part of the action is [2]

$$\begin{aligned} S_{\text{IIA}} = & \frac{1}{2} \int e^{-2\hat{\varphi}} \left(d^{10}x \sqrt{-\hat{g}} (\hat{R} + 4\partial_M \hat{\varphi} \partial^M \hat{\varphi}) + \frac{1}{2} d\hat{B} \wedge *d\hat{B} \right) \\ & + \frac{1}{4} \int \left(d\hat{A} \wedge *d\hat{A} + \tilde{F}_4 \wedge *\tilde{F}_4 \right) + \frac{1}{4} \int \hat{B} \wedge d\hat{C} \wedge d\hat{C}, \end{aligned} \tag{2.1}$$

where the field strength \tilde{F}_4 is given by

$$\tilde{F}_4 = d\hat{C} - \hat{A} \wedge d\hat{B}. \tag{2.2}$$

The action is invariant under p -form gauge transformations, with a modified gauge transformation for \hat{C} that keeps \tilde{F}_4 invariant. Together, the transformations take the following form

$$\begin{aligned}\delta\hat{A} &= d\lambda_0, \\ \delta\hat{B} &= d\lambda_1, \\ \delta\hat{C} &= d\lambda_2 + \lambda_0 d\hat{B},\end{aligned}\tag{2.3}$$

where the λ_p are p -form transformation parameters.

The full type IIA theory also contains fermionic fields, given by the two dilatinos and the two gravitinos. The complete action has a local $\mathcal{N} = 2$ supersymmetry relating the bosons and the fermions. The supersymmetry transformations are parametrized by two ten-dimensional Majorana-Weyl spinors of opposite chirality $\varepsilon^{\text{I}}, \varepsilon^{\text{II}}$ accounting for 32 supercharges in total.

In a dimensional reduction on a background $\mathcal{M}_{1,3} \times \mathcal{Y}_6$, the ten-dimensional supersymmetry transformations will descend to supersymmetries of the four-dimensional theory. We can decompose the parameters of the ten-dimensional supersymmetry transformations $\varepsilon_{10}^{\text{I,II}}$ with respect to the tensor product $\text{Spin}(1,3) \otimes \text{Spin}(6)$ as follows:

$$\begin{aligned}\varepsilon_{10}^{\text{I}} &= \varepsilon_{+i}^{\text{I}} \otimes \eta_+^i + \varepsilon_{-i}^{\text{I}} \otimes \eta_-^i, \\ \varepsilon_{10}^{\text{II}} &= \varepsilon_{+i}^{\text{II}} \otimes \eta_-^i - \varepsilon_{-i}^{\text{II}} \otimes \eta_+^i,\end{aligned}\tag{2.4}$$

where $\varepsilon_{+i}^{\text{I,II}}$ and η_+^i are four-dimensional and six-dimensional Weyl spinors. $\varepsilon_{-i}^{\text{I,II}}$ and η_-^i are the opposite chirality spinors, related to their positive chirality counterparts by complex conjugation (more details on our spinor conventions can be found in appendix A). Thus we obtain two four-dimensional supersymmetry transformations for each globally defined internal spinor η^i , provided the chosen background $\mathcal{M}_{1,3} \times \mathcal{Y}_6$ is invariant under the corresponding transformations $\varepsilon_{10}^{\text{I,II}}$.

In general, the construction of a concrete background which solves the equations of motion and satisfies the supersymmetry conditions requires the correct balancing of fluxes, as well as localized sources in the form of D-branes and O-planes. We will, however, not consider these factors explicitly. Instead, we assume that a suitable background has been chosen, and expand the action of the closed string sector on this background, setting fluxes to zero.

On an SU(2)-structure manifold, one can construct two independent global spinors. In other words, the index i in equation (2.4) takes the values $i = 1, 2$. The four-dimensional effective theory obtained from a compactification should therefore possess $\mathcal{N} = 4$ supersymmetry.

2.2 SU(2)-structures in 6 dimensions

By the isomorphism $\text{Spin}(6) \cong \text{SU}(4)$, spinors on \mathcal{Y}_6 transform in the fundamental representation $\mathbf{4}$ of $\text{SU}(4)$. The existence of two global sections of the spin bundle, the spinors η^i , then implies that a set of open neighborhoods on \mathcal{Y}_6 exists, such that the transition functions between these neighborhoods preserves two singlets. In other words, the transition functions do not take values in the full $\text{SU}(4)$ but we have instead the reduction

$$\text{SU}(4) \rightarrow \text{SU}(2), \quad \mathbf{4} \rightarrow \mathbf{2} \oplus \mathbf{1} \oplus \mathbf{1}. \quad (2.5)$$

Alternatively, an $\text{SU}(2)$ -structure on a 6-dimensional manifold can be characterized by the existence of a set of differential forms, subject to the appropriate constraints. Both pictures will be useful to us, since the orientifold projection is more easily discussed in terms of the spinors η^i , whereas the derivation of the effective action for the deformations of \mathcal{Y}_6 makes use of the differential forms.

Thus, an $\text{SU}(2)$ -structure on a six-dimensional manifold \mathcal{Y}_6 is defined by a real two-form J , a complex two-form Ω and a complex one-form K satisfying the constraints [14, 16, 17]

$$\begin{aligned} \Omega \wedge \bar{\Omega} &= 2J \wedge J \neq 0, & K \wedge \bar{K} &\neq 0, \\ \Omega \wedge J &= 0, & \Omega \wedge \Omega &= 0, \\ K^m \Omega_{mn} dY^n &= K^m \bar{\Omega}_{mn} dY^n = 0, & K^m J_{mn} dY^n &= 0. \end{aligned} \quad (2.6)$$

Here, $Y^m, m = 1, \dots, 6$ are coordinates on \mathcal{Y}_6 . The constraints (2.6) imply that the tangent bundle $T\mathcal{Y}_6$ splits into two orthogonal components: the two-dimensional component $T_2\mathcal{Y}_6$ spanned by the real and imaginary parts of K , and the remaining 4-dimensional component $T_4\mathcal{Y}_6$, on which Ω and J act.

The $\text{SU}(2)$ -structure defines a canonical metric on \mathcal{Y}_6 . It is determined by the following relations:

$$K_m \bar{K}^m = 2, \quad K_m K^m = 0, \quad (2.7)$$

which determines the metric on $T_2\mathcal{Y}_6$, and

$$\begin{aligned} *J &= \frac{i}{2} K \wedge \bar{K} \wedge J, \\ *\Omega &= \frac{i}{2} K \wedge \bar{K} \wedge \Omega, \end{aligned} \quad (2.8)$$

which is the statement that J and the real and imaginary parts of Ω span the Hodge-self-dual two-forms on $T_4\mathcal{Y}_6$. Finally,

$$\text{vol}_{\mathcal{Y}_6} = \text{vol}_2 \otimes \text{vol}_4 = \left(\frac{i}{2} K \wedge \bar{K}\right) \wedge \left(\frac{1}{2} J \wedge J\right), \quad (2.9)$$

which tells us that the volume form on \mathcal{Y}_6 splits into the product of the volume forms on the two components of the tangent space.

Note that one can define a triplet of two-forms $J^x, x = 1, \dots, 3$ as follows

$$J^x = (J, \text{Re}\Omega, \text{Im}\Omega), \quad x = 1, \dots, 3. \quad (2.10)$$

In terms of the J^x , the constraints on J and Ω can be written as

$$\begin{aligned} J^x \wedge J^y &= 2\delta^{xy} \text{vol}_4, \\ K^m J_{mn}^x &= 0, \\ *J^x &= \frac{i}{2} K \wedge \bar{K} \wedge J^x, \end{aligned} \quad (2.11)$$

which is manifestly invariant under an SO(3) rotation of the triplet J^x . More trivially, multiplication of K by a U(1) factor also leaves the SU(2)-structure invariant, since it leads to an SO(2) rotation of the orthogonal frame defined by the real and imaginary components of K .

The characterization of an SU(2)-structure manifold by the two spinors η^i and the characterization by the above set of differential forms, are equivalent. The differential forms defining an SU(2)-structure can be expressed in terms of the two spinors η^i as follows [14, 16, 17]

$$\begin{aligned} J &= \frac{i}{4} (\eta_-^{1\dagger} \gamma_{mn} \eta_-^1 - \eta_-^{2\dagger} \gamma_{mn} \eta_-^2) dY^m \wedge dY^n, \\ \Omega &= \frac{i}{2} \eta_-^{1\dagger} \gamma_{mn} \eta_-^2 dY^m \wedge dY^n, \\ K &= \eta_-^{2\dagger} \gamma_m \eta_+^1 dY^m = K^1 + iK^2, \end{aligned} \quad (2.12)$$

where the dagger symbol indicates Hermitian conjugation. Using Fierz identities, one can show that J, Ω and K as defined in (2.12) satisfy the equations (2.6)-(2.9). An SU(2) rotation of the doublet η^i leads to an SO(3) rotation of the triplet of two-forms J^x defined earlier, whereas a multiplication of both η^i by the same U(1) phase factor gives rise to a U(1) rotation of K .

We have introduced the defining properties of a general SU(2)-structure manifold. To gain some intuition, we return to the special case where $\mathcal{Y}_6 = \text{K3} \times T^2$. Coordinates on K3 are labeled $z^a, a = 1, \dots, 4$, and the coordinates on the torus T^2 are $y^i, i = 1, 2$. In this case we are dealing with a product manifold, and thus the global splitting of the tangent bundle $T\mathcal{Y}_6 = T_4\mathcal{Y}_6 \oplus T_2\mathcal{Y}_6$ is the familiar fact that the tangent bundle is the direct sum of the tangent bundles on K3 and T^2 . The real and imaginary parts of K are just the basis one-forms on the torus $K = dy^1 + idy^2$. Ω and J are the holomorphic two-form and Kähler form on K3. Indeed, the second line of (2.12) states that Ω is a (2, 0)-form with respect to the complex structure defined by J . In addition to the algebraic constraints (2.6), J and Ω satisfy the extra relation $dJ = 0 = d\Omega$, which can be seen as a consequence of the fact that the spinors η^i are parallel with respect to the Levi-Civita connection.

In the general SU(2)-structure case, \mathcal{Y}_6 is not a direct product, but the splitting of the tangent bundle is still globally defined. This is known as an almost-product

structure. Furthermore, \mathcal{Y}_6 is not necessarily Kähler, or even complex. In the presence of torsion, the two-forms J and Ω are no longer necessarily closed.

We will now briefly review the derivation of the moduli space of SU(2)-structure manifolds, following [19, 38]. We are interested in the effective four-dimensional action for perturbations of the metric on \mathcal{Y}_6 , which we formally write as

$$g_{\mathcal{Y}_6}(x, Y) = g(Y) + \sum_{n=1}^{\infty} \varphi_n(x) \delta g^n(Y), \quad (2.13)$$

where generic perturbations of the internal metric would require us to include an infinite sum of modes δg^n . However, we can truncate the generic action to an action for all φ_n up to a certain mass level, which amounts to truncating the expansion (2.13) to a corresponding finite set of modes δg^n . Such a truncation can be trusted as long as the separation between the different mass levels is large. The action for the fields $\varphi_n(x)$ is then obtained by substituting the ansatz (2.13) for g_{mn} into the ten-dimensional action and integrating over the six compact directions Y^m .

2.2.1 Metric moduli of $K3 \times T^2$

It is instructive to look at the case of a compactification on $K3 \times T^2$ in some detail. Here, the internal manifold is Ricci-flat, and there is a set of perturbations to the metric which leads to massless four-dimensional fields. As we shall illustrate, these perturbations correspond to the harmonic forms on $K3 \times T^2$. At this point, we are interested in the terms in the effective action obtained from the reduction of the ten-dimensional Ricci scalar, that describe the metric moduli space. Restricting ourselves to the components of the internal metric $g_{mn} dY^m dY^n$, the contribution from the ten-dimensional Einstein-Hilbert term to the four-dimensional Lagrangian is given by¹

$$e^{-1} \mathcal{L}_4 = -\frac{1}{8} \int_{\mathcal{Y}_6} \text{vol}_{\mathcal{Y}_6} g^{mq} g^{np} \partial_M g_{mn} \partial^M g_{pq}, \quad (2.14)$$

where the index $M = 0, \dots, 9$ labels derivatives with respect to all ten-dimensional coordinates.

From a four-dimensional perspective, derivatives ∂_μ with respect to the non-compact directions x^μ give rise to kinetic terms, whereas terms with derivatives along the compact directions ∂_m lead to potential terms in the four-dimensional action. The terms containing internal derivatives $(\partial_m g_{np})^2$ will vanish precisely for harmonic functions g_{np} .

At this point, we want to split the action into a component $g_{ab}(x, z)$ describing the metric on K3, with coordinates z^a , and a component $g_{ij}(x, y)$ describing the

¹We have set the dilaton $\hat{\varphi}$ to zero since we are only interested in the metric moduli for the moment. Furthermore, the reduction of the Ricci scalar gives rise to further terms that can be absorbed into the four-dimensional dilaton field.

metric on T^2 , with coordinates y^i . Therefore we assume that the perturbed metric remains block-diagonal. This is consistent as long as we are studying the massless perturbations to the metric: mixed terms g_{ai} can not appear at the massless level, since no harmonic 1-forms exist on K3 [43]. Similarly, since the only harmonic function on the compact spaces T^2 and K3 is the constant function, g_{ab} can not depend on the coordinates y^i , and g_{ij} can not depend on the z^a . Thus, the right hand side of (2.14) splits into two terms

$$\mathcal{L}_4 = \mathcal{L}_{T^2} + \mathcal{L}_{K3},$$

$$e^{-1}\mathcal{L}_{T^2} = -\frac{1}{8}e^{-\rho} \int_{T^2} \text{vol}_{T^2} g^{ik} g^{jl} \partial_M g_{ij} \partial^M g_{kl}, \quad (2.15a)$$

$$e^{-1}\mathcal{L}_{K3} = -\frac{1}{8}e^{-\eta} \int_{K3} \text{vol}_{K3} g^{ac} g^{bd} \partial_M g_{ab} \partial^M g_{cd}, \quad (2.15b)$$

where $e^{-\eta}$ represents the volume of the torus T^2 , and the factor $e^{-\rho}$ is the volume of K3.

The massless modes on T^2 , for which the internal derivatives $(\partial_i g_{kl})^2$ in (2.15a) vanish, are just the modes that do not depend on the T^2 coordinates y^i . Therefore the ansatz becomes $g_{ij}(x, y) = g_{ij}(x)$, and (2.15a) reduces to

$$e^{-1}\mathcal{L}_{T^2} = -\frac{1}{8}e^{-(\rho+\eta)} g^{ik} g^{jl} \partial_\mu g_{ij} \partial^\mu g_{kl}. \quad (2.16)$$

The moduli space associated to this action is the well-known moduli space of flat torus metrics

$$\mathcal{M}_{T^2} = \frac{\text{SI}(2, \mathbb{R})}{\text{SO}(2)} \times \mathbb{R}^+. \quad (2.17)$$

It is not immediately obvious what ansatz one should make for the components g_{ab} in (2.15b). It turns out that the metric on K3 is best described in terms of perturbations to the two-forms J and Ω described in (2.10). As it was shown explicitly in [38], (2.15b) can be rewritten as

$$-\frac{1}{8}e^{-\eta} \int_{K3} \text{vol}_{K3} g^{ac} g^{bd} \partial_M g_{ab} \partial^M g_{cd} = -\frac{1}{4}e^{-\eta} \int_{K3} \text{vol}_{K3} g^{ac} g^{bd} \partial_M J_{ab}^x \partial^M J_{cd}^x, \quad (2.18)$$

where the three two-forms J^x are now both x - and z -dependent. We now recall the fact that the Kähler form J and holomorphic 2-form Ω on K3 are harmonic forms. This will make the moduli corresponding to their variation massless. Indeed, terms with internal derivatives $(\partial_a J_{bc}^x)^2$ will vanish if the J^x are zero modes of the Laplacian on K3. Hodge theory tells us that the number of harmonic two-forms on K3 is equal to the dimension of the second cohomology $H^2(K3, \mathbb{R})$, which is 22.

$$J_{ab}^x(x, z) = e^{-\frac{1}{2}\rho(x)} \xi_\alpha^x(x) \omega_{ab}^\alpha(z), \quad \alpha = 1, \dots, 22, \quad (2.19)$$

where we have extracted the overall volume factor $e^{-\rho}$. In order to better understand the nature of the (3×22) matrix ξ_α^x that was just introduced, we need some information

on the space of harmonic forms on K3. Namely, the intersection form $\eta^{\alpha\beta}$ on K3, which is defined as

$$\eta^{\alpha\beta} = (\omega^\alpha, \omega^\beta) = \int_{\text{K3}} \omega^\alpha \wedge \omega^\beta, \quad (2.20)$$

defines a metric of signature (3, 19) on the space of harmonic two-forms.

With this information, one sees that the first constraint in (2.11) leads to the following 9 constraints on the matrix ξ_α^x

$$\eta^{\alpha\beta} \xi_\alpha^x \xi_\beta^y = 2\delta^{xy}. \quad (2.21)$$

These constraints imply that ξ_α^x is parametrized by 57 physical moduli fields. We will not choose an explicit parametrization, but instead keep the constraints (2.21) in mind. The 57 degrees of freedom of ξ_α^x represent the choice of a three-dimensional subspace of positive-normed vectors² in $H^2(\text{K3}, \mathbb{R}) \cong \mathbb{R}^{3,19}$. We can therefore identify the moduli space of Ricci-flat K3 metrics with the Grassmannian

$$\mathcal{M}_{\text{K3}} = \frac{\text{SO}(3, 19)}{\text{SO}(3) \times \text{SO}(19)} \times \mathbb{R}^+, \quad (2.22)$$

where the factor \mathbb{R}^+ represents the volume modulus ρ . A representative of the element in \mathcal{M}_{K3} determined by ξ_α^x is given by

$$H^\alpha_\beta = -\delta^\alpha_\beta + \xi^{x\alpha} \xi_\beta^x. \quad (2.23)$$

It follows from the fact that the J^x span the space of self-dual harmonic two-forms on K3, that the action for the Hodge *-operator on the ω^α is determined by H^α_β as follows:

$$* \omega^\alpha = H^\alpha_\beta \omega^\beta. \quad (2.24)$$

Using (2.20) and (2.21), the action (2.18) becomes

$$e^{-1} \mathcal{L}_{\text{K3}} = -\frac{1}{8} e^{-\rho-\eta} \partial_\mu \rho \partial^\mu \rho + \frac{1}{4} e^{-\rho-\eta} \partial_\mu \xi_\alpha^x \partial^\mu \xi_\beta^x (\eta^{\alpha\beta} - \frac{1}{2} \xi^{y\alpha} \xi^{y\beta}), \quad (2.25)$$

which can be written in terms of H^α_β as follows:

$$e^{-1} \mathcal{L}_{\text{K3}} = -\frac{1}{8} e^{-\rho-\eta} \partial_\mu \rho \partial^\mu \rho + \frac{1}{16} e^{-\rho-\eta} \partial_\mu H^\alpha_\beta \partial^\mu H^\beta_\alpha. \quad (2.26)$$

This concludes our condensed review of the derivation of the moduli space of metrics on $\text{K3} \times T^2$. We have chosen to go into some detail because the derivation of the result for a general SU(2)-structure manifold essentially follows the same steps. However, the general calculation involves a number of technicalities, mainly due to the fact that a general SU(2)-structure manifold is no longer a direct product. The previous discussion contains the essential ingredients of the derivation, and the following section will only discuss the necessary modifications to our ansatz, and the corresponding changes to the effective Lagrangian.

²We will call such spaces spacelike three-planes in the remainder of the text.

2.2.2 Metric moduli of general SU(2)-structure manifolds

The results obtained for $K3 \times T^2$ in the previous section have to be extended to a general class of SU(2)-structure manifolds. A metric on a general SU(2)-structure manifold is still specified by the set of differential forms K, J and Ω . Thus, we can parametrize the deformations of the metric by the deformations of these differential forms.

However, it turns out to be more difficult to consistently truncate the infinite space of metric deformations on \mathcal{Y}_6 to a finite set, reducing the action (2.14) to an action containing only a finite number of four-dimensional fields. As we have seen, on $K3 \times T^2$, the lightest perturbations are given by harmonic forms. These forms are well understood, and the topology of the internal space provides us with enough information to determine the effective action, even though the Calabi-Yau metric is not known explicitly. Assuming that the first non-zero eigenvalues of the Laplacian are of the same order as the inverse length scale of the manifold, the truncated modes can be made sufficiently massive when the compact space is small enough.

On a general SU(2)-structure manifold, the forms J, Ω and K are no longer required to be closed, as a consequence of the torsion of the internal manifold. Consequently, we can no longer expand them in a set of harmonic forms. Furthermore, the distinction between “light” and “heavy” modes coming from various perturbations to the metric is no longer as clear-cut, since all deformation to the metric can in principle lead to massive four-dimensional fields. Obtaining mass terms for the perturbations of the internal metric was one of the motivations of the endeavor in the first place, but the point is that one must take care that one is working with a consistent truncation.

As a consequence, there is currently no explicit characterization of the set of modes with respect to which one can expand the deformations of the internal metric on SU(2)-structure manifolds. To make progress, the current approach is to assume that there exists a consistent truncation to finite set of modes, determining the low-energy theory. One then makes a general ansatz and tries to characterize the differential forms in this ansatz via various consistency conditions [44–49]. Using these assumptions, the moduli space of SU(2)-structure manifolds was determined in [18, 37, 19, 38].

Imposing that no massive gravitinos are present in the low-energy theory, also requires that one removes from the spectrum those perturbations that change the splitting of the tangent bundle [19]

$$T\mathcal{Y}_6 = T_2\mathcal{Y}_6 \oplus T_4\mathcal{Y}_6. \quad (2.27)$$

This means that, in the low-energy limit, the two orthogonal subspaces $T_2\mathcal{Y}_6$ and $T_4\mathcal{Y}_6$ are fixed, and only the separate metrics within these two components are dynamical. This is the generalization of the statement that perturbations on $K3 \times T^2$ preserve the block-diagonal structure of the metric. However, due to the fact that \mathcal{Y}_6 is no longer a global product manifold, the action can no longer be written in the simple

form (2.15). Furthermore, it was shown that, to lowest energy, the set of forms that appear in our expansion is limited to the complex one-form K and a finite space of two-forms $\Lambda^{2,\text{finite}}\mathcal{Y}_6$, with a basis $\omega^\alpha, \alpha = 1, \dots, n$. The two-forms ω^α are now no longer required to be harmonic, and we leave their total number n as an unknown, since it would depend on the concrete background one is considering.

As was mentioned before, the external derivatives of the forms K, J and Ω no longer vanish for an SU(2)-structure with torsion. Therefore, we must allow that the differential forms used in our Kaluza-Klein expansion have non-zero exterior derivatives. Consistency then requires that exterior derivatives of these forms close among each other [44], and we can introduce constant matrices $\tilde{T}_{i\beta}^\alpha$ and constants t^i parametrizing the exterior derivatives as follows [18, 37, 38]:

$$\begin{aligned} d\omega^\alpha &= \tilde{T}_{i\beta}^\alpha K^i \wedge \omega^\beta, \\ dK^i &= t^i K^1 \wedge K^2. \end{aligned} \tag{2.28}$$

here, our treatment is not completely general, since we have not included possible derivatives $dK^i \equiv \theta_\alpha^i \omega^\alpha$.³

Using $d^2 = 0$ and Stokes' theorem, specifically the equation $\int dK^i \wedge \omega^\alpha \wedge \omega^\beta = 0$, one finds the following constraints on the torsion coefficients:

$$\tilde{T}_{i\gamma}^\alpha \tilde{T}_{j\beta}^\gamma - \tilde{T}_{j\gamma}^\alpha \tilde{T}_{i\beta}^\gamma = -\epsilon_{ij} t^k \tilde{T}_{k\beta}^\alpha, \tag{2.29a}$$

$$\eta^{\alpha\gamma} \tilde{T}_{i\gamma}^\beta + \tilde{T}_{i\gamma}^\alpha \eta^{\gamma\beta} = \eta^{\alpha\beta} \epsilon_{ij} t^j, \tag{2.29b}$$

where the matrix $\eta^{\alpha\beta}$ is now defined as

$$\eta^{\alpha\beta} = \int_{\mathcal{Y}_6} K^1 \wedge K^2 \wedge \omega^\alpha \wedge \omega^\beta. \tag{2.30}$$

It was shown in [19] that in the general case, $\eta^{\alpha\beta}$ still defines a metric of signature $(3, n-3)$.

Instead of using the matrices \tilde{T} , we choose a new basis of traceless matrices $T_{i\beta}^\alpha$, in terms of which the constraint (2.29b) takes a simpler form:

$$T_{i\beta}^\alpha := \tilde{T}_{i\beta}^\alpha - \frac{1}{2} \epsilon_{ij} t^j \delta_{\beta}^{\alpha}, \tag{2.31}$$

Since the term we are subtracting is proportional to the identity matrix, this does not change the commutation property (2.29a). However, the relation (2.29b) is modified. Equations (2.29) therefore become

$$T_{i\gamma}^\alpha T_{j\beta}^\gamma - T_{j\gamma}^\alpha T_{i\beta}^\gamma = -\epsilon_{ij} t^k T_{k\beta}^\alpha, \tag{2.32a}$$

$$\eta^{\alpha\gamma} T_{i\gamma}^\beta + T_{i\gamma}^\alpha \eta^{\gamma\beta} = 0, \tag{2.32b}$$

³So far, it has not been possible to calculate the effective action in the most general case. Instead, we assume that the almost product structure defined by the K^i is integrable, which amounts to the fact that, in every local neighborhood of \mathcal{Y}_6 , coordinates (y^i, z^a) exist such that $T_2\mathcal{Y}_6$ is spanned by the directions $\partial/\partial y^i$ and $T_4\mathcal{Y}_6$ is spanned by $\partial/\partial z^a$ [50].

and the exterior derivative of the two-forms ω^α becomes

$$d\omega^\alpha = T_{i\beta}^\alpha K^i \wedge \omega^\beta + \frac{1}{2} K^i \epsilon_{ij} t^j \delta^\alpha_\beta \omega^\beta. \quad (2.33)$$

Now that we have discussed how our Kaluza-Klein ansatz for the metric on $K3 \times T^2$ can be adapted to the case of general SU(2)-structure manifolds \mathcal{Y}_6 , we can discuss the resulting moduli space and effective action.

Since the space $T_2\mathcal{Y}_6$ is fixed, and spanned by the real and imaginary components K^i of the one-form K , x^μ -dependent perturbations to the metric on $T_2\mathcal{Y}_6$ can be written as [19]

$$g_{mn}(x, Y) = g_{ij}(x) K_m^i K_n^j, \quad (2.34)$$

where we note that the components g_{ij} are no longer the components of the metric with respect to a coordinate basis. Nevertheless, the g_{ij} have the same degrees of freedom as a 2-dimensional constant metric, and thus describe the same moduli space as in the previous section

$$\mathcal{M}_2 = \frac{\text{SI}(2, \mathbb{R})}{\text{SO}(2)} \times \mathbb{R}^+, \quad (2.35)$$

though the fact that the K^i now depend on the internal coordinates gives rise to modifications of the effective action.

Variations of the metric on $T_4\mathcal{Y}_6$ are again determined by the variations of the two-forms J^x . As before, the J^x are expanded with respect to the forms ω^α

$$J^x = e^{-\frac{1}{2}\rho} \xi_\alpha^x \omega^\alpha, \quad (2.36)$$

where ξ_α^x and the volume ρ are x^μ -dependent. The ξ_α^i now determine a spacelike three-plane in the vector space spanned by the two-forms ω^α . We can identify this vector space with $\mathbb{R}^{3, n-3}$, since it is equipped with a metric via $\eta^{\alpha\beta}$, which has signature $(3, n-3)$. Therefore, the moduli space remains of the same form as in the case of $K3 \times T^2$

$$\mathcal{M}_4 = \frac{\text{SO}(3, n-3)}{\text{SO}(3) \times \text{SO}(n-3)} \times \mathbb{R}^+, \quad (2.37)$$

which has dimension $3(n-3)$.

We conclude that the space of light moduli of a general SU(2)-structure manifold has a similar form as the moduli space of $K3 \times T^2$. Bigger differences arise at the level of the effective action, however. Due to the non-closedness of the forms K^i and ω^α , terms with derivatives $\partial_i g_{mn}$ of the metric with respect to the internal coordinates are no longer required to vanish. Therefore, extra terms arise in the dimensional reduction of the Ricci scalar (2.14), which in turn leads to extra terms in the effective action besides the kinetic terms (2.16) and (2.26). The computation of the action for this general case was carried through in [38], where it was found that the terms containing internal derivatives of the metric assemble into the potential

$$\mathcal{V} = \frac{5}{8} e^{-\rho-\eta} g_{ij} t^i t^j + \frac{1}{16} g^{ij} [M, T_i]^\alpha_\beta [M, T_j]^\beta_\alpha. \quad (2.38)$$

This analysis did not take into account the remaining components of the metric, as well as the other fields in the spectrum of type IIA string theory. Upon including also these remaining degrees of freedom, one finds that the torsion coefficients give rise to further changes in the effective action. In particular, the derivatives in the kinetic terms (2.16) and (2.26) become covariant derivatives, reflecting the fact that the scalar fields are now charged with respect to deformations of the internal manifold. These issues are the subject of the next section.

2.3 Reduction of the type IIA action

We gave a fairly detailed account of the derivation of the effective action for the low-energy dynamics of the metric on \mathcal{Y}_6 and the structure of the resulting moduli space, since many of those details will be useful to us when we impose the orientifold projection on these fields in chapter 3. During this discussion, we focused our attention on the internal metric g_{mn} . We now return to the complete action and the complete spectrum of type IIA supergravity (2.1). Thus, we make a Kaluza-Klein ansatz for the dependence on the internal coordinates Y^m of the remaining fields, by expanding them in the same set of differential forms K^i, ω^α we used in our expansion of the metric g_{mn} .

The ansatz for the complete ten-dimensional metric \hat{g}_{MN} , including components on the non-compact directions x^μ then reads [37]

$$\hat{g}_{MN} dx^M dx^N = g_{\mu\nu} dx^\mu dx^\nu + g_{ij} \mathcal{E}^i \mathcal{E}^j + g_{ab} [\rho; \xi_\alpha^x \omega^\alpha] dz^a dz^b, \quad (2.39)$$

where, instead of working with the K^i directly, we used an expansion with respect to the one-forms

$$\mathcal{E}^i = K^i - G_\mu^i dx^\mu, \quad (2.40)$$

The $G_\mu^i = -g^{ij} g_{j\mu}$ depend on the off-diagonal components of the metric. This expansion is convenient because \mathcal{E}^i is invariant under the symmetry transformation

$$\begin{aligned} K^i &\rightarrow K^i + \partial_\mu \lambda^i(x) dx^\mu, \\ G_\mu^i &\rightarrow G_\mu^i + \partial_\mu \lambda^i(x), \end{aligned} \quad (2.41)$$

which is present in our four-dimensional effective theory as a remnant of the ten-dimensional diffeomorphism symmetry of the original theory. Expanding with respect to the \mathcal{E}^i then guarantees that the thus defined four-dimensional fields have canonical transformation properties with respect to the diffeomorphism symmetry (2.41).

The expansion (2.39) contains the four-dimensional metric $g_{\mu\nu}$, as well as off-diagonal components G_μ^i . The components G_μ^i will appear as four-dimensional vector fields in the effective action. We ignored these fields in the discussion of sections 2.2.1 and 2.2.2, and their presence will give rise to additional terms in the effective action. There are no components $g_{\mu a} dx^\mu dz^a$, for the same reasons that cause the

components $g_{ia}K^iz^a$ to vanish, discussed in the previous sections. The components of the metric on $T_4\mathcal{Y}_6$, finally, are labeled by $g_{ab}[\rho; \xi_\alpha^x\omega^\alpha]$, to indicate their dependence on the overall volume $\rho(x)$, the $(3n-9)$ moduli encoded in the matrix $\xi_\alpha^x(x)$, and the basis of two-forms ω^α , as was also discussed in the previous sections.

The ten-dimensional dilaton field $\hat{\varphi}(x^M)$ just reduces to a four-dimensional scalar field $\varphi(x^\mu)$. The Neveu-Schwarz two-form field \hat{B} is decomposed as follows:

$$\begin{aligned} \frac{1}{2}\hat{B}_{MN}dx^M \wedge dx^N &= \frac{1}{2}B_{\mu\nu}dx^\mu \wedge dx^\nu \\ &+ B_{i\mu}dx^\mu \wedge \mathcal{E}^i \\ &+ b_{12}\mathcal{E}^1 \wedge \mathcal{E}^2 + b_\alpha\omega^\alpha, \end{aligned} \quad (2.42)$$

whereas the Ramond-Ramond one- and three-form \hat{A} and \hat{C} have the expansion

$$\begin{aligned} \hat{A}_M dx^M &= A_\mu dx^\mu + a_i \mathcal{E}^i, \\ \frac{1}{3!}\hat{C}_{MNP}dx^M \wedge dx^N \wedge dx^P &= \left(\frac{1}{3!}C_{\mu\nu\rho} - \frac{1}{2}A_\mu B_{\nu\rho}\right)dx^\mu \wedge dx^\nu \wedge dx^\rho \\ &+ \left(\frac{1}{2}C_{i\mu\nu} - A_\mu B_{i\nu}\right)dx^\mu \wedge dx^\nu \wedge \mathcal{E}^i \\ &+ (C_{12\mu} - A_\mu b_{12})dx^\mu \wedge \mathcal{E}^1 \wedge \mathcal{E}^2 \\ &+ (C_{\alpha\mu} - A_\mu b_\alpha)dx^\mu \wedge \omega^\alpha \\ &+ c_{i\alpha}\mathcal{E}^i \wedge \omega^\alpha. \end{aligned} \quad (2.44)$$

The subtractions in the first four lines of (2.44) assure that the field strengths of the four-dimensional form fields dC, dC_i, dC_{12} and dC_α remain invariant under the ten-dimensional p -form gauge transformations (2.3).⁴ For clarity, we have explicitly written the components of the four-dimensional fields with respect to the four-dimensional differentials dx^μ . In the remainder of this thesis, we will mostly suppress four-dimensional indices μ , writing $C_{\mu\nu\rho}dx^\mu \wedge dx^\nu \wedge dx^\rho \equiv C$, $B_{i\mu}dx^\mu \equiv B_i$, and so on.

With these preparations, we can give the full effective action obtained by substituting the Kaluza-Klein expansions for the fields above into the action (2.1), and integrating over the compact directions. After the formulation of the right Kaluza-Klein ansatz, the calculation of the effective action is a rather tedious procedure. Therefore, we do not go into further detail here, and instead refer to [38] for the derivation of the action for the metric moduli, and to [37, 20] for details on the reduction of the action for the RR fields. We choose to work with a bosonic spectrum consisting only of scalar and vector fields, and therefore higher p -forms are eliminated from the action.

⁴Of course, as in the case of the ten-dimensional diffeomorphism symmetry, only part of the ten-dimensional p -form gauge freedom (2.3) remains in our effective theory. The remaining symmetry transformations are obtained by expanding the p -form transformation parameters from (2.3) with respect to the differential forms K^i and ω^α in the same manner as the p -form fields.

	$j = 2$	$j = 1$	$j = 0$
\hat{g}	$g_{\mu\nu}$	G_{μ}^i	g_{ij} H^{α}_{β} ρ
$\hat{\varphi}$			φ
\hat{B}		$B_{i\mu}$	$B_{\mu\nu}/\beta$ b_{12} b_{α}
\hat{A}		A_{μ}	a_i
\hat{C}		$C_{12\mu}/\tilde{C}$ $C_{\alpha\mu}$	$C_{i\mu\nu}/\gamma_i$ $c_{i\alpha}$

Table 2.1: The full spectrum of the $\mathcal{N} = 4$ supergravity theory obtained from compactification on an $SU(2)$ -structure manifold. The first column indicates the field in the original ten-dimensional theory, the other columns list the fields in the effective four-dimensional theory, grouped by helicity. For fields that are dualized, we indicate the original field and its dual separated by a $/$.

A three-form field in four dimensions has no degrees of freedom, therefore C was integrated out. Then, the two-form fields B and C_i were replaced by their Hodge dual scalar fields β and γ_i . Due to the specific couplings of C_i and C_{12} , it is necessary to replace C_{12} by a dual vector field \tilde{C} as well. Schematically, we can summarize the relationship between the various fields as follows:

$$\begin{aligned}
*dC_i &\sim g_{ij}\epsilon^{jk}d\gamma_k, \\
*dB &\sim d\beta, \\
*dC_{12} &\sim d\tilde{C}.
\end{aligned}
\tag{2.45}$$

In order to make the distinction between the various types of fields easier, we denote scalar fields using lower case roman or greek letters, whereas capital roman letters denote vector fields. The only exception is the matrix H^{α}_{β} , which consists of scalar fields. An overview of all the fields in the effective theory is given in table 2.1. In total, the spectrum consists of the graviton, $(6+n)$ vector fields, and $(2+6n)$ scalar fields, which corresponds to the bosonic content of one $\mathcal{N} = 4$ gravity multiplet and n vector multiplets.

In terms of these fields, the bosonic part of the action can be written as

$$S_{\mathcal{N}=4} = S_{\text{scalar}} + S_{\text{vector}} + S_{\text{potential}}, \tag{2.46}$$

where the scalar kinetic term is

$$\begin{aligned}
S_{\text{scalar}} = & \int \frac{1}{2} \left[- *R + 2d(\varphi + \frac{1}{2}\eta + \frac{1}{2}\rho) \wedge *d(\varphi + \frac{1}{2}\eta + \frac{1}{2}\rho) \right. \\
& - \frac{1}{8}DH^\alpha_\beta \wedge *DH^\beta_\alpha + \frac{1}{2}e^{2\varphi} g^{ij} Da_i \wedge *Da_j \\
& + \frac{1}{2}e^\rho H_{\alpha\beta} Db^\alpha \wedge *Db^\beta + \frac{1}{2}e^{2\eta} Db_{12} \wedge *Db_{12} \\
& \left. + \frac{1}{4}(g^{ik}g^{jl}Dg_{ij} \wedge *Dg_{kl} + D\rho \wedge *D\rho) \right] \\
& + \frac{1}{4}e^{4\varphi+2\rho+2\eta} \left[D\beta - \epsilon^{ij}a_i D\gamma_j \right. \\
& \left. + \epsilon^{ij}\frac{1}{2}(c_i^\alpha Dc_{j\alpha} + a_i b^\alpha Dc_{j\alpha} + c_i^\alpha Da_j b_\alpha + c_i^\alpha a_j Db_\alpha) \right] \\
& \wedge * \left[D\beta - \epsilon^{kl}a_k D\gamma_l \right. \\
& \left. + \epsilon^{kl}\frac{1}{2}(c_k^\beta Dc_{l\beta} + a_k b^\beta Dc_{l\beta} + c_k^\beta Da_l b_\beta + c_k^\beta a_l Db_\beta) \right] \\
& + \frac{1}{4}e^{2\varphi+2\rho} g^{ij} (D\gamma_i - b^\alpha Dc_{i\alpha}) \wedge *(D\gamma_j - b^\beta Dc_{j\beta}) \\
& + \frac{1}{4}e^{2\varphi+\rho} H_{\alpha\beta} g^{ij} (Dc_i^\alpha + a_i Db^\alpha) \wedge *(Dc_j^\beta + a_j Db^\beta),
\end{aligned} \tag{2.47}$$

the kinetic term and topological terms for the vector fields are

$$\begin{aligned}
S_{\text{vector}} = & \frac{1}{4} \int e^{-(2\varphi+\eta+\rho)} \left[g_{ij} \mathcal{D}G^i \wedge *\mathcal{D}G^j \right. \\
& \left. + g^{ij} (\mathcal{D}B_i + \mathcal{D}G^k b_{ik}) \wedge *(\mathcal{D}B_j + \mathcal{D}G^l b_{jl}) \right] \\
& + e^{\rho-\eta} (\mathcal{D}\tilde{C} - b_\alpha \mathcal{D}C^\alpha + \frac{1}{2}b_\alpha b^\alpha \mathcal{D}A - (\gamma_k - b^\alpha c_{k\alpha}) \mathcal{D}G^k) \\
& \wedge *(\mathcal{D}\tilde{C} - b_\beta \mathcal{D}C^\beta + \frac{1}{2}b_\beta b^\beta \mathcal{D}A - (\gamma_l - b^\beta c_{l\beta}) \mathcal{D}G^l) \\
& + e^{-(\rho+\eta)} (\mathcal{D}A - \mathcal{D}G^k a_k) \wedge *(\mathcal{D}A - \mathcal{D}G^l a_l) \\
& + e^{-\eta} H_{\alpha\beta} (\mathcal{D}C^\alpha - \mathcal{D}A b^\beta - \mathcal{D}G^k c_k^\alpha) \\
& \wedge *(\mathcal{D}C^\beta - \mathcal{D}A b^\beta - \mathcal{D}G^l c_l^\beta) \\
& - \frac{1}{4} \int b_{12} \eta_{\alpha\beta} \mathcal{D}C^\alpha \wedge \mathcal{D}C^\beta - 2b_{12} \mathcal{D}A \wedge \mathcal{D}\tilde{C} \\
& + \frac{1}{2} \int \epsilon^{ij} (\mathcal{D}B_i + \mathcal{D}G^k b_{ik}) \\
& \wedge \left[(c_{j\alpha} + a_j b_\alpha) \mathcal{D}C^\alpha - a_j \mathcal{D}\tilde{C} - (\gamma_j + \frac{1}{2}a_j b_\alpha b^\alpha) \mathcal{D}A \right. \\
& \left. + (\epsilon_{jk} \beta + a_j \gamma_k - \frac{1}{2}c_j^\alpha c_{k\alpha} - \frac{1}{2}a_j b^\alpha c_{k\alpha} - \frac{1}{2}a_k b^\alpha c_{j\alpha}) \mathcal{D}G^k \right] \\
& + \frac{1}{2} (\epsilon^{ij} \mathcal{D}B_i \wedge C_\alpha \wedge T_{j\beta}^\alpha C^\beta + 2B_i t^i \wedge \mathcal{D}A \wedge \tilde{C} - B_i t^i \wedge \mathcal{D}C^\alpha \wedge C_\alpha),
\end{aligned} \tag{2.48}$$

and the potential is given by

$$\begin{aligned}
S_{\text{potential}} = & \int e^{2\varphi+\eta+\rho} * \left[\frac{5}{8} e^{2\eta} g_{ij} t^i t^j - \frac{1}{16} g^{ij} [H, T_i]^\alpha{}_\beta [H, T_j]^\beta{}_\alpha \right. \\
& \left. + \frac{1}{4} g^{ij} e^\rho H_{\alpha\beta} (T_{i\gamma}^\alpha - \frac{1}{2} \epsilon_{ik} t^k \delta_\gamma^\alpha) b^\gamma (T_{j\delta}^\beta - \frac{1}{2} \epsilon_{jl} t^l \delta_\delta^\beta) b^\delta \right] \\
& + \frac{1}{4} e^{4\varphi+3\eta+3\rho} * \left(b^\alpha (T_{1\alpha}^\beta c_{2\beta} - T_{2\alpha}^\beta c_{1\beta} + \frac{1}{2} t^i c_{i\alpha}) \right)^2 + * \frac{1}{4} e^{4\varphi+3\eta+\rho} t^i a_i t^j a_j \\
& + * \frac{1}{4} e^{4\varphi+3\eta+2\rho} H_{\alpha\beta} (\epsilon^{ij} T_{i\gamma}^\alpha (c_j{}^\gamma + a_j b^\gamma) - \frac{1}{2} t^k c_k{}^\alpha + \frac{1}{2} t^k a_k b^\alpha) \\
& \cdot (\epsilon^{kl} T_{k\delta}^\beta (c_l{}^\delta + a_l b^\delta) - \frac{1}{2} t^k c_k{}^\beta + \frac{1}{2} t^l a_l b^\beta).
\end{aligned} \tag{2.49}$$

Almost all kinetic terms in the action (2.47),(2.48) contain covariant derivatives. We use the notation Ds for the covariant derivative of a scalar field s , whereas the covariant derivative $\mathcal{D}V$ denotes the non-Abelian field strength of the vector V . The covariant derivatives of the various scalar fields are given by

$$\begin{aligned}
Dg_{ij} &= dg_{ij} + G^l (\epsilon_{il} t^k g_{kj} + \epsilon_{jl} t^k g_{ik}), \\
D\varphi &= d\varphi, \\
D\eta &= d\eta + G^k \epsilon_{kl} t^l, \\
D\rho &= d\rho - G^k \epsilon_{kl} t^l, \\
D\gamma_i &= d\gamma_i - \tilde{C} \epsilon_{ij} t^j + G^k \epsilon_{ij} t^j \gamma_k, \\
D\beta &= d\beta + \frac{1}{2} C_\alpha (\epsilon^{ij} T_{i\beta}^\alpha c_j{}^\beta - \frac{1}{2} t^i c_i{}^\alpha), \\
Da_i &= da_i - G^j \epsilon_{ji} t^k a_k, \\
Db_{12} &= db_{12} - B_j t^j - G^j \epsilon_{jk} t^k b_{12}, \\
Db_\alpha &= db_\alpha + G^j (T_{j\alpha}^\beta + \frac{1}{2} \epsilon_{jk} t^k \delta_\alpha^\beta) b_\beta, \\
Dc_{i\alpha} &= dc_{i\alpha} + G^j (T_{j\alpha}^\beta + \frac{1}{2} \epsilon_{jk} t^k \delta_\alpha^\beta) c_{i\beta} - G^j \epsilon_{ji} t^k c_{k\alpha} \\
&\quad - C_\beta (T_{i\alpha}^\beta + \frac{1}{2} \epsilon_{ij} t^j \delta_\alpha^\beta), \\
DH^\alpha{}_\beta &= dH^\alpha{}_\beta - G^j (T_{j\gamma}^\alpha H^\gamma{}_\beta - H^\alpha{}_\gamma T_{j\beta}^\gamma), \\
D\xi^{x\alpha} &= d\xi^{x\alpha} - G^i T_{i\beta}^\alpha \xi^{x\beta}.
\end{aligned} \tag{2.50}$$

We note that all scalar fields except the dilaton φ are charged, and that all vector fields except A take part in the gauging. The last line of (2.50) indicates the covariant derivatives of the $\xi^{x\alpha}$ which are implicit in the $H^\alpha{}_\beta$ via (2.23). The corresponding

vector field strengths are

$$\begin{aligned}
\mathcal{D}G^i &= dG^i + t^i G^1 \wedge G^2, \\
\mathcal{D}B_i &= dB_i + \epsilon_{ij} t^k G^j \wedge B_k, \\
\mathcal{D}A &= dA, \\
\mathcal{D}\tilde{C} &= d\tilde{C} + \epsilon_{jk} t^k G^j \wedge \tilde{C}, \\
\mathcal{D}C^\alpha &= dC^\alpha - T_{j\beta}^\alpha G^j \wedge C^\beta + \frac{1}{2} \epsilon_{jk} t^k G^j \wedge C^\alpha.
\end{aligned} \tag{2.51}$$

The charges of the fields all depend on the torsion parameters $T_{i\beta}^\alpha$ and t^i . The action for compactification on $K3 \times T^2$ can be obtained by setting all the torsion parameters to zero, and the number of two-forms ω^α to $n = 22$, the second Betti number of K3. In this case, all covariant derivatives D and \mathcal{D} become simple exterior derivatives d , and the potential term (2.49) vanishes.

It was shown in [37] that the action (2.47)-(2.49) describes the bosonic fields of a gauged $\mathcal{N} = 4$ supergravity theory, which puts a strong constraint on the theory. In particular, the scalar field space of an $\mathcal{N} = 4$ theory is required to be of the form

$$\mathcal{M}_{\text{scalar}} = \frac{\text{SU}(1, 1)}{\text{U}(1)} \times \frac{\text{SO}(6, n)}{\text{SO}(6) \times \text{SO}(n)}. \tag{2.52}$$

For the case at hand, the first factor is described by the complex field

$$\tau = b_{12} + i e^{-\eta}, \tag{2.53}$$

and the second factor is a coset space similar to the space of metric moduli (2.37) (the moduli space (2.37) is now, of course, a subspace of (2.52)). As we show explicitly in section C.1 of the appendix, the $6n$ remaining scalar fields can be used to parametrize a matrix $V^{I\bar{a}}, I = 1, \dots, n+6, \bar{a} = 1, \dots, 6$ subject to the constraints

$$V^{\bar{a}I} \eta_{IJ} V^{\bar{b}J} = \delta^{\bar{a}\bar{b}}, \tag{2.54}$$

for a constant metric η_{IJ} of signature $(6, n)$. In other words, the columns $V^{\bar{a}}$ may be thought of as (pseudo-)orthonormal vectors in an internal vector space $\mathbb{R}^{6,n}$. Together, they determine a 6-dimensional subspace of spacelike vectors in $\mathbb{R}^{6,n}$. Similarly to the kinetic term (2.26) for the metric moduli on K3, the total kinetic term can now be written in terms of a matrix

$$M^I{}_J = -\delta^I{}_J + V^{\bar{a}I} V^{\bar{a}}{}_J. \tag{2.55}$$

Indeed, in terms of τ and $M^I{}_J$, the scalar action (2.47) takes the simple form

$$S_{\text{scalar}} = \int \frac{1}{4 \text{Im}(\tau)^2} D\tau \wedge *D\bar{\tau} - \frac{1}{16} DM^I{}_J \wedge *DM^J{}_I, \tag{2.56}$$

for appropriately defined covariant derivatives. We refer to appendix C for more details on such coset Lagrangians.

We have now reviewed the necessary background information in order to be able to construct an orientifold projection on the SU(2)-structure background, and give the effective action.

Chapter 3

The Orientifold Projection

In the previous chapter we have introduced the effective theory of type IIA string theory compactified on a six-dimensional background manifold \mathcal{Y}_6 with $SU(2)$ -structure. The result is a gauged $\mathcal{N} = 4$ supergravity theory. The aim of this chapter is to find the \mathbb{Z}_2 orientifold projections compatible with this background, and to derive the resulting $\mathcal{N} = 2$ supergravity theory. The implementation of the $O6$ orientifold projection and its results, mainly discussed in section 3.2, are the subject of [36].

3.1 Orientifold Action

The orientifold procedure consists of truncating the spectrum of the theory to those states invariant under the action of a discrete symmetry \mathcal{O} . The discrete symmetry from which the orientifold projection receives its name, is orientation reversal Ω_p of the two-dimensional string world sheet. Labeling the coordinates on the world sheet (σ, τ) , Ω_p acts as

$$\Omega_p(\sigma, \tau) = (\sigma, 2\pi - \tau). \quad (3.1)$$

The complete orientifold action further contains a spacetime involution S , and a possible factor $(-1)^{F_L}$, where F_L is the number of left-moving fermionic degrees of freedom. Depending on the action of S , it can be necessary to include the operator $(-1)^{F_L}$ to assure that \mathcal{O} squares to unity on fermionic states. The transformation Ω_p interchanges left-moving and right-moving modes on the world-sheet. In type IIA string theory, the left- and right-moving fermionic fields have opposite spacetime chirality. Thus, a consistent orientifold transformation \mathcal{O} must contain a spacetime involution S which relates fermions of different chirality, and therefore also inverts the spacetime orientation. The background vacuum should be invariant under the orientifold map \mathcal{O} , and therefore S should be an isometry of the background metric. This translates into the requirement that S conserves the space spanned by the spinors η^i that define the $SU(2)$ -structure (2.12). As we explain in detail in appendix A, this leaves us with essentially two different types of involution S , and corresponding orientifold projections:

- Orientifolds with $O6$ -planes: S is an involution with $(1+6)$ -dimensional fixed-point loci, and acts on the spinors η^i as follows:

$$S(\eta_{\pm}^i) = \pm \eta_{\mp}^i. \quad (3.2)$$

This action of S squares to minus the identity, and therefore the factor of $(-1)^{F_L}$ must be added. The orientifold action takes the form

$$\mathcal{O}_{O6} = S\Omega_p(-1)^{F_L}. \quad (3.3)$$

- $O5$ - and $O8$ -planes: The involution S has $(1+5)$ - or $(1+7)$ -dimensional fixed-point loci. The action of S is given by

$$\begin{aligned} S(\eta_{\pm}^1) &= \pm \eta_{\mp}^2, \\ S(\eta_{\pm}^2) &= \mp \eta_{\mp}^1. \end{aligned} \quad (3.4)$$

Now the action of S squares to the identity, and the complete orientifold action is

$$\mathcal{O}_{O4/O8} = S\Omega_p. \quad (3.5)$$

As we shall see, characterization of S by its action on the internal spinors η^i gives us the information we need in order to determine the action of S on all of the fields in our theory. We will call a field F “even”, respectively “odd”, if it transforms as $S(F) = \pm F$ under the action of S . However, in order to determine the complete orientifold action \mathcal{O} , we still need to know the effect of the operators Ω_p and $(-1)^{F_L}$. Using the world-sheet description of the various type IIA fields, one finds [32, 33]¹

$$\Omega_p S : \begin{cases} \hat{\varphi} \rightarrow S(\hat{\varphi}) \\ \hat{g} \rightarrow S(\hat{g}) \\ \hat{B} \rightarrow -S(\hat{B}) \\ \hat{A} \rightarrow S(\hat{A}) \\ \hat{C} \rightarrow -S(\hat{C}) \end{cases}, \quad (-1)^{F_L} : \begin{cases} \hat{\varphi} \rightarrow \hat{\varphi} \\ \hat{g} \rightarrow \hat{g} \\ \hat{B} \rightarrow \hat{B} \\ \hat{A} \rightarrow -\hat{A} \\ \hat{C} \rightarrow -\hat{C} \end{cases}. \quad (3.6)$$

In other words, Ω_p and $(-1)^{F_L}$ are extra internal symmetries of the effective field theory, multiplying fields by a factor of (± 1) .

The orientifold projection now consists of truncating all modes which are not invariant under the total action of \mathcal{O} . In the NS sector, \mathcal{O}_{O6} and $\mathcal{O}_{O4/O8}$ have a similar effect: the dilaton $\hat{\varphi}$ and metric \hat{g} must be even

$$\begin{aligned} S(\hat{\varphi}) &= \hat{\varphi}, \\ S(\hat{g}) &= \hat{g}, \end{aligned} \quad (3.7)$$

¹We use the shorthand $S(F)$ to denote the action of S on a p -form field $F_{M\dots P}dx^M \wedge \dots \wedge dx^P$ by its pullback, i.e. $S(F) = F(S(x))_{M\dots P} \frac{\partial S^M}{\partial x^Q} \dots \frac{\partial S^P}{\partial x^R} dx^M \wedge \dots \wedge dx^Q$, for a transformation $S : x^M \rightarrow S^M(x)$. Equation (3.6) gives the combined action of $\Omega_p S$, since an action of Ω_p alone can not be given meaning in type IIA theory, due to the different chiralities of the left- and right-moving sector.

whereas for the two-form \hat{B} , only odd modes survive:

$$S(\hat{B}) = -\hat{B}. \quad (3.8)$$

For the RR fields, the situation is different in each case, since the factor $(-1)^{F_L}$ is not present in $\mathcal{O}_{O4/O8}$. Thus in the $O6$ projection, combining the two columns of (3.6), one finds that the one-form \hat{A} must transform with eigenvalue (-1) , and the three-form \hat{C} must be invariant. The opposite holds in the case of an $O4/O8$ projection:

$$\begin{array}{cc} \underline{O6} & \underline{O4/O8} \\ S(\hat{A}) = -\hat{A}, & S(\hat{A}) = \hat{A}, \\ S(\hat{C}) = \hat{C}, & S(\hat{C}) = -\hat{C}. \end{array} \quad (3.9)$$

This restriction on the possible internal coordinate dependence of the different fields leads to a truncation of the effective four-dimensional spectrum, since, in the Kaluza-Klein expansion (2.39)-(2.44), each ten-dimensional field may only be expanded with respect to Kaluza-Klein modes with the correct transformation properties under the action of S . We discuss the adaption to the Kaluza-Klein ansatz and the corresponding low-energy effective theory for the two cases separately.

3.2 $O6$ orientifolds

The action (3.2) of the $O6$ orientifold on the internal spinors, leads to the following transformation of the two-forms Ω and J [32, 33]:

$$\begin{aligned} S(J) &= -J, \\ S(\Omega) &= -\bar{\Omega}, \end{aligned} \quad (3.10)$$

and the transformation of the complex one-form K is

$$S(K) = \bar{K}. \quad (3.11)$$

Thus, $J, \text{Re}(\Omega)$ and $\text{Im}(K)$ are odd forms, while $\text{Im}(\Omega)$ and $\text{Re}(K)$ are even.

Since $S^2 = 1$ when acting on two-forms, the space of two-forms ω^α introduced in section 2.2.2 splits into an even and an odd eigenspace. We choose a basis of two-forms in each of these spaces. The space of even forms is labeled $H^{2,+}$, with a basis $\omega^A, A = 1, \dots, n_+$, and the space of odd two-forms is labeled $H^{2,-}$, with a basis $\omega^P, P = 1, \dots, n_-$.

$$\begin{aligned} H^{2,+} &= \{\omega \in \Lambda^{2,\text{finite}} \mid S(\omega) = \omega\} = \text{span}\{\omega^A, A = 1, \dots, n_+\}, \\ H^{2,-} &= \{\omega \in \Lambda^{2,\text{finite}} \mid S(\omega) = -\omega\} = \text{span}\{\omega^P, P = 1, \dots, n_-\}. \end{aligned} \quad (3.12)$$

We now use the fact that²

$$\begin{aligned}\eta^{AP} &= \int K^1 \wedge K^2 \wedge \omega^A \wedge \omega^P = \int K^1 \wedge K^2 \wedge S(\omega^A) \wedge S(\omega^P) \\ &= - \int K^1 \wedge K^2 \wedge \omega^A \wedge \omega^P = -\eta^{AP},\end{aligned}\tag{3.13}$$

to show that the intersection form $\eta^{\alpha\beta}$ reduces to the block-diagonal form

$$\eta^{\alpha\beta} = \begin{pmatrix} \eta^{AB} & 0 \\ 0 & \eta^{PQ} \end{pmatrix}\tag{3.14}$$

under the splitting $\Lambda^{2,\text{finite}}\mathcal{Y}_6 = H^{2,+} \oplus H^{2,-}$. From (3.10) we can see that the forms J and $\text{Re}(\Omega)$ are in the space $H^{2,-}$, whereas $\text{Im}(\Omega)$ is in $H^{2,+}$. In other words, $H^{2,-}$ contains two linearly independent positive-normed vectors, whereas $H^{2,+}$ contains only one. Therefore η^{PQ} is an inner product of signature $(2, n_-)$ on $H^{2,-}$, and η^{AB} is an inner product of signature $(1, n_+)$ on $H^{2,+}$.

We note that the transformation properties of the differential forms ω^A, ω^P, K^1 and K^2 also hold for their exterior derivatives. Since $d(S(F)) = S(dF)$ holds for every map S and every differential form F , the exterior derivative of a differential form has the same parity with respect to S , as F itself. Therefore, the exterior derivatives (2.28), (2.33) must reduce to

$$\begin{aligned}d\omega^A &= T_{1B}^A K^1 \wedge \omega^B - \frac{1}{2}tK^1 \wedge \omega^A + T_{2P}^A K^2 \wedge \omega^P, \\ d\omega^P &= T_{1Q}^P K^1 \wedge \omega^Q - \frac{1}{2}tK^1 \wedge \omega^P + T_{2B}^P K^2 \wedge \omega^B, \\ dK^1 &= 0, \\ dK^2 &= tK^1 \wedge K^2.\end{aligned}\tag{3.15}$$

3.2.1 The spectrum

We are now prepared to discuss the resulting low-energy spectrum of the theory. We begin by determining the low-energy moduli space of metrics on \mathcal{Y}_6 after orientifold projection. The projection imposes the transformations (3.10) on J and Ω . Therefore, we expand fluctuations of J and $\text{Re}(\Omega)$ with respect to the odd forms ω^P , and fluctuations of $\text{Im}(\Omega)$ with respect to the even forms ω^A . The parametrization (2.19) becomes

$$\begin{aligned}J^{1,2} &= e^{-\frac{1}{2}\rho} \xi_P^{1,2} \omega^P, P = 1, \dots, n_-, \\ J^3 &= e^{-\frac{1}{2}\rho} \xi_A^3 \omega^A, A = 1, \dots, n_+.\end{aligned}\tag{3.16}$$

In other words, imposing the orientifold symmetry (3.10) reduces the parameter space of metric fluctuations to the choice of two orthogonal fixed-norm vectors ξ^1, ξ^2 in

² To see that the first line of (3.13) holds, note that the pull-back by S enjoys the property $\int K^1 \wedge K^2 \wedge \omega^A \wedge \omega^P = - \int S(K^1 \wedge K^2 \wedge \omega^A \wedge \omega^P)$, where the minus sign appears because S inverts the orientation on the manifold \mathcal{Y}_6 , and $S(K^2) = S(\text{Im}(K)) = -K^2$

$\mathbb{R}^{2,(n-2)}$, and one fixed-norm vector ξ^3 in $\mathbb{R}^{1,(n+1)}$. This corresponds to the reduction of the moduli space (2.37) to the subspace

$$\frac{\mathrm{SO}(2, n_- - 2)}{\mathrm{SO}(2) \times \mathrm{SO}(n_- - 2)} \times \frac{\mathrm{SO}(1, n_+ - 1)}{\mathrm{SO}(n_+ - 1)} \times \mathbb{R}^+. \quad (3.17)$$

This is consistent with the constraint that the Hodge $*$ operation on $\Lambda^{2,\mathrm{finite}}\mathcal{Y}_6$, defined by the matrix H^α_β , is invariant under the involution S . This implies that the Hodge $*$ operator can only act within the spaces $H^{2,+}$ and $H^{2,-}$, i.e. H^α_β reduces to block-diagonal form as well:

$$H^\alpha_\beta = \begin{pmatrix} H^A_B & 0 \\ 0 & H^P_Q \end{pmatrix}, \quad (3.18)$$

where the matrices H^A_B and H^P_Q depend on the parameters ξ^x as follows:

$$H^A_B = -\delta^A_B + \xi^{3A}\xi^3_A, \quad H^P_Q = -\delta^P_Q + (\xi^{1P}\xi^1_Q + \xi^{2P}\xi^2_Q). \quad (3.19)$$

The action of S on the remaining components of the metric can be understood by looking at the explicit expansion of the metric with respect to the basis $\{dx^\mu, K^i\}$. This leads to the components $\hat{g}_{\mu\nu}$, $\hat{g}_{\mu j}$ and \hat{g}_{ij} , defined as follows:

$$\begin{aligned} \hat{g}_{MN}dx^M dx^N = & \hat{g}_{\mu\nu}(x)dx^\mu dx^\nu + \hat{g}_{ij}(x)K^i K^j + \hat{g}_{i\nu}(x)K^i dx^1\nu + \hat{g}_{\mu j}(x)dx^\mu K^j \\ & + g_{ab}dz^a dz^b. \end{aligned} \quad (3.20)$$

Comparing this to the expansion (2.39), we see that the components g and \hat{g} are related by

$$\hat{g}_{\mu\nu} = g_{\mu\nu} + g_{ij}G^i_\mu G^j_\nu, \quad \hat{g}_{i\nu} = -g_{ij}G^j_\nu, \quad \hat{g}_{ij} = g_{ij}. \quad (3.21)$$

Since dx^μ and K^1 are even, and K^2 is odd, the components of \hat{g} proportional to $dx^\mu dx^\nu$, $dx^\mu K^1$, $(K^1)^2$ and $(K^2)^2$ are even, components proportional to $K^2 dx^\mu$ or $K^1 K^2$ are odd, and are therefore projected out. Thus we are left with the components $\hat{g}_{\mu\nu}$, $\hat{g}_{i\mu}$, \hat{g}_{11} and \hat{g}_{22} , or, returning to the variables from (2.39), the components $g_{\mu\nu}$, G^1_μ , g_{11} and g_{22} .

The modification of the Kaluza-Klein ansatz for the remaining fields proceeds in a similar fashion. We expand fields with respect to the differential forms

$$dx^\mu, \quad \mathcal{E}^1 = K^1 - G^1_\mu dx^\mu, \quad \mathcal{E}^2 = K^2, \quad \omega^A, \omega^P, \quad (3.22)$$

as in equations (2.42)-(2.44). Each term in the expansion is even or odd, depending on the number of odd differential forms it contains, for example, terms proportional to dx^μ, ω^A or K^1 are even, terms proportional to K^2 or ω^P are odd, and terms proportional to the product of two odd forms, such as $K^2 \wedge \omega^P$ are even again. Then, depending on the action (3.6) of \mathcal{O} , either odd modes or even modes are truncated from the Kaluza-Klein expansion.

The orientifold action (3.6) implies that the two-form field \hat{B} has to transform as $S(\hat{B}) = -\hat{B}$, so that only odd modes survive in the expansion of \hat{B} in (2.42). The odd

	$j = 2$	$j = 1$	$j = 0$
\hat{g}	$g_{\mu\nu}$	G_μ^1	g_{11}, g_{22} $H^A_B [\times(n_+ - 1)]$ $H^P_Q [\times(2n_- - 4)]$ ρ
$\hat{\varphi}$			φ
\hat{B}		$B_{2\mu}$	b_{12} b_P
\hat{A}			a_2
\hat{C}		$C_{A\mu}$	c_{1A} c_{2P} $C_{1\mu\nu}/\gamma_2$

Table 3.1: This table lists the massless fields which survive the $O6$ orientifold projection. The hatted fields are the massless ten-dimensional fields while the unhatted fields are the massless modes in four space-time dimensions with j indicating their (four-dimensional) spin. The indices $A, B = 1, \dots, n_+$ label components from the expansion in even two-forms, the indices $P, Q = 1, \dots, n_-$ correspond to odd two-forms. For convenience, we have indicated the number of parameters contained in the matrices H in square brackets. As in the case without orientifold projection, the four-dimensional two-form is exchanged for its dual scalar field γ_2 .

modes are the coefficients of $\mathcal{E}^1 \wedge dx^\mu$, $\mathcal{E}^1 \wedge \mathcal{E}^2$ and ω^P or in other words the components $B_{1\mu}, b_{12}$ and b_P . \hat{A} similarly transforms as $\mathcal{O}(\hat{A}) = -S(\hat{A})$, so that only the odd component a_2 is kept. The three-form \hat{C} transforms as $\mathcal{O}(\hat{C}) = S(\hat{C})$ which implies that the even modes $C_{2\mu\nu}, C_{A\mu}, c_{1A}, c_{2P}$ and $C_{\mu\nu\rho}$ remain in the spectrum. Finally, the dilaton should be invariant under the action of S as well, which is satisfied as the low-energy mode coming from the ten-dimensional dilaton has no internal coordinate dependence.

As summarized in table 3.1, we have reduced the spectrum of the original $\mathcal{N} = 4$ theory to $(2 + n_+)$ vector fields, $4n_- + 2n_+ + 2$ scalar fields, and the metric. This corresponds to the bosonic field content of $\mathcal{N} = 2$ supergravity with $n_+ + 1$ vector multiplets and n_- hypermultiplets. In the next section we will examine the effective action for these fields in detail, and show that it corresponds to the (bosonic) action of a gauged $\mathcal{N} = 2$ supergravity theory. For this to be true, the scalar sector of the theory must be a sigma model where the target space is a direct product of a special Kähler manifold $\mathcal{M}_{S.K.}$, spanned by the $2(n_+ + 1)$ complex scalars in the vector multiplets, and a quaternion-Kähler manifold $\mathcal{M}_{Q.K.}$, spanned by the $4n_-$ scalars in the hypermultiplets:

$$\mathcal{M}_{\mathcal{N}=2} = \mathcal{M}_{S.K.} \times \mathcal{M}_{Q.K.} \quad (3.23)$$

In this case, this structure is inherited from the $\mathcal{N} = 4$ theory we started out with. As we recalled in section 2.3, the scalar fields of the $\mathcal{N} = 4$ theory live in a target

space

$$\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(6,n)}{\mathrm{SO}(6) \times \mathrm{SO}(n)} \quad (3.24)$$

where the second factor is described by 6 orthonormal vectors $V_I^{\bar{a}}$ in $\mathbb{R}^{6,n}$, $\bar{a} = 1, \dots, 6$; $I = 1, \dots, 6+n$; $n = n_+ + n_-$. As we show explicitly in section C.2, the orientifold projection leaves the first factor of (3.24) intact, and projects the $V^{\bar{a}}$ onto two orthogonal subspaces. Two of the $V^{\bar{a}}$ are reduced to orthonormal vectors in \mathbb{R}^{2,n_+} . They combine with the first factor from (3.24) to form the moduli space

$$\mathcal{M}_{\mathrm{S.K.}} = \frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \times \frac{\mathrm{SO}(2,n_+)}{\mathrm{SO}(2) \times \mathrm{SO}(n_+)}. \quad (3.25)$$

The four remaining $V^{\bar{a}}$ form the Grassmannian

$$\mathcal{M}_{\mathrm{Q.K.}} = \frac{\mathrm{SO}(4,n_-)}{\mathrm{SO}(4) \times \mathrm{SO}(n_-)}. \quad (3.26)$$

As discussed in, for example, [41], it is a well-known fact that the symmetric spaces $\mathcal{M}_{\mathrm{S.K.}}$ and $\mathcal{M}_{\mathrm{Q.K.}}$ in equations (3.25) and (3.26) are special Kähler, respectively quaternion-Kähler manifolds.

We will now look at the $\mathcal{N} = 2$ moduli space more explicitly, by reducing also the effective action (2.47)-(2.49). This will allow us to identify the canonical data of the $\mathcal{N} = 2$ supergravity theory, such as holomorphic variables for the special Kähler manifold and the corresponding prepotential, the gaugings and their Killing prepotentials.

3.2.2 The $\mathcal{N} = 2$ theory

Projecting out the odd modes, the scalar, vector and potential terms from the effective action (2.47)-(2.49) are reduced. The kinetic term (2.47) for the scalars now takes the form

$$\begin{aligned} S_{\mathrm{scalar}} = & \int -\frac{1}{2} * R + d(\varphi + \frac{1}{2}\eta + \frac{1}{2}\rho) \wedge *d(\varphi + \frac{1}{2}\eta + \frac{1}{2}\rho) \\ & -\frac{1}{16}(DH^A_B \wedge *DH^B_A + DH^P_Q \wedge *DH^Q_P) + \frac{1}{4}e^{2\varphi}g^{22}Da_2 \wedge *Da_2 \\ & +\frac{1}{4}e^\rho H_{PQ}Db^P \wedge *Db^Q + \frac{1}{4}e^{2\eta}Db_{12} \wedge *Db_{12} \\ & +\frac{1}{8}(g^{ik}g^{jl}Dg_{ij} \wedge *Dg_{kl} + D\rho \wedge *D\rho) \\ & +\frac{1}{4}e^{2(\varphi+\rho)}g^{22}(d\gamma_2 - b^P Dc_{2P}) \wedge *(d\gamma_2 - b^Q Dc_{2Q}) \\ & +\frac{1}{4}e^{2\varphi+\rho}g^{11}H_{AB}Dc_1^A \wedge *Dc_1^B \\ & +\frac{1}{4}e^{2\varphi+\rho}g^{22}H_{PQ}D(c_2^P + a_2Db^P) \wedge *(Dc_2^Q + a_2Db^Q), \end{aligned} \quad (3.27)$$

the term (2.48) for the vector fields becomes

$$\begin{aligned}
S_{\text{vector}} = & \frac{1}{4} \int e^{-(2\varphi+\eta+\rho)} g_{11} dG^1 \wedge *dG^1 \\
& + e^{-(2\varphi+\eta+\rho)} g^{22} (\mathcal{D}B_2 - b_{12}dG^1) \wedge *(\mathcal{D}B_2 - b_{12}dG^1) \\
& + e^{-\eta} H_{AB} (\mathcal{D}C^A - dG^1 c_1^A) \wedge *(\mathcal{D}C^B - dG^1 c_1^B) \\
& - b_{12} \eta_{AB} \mathcal{D}C^A \wedge \mathcal{D}C^B \\
& - 2(\mathcal{D}B_2 - b_{12}dG^1) \wedge (c_{1A} \mathcal{D}C^A - \frac{1}{2} c_1^A c_{1A} dG^1) \\
& - T_{1B}^A dB_2 \wedge C_A \wedge C^B - tB_2 \wedge dC^A \wedge C_A,
\end{aligned} \tag{3.28}$$

and the remaining terms from the potential (2.49) are given by

$$\begin{aligned}
S_{\text{potential}} = & \int * \left(\frac{5}{8} e^{2\varphi+3\eta+\rho} g_{22}(t)^2 + \frac{1}{4} e^{4\varphi+3\eta+\rho} (ta_2)^2 \right. \\
& - \frac{1}{16} e^{2\varphi+\eta+\rho} (g^{11} [H, T_1]^A{}_B [H, T_1]^B{}_A + g^{11} [H, T_1]^P{}_Q [H, T_1]^Q{}_P \\
& \quad \left. + 2g^{22} [H, T_2]^P{}_A [H, T_2]^A{}_P) \right. \\
& + \frac{1}{4} e^{2\varphi+2\rho+\eta} b^R b^S (g^{11} H_{PQ} (T_{1R}^P + \frac{1}{2} t \delta^P{}_R) (T_{1S}^Q + \frac{1}{2} t \delta^Q{}_S) \\
& \quad \left. + g^{22} H_{AB} T_{2R}^A T_{2S}^B) \right. \\
& + \frac{1}{4} e^{4\varphi+3\eta+3\rho} (b^P (c_{2Q} T_{1P}^Q - c_{1A} T_{2P}^A + \frac{1}{2} t c_{2P}))^2 \\
& + \frac{1}{4} e^{4\varphi+3\eta+2\rho} H_{PQ} (T_{1R}^P (c_2^R + a_2 b^R) - \frac{1}{2} t (c_2^P - a_2 b^P) - T_{2A}^P c_1^A) \\
& \quad \left. \cdot (T_{1S}^Q (c_2^S + a_2 b^S) - \frac{1}{2} t (c_2^Q - a_2 b^Q) - T_{2B}^Q c_1^B) \right).
\end{aligned} \tag{3.29}$$

The couplings of the vector fields in (3.28) may only depend on scalars in the vector multiplets. This allows us to determine which are the scalar degrees of freedom in the vector multiplets, and we find that they are given by the $n_+ + 1$ complex scalars

$$\begin{aligned}
z^A &= c_1^A + i e^{-(\varphi+\frac{1}{2}\rho)} \sqrt{g_{11}} \xi^{3A}, \\
\tau &= b_{12} + i e^{-\eta}.
\end{aligned} \tag{3.30}$$

The complex scalar τ describes a $SU(1,1)/U(1)$ coset as before. We recall that it follows from (2.21) that ξ^{3A} is restricted to have norm $\xi^{3A} \xi_A^3 = 2$, and thus contains only $(n_+ - 1)$ degrees of freedom. The required extra degree of freedom turns out to be the scalar factor $e^{-(\varphi+\frac{1}{2}\rho)} \sqrt{g_{11}}$.

In the hypermultiplet sector, it is convenient to use the following variables

$$\begin{aligned}
T &\equiv a_2 + i e^{-\varphi} \sqrt{g_{22}}, \\
\mathcal{M}_{PQ} &\equiv \text{Re}(T) \eta_{PQ} + i \text{Im}(T) H_{PQ}, \\
\phi &\equiv 2 e^{-\varphi-\rho} \sqrt{g_{22}}, \\
\tilde{\phi} &\equiv -2\gamma_2 - b^P c_{2P}.
\end{aligned} \tag{3.31}$$

The fields in (3.31), together with the scalars b^P and c_{2P} , which undergo no field redefinition, account for $4n_-$ degrees of freedom. H_{PQ} contains $(2n_- - 4)$ degrees of freedom, the c_{2P} and b^P contribute another $2n_-$ degrees of freedom, and the 4 remaining degrees of freedom come from ϕ , $\tilde{\phi}$ and the complex scalar T .

Thus, the spectrum of the theory contains the following set of $\mathcal{N} = 2$ multiplets:

- The gravity multiplet, whose bosonic degrees of freedom are the metric/graviton $g_{\mu\nu}$ and the graviphoton G_μ^1 .
- $(n_+ + 1)$ vector multiplets, which contain, as bosonic fields, one vector and one complex scalar each. These are given by $(B_{2\mu}, \tau)$ and the n_+ pairs (C_μ^A, z^A) .
- n_- hypermultiplets, consisting of the $4n_-$ scalar fields H^P_Q , c_{2P} , b_P , ϕ , $\tilde{\phi}$ and T .

In terms of the sets of variables (3.30) and (3.31), the scalar kinetic term (3.27) indeed decouples into a separate term for the scalars in the vector multiplets, and one for the scalars in the hypermultiplets, which are discussed in detail in the following subsections. The separation of the scalar fields into hyper- and vector multiplets involves a rather complicated redefinition of the four fields φ , g_{11} , g_{22} and ρ , but one can see that the number of degrees of freedom is preserved by this redefinition. We also note that the dilaton, which is the expansion parameter in string perturbation theory, is a combination of fields from both hyper- and vector multiplets. This implies that both vector- and hypermultiplet moduli spaces are sensitive to string loop corrections [51]. This is in contrast to $\mathcal{N} = 2$ theories obtained from compactifications of type IIA theory on Calabi-Yau threefolds, where the dilaton is entirely part of a hypermultiplet, and the moduli space of the vector multiplets therefore does not receive string loop contributions [52].

Vector multiplets

Rewriting (3.27) in terms of the scalars defined in equations (3.30) and (3.31), the action for the scalars (3.30) can be written as

$$S_{\text{vector}} = \int \frac{-1}{(\tau - \bar{\tau})^2} D\tau \wedge *D\bar{\tau} + G_{A\bar{B}} Dz^A \wedge *D\bar{z}^B, \quad (3.32)$$

where the coupling $G_{A\bar{B}}$ is given by the expression

$$G_{A\bar{B}} = -4 \frac{(z - \bar{z})_A (z - \bar{z})_B}{((z - \bar{z})^C (z - \bar{z})_C)^2} + \frac{2\eta_{AB}}{(z - \bar{z})^C (z - \bar{z})_C}. \quad (3.33)$$

The combined metric defined by the couplings in (3.32) is Kähler with the Kähler potential

$$\mathcal{K} = -\ln i(\bar{\tau} - \tau) - \ln[-\frac{1}{4}\eta_{AB}(z - \bar{z})^A(z - \bar{z})^B] = \ln(\frac{1}{4}e^{2\varphi+\rho+\eta}g^{11}). \quad (3.34)$$

\mathcal{K} is a Kähler potential for the coset space [41]

$$\mathcal{M}_{\text{S.K.}} = \frac{\text{SU}(1,1)}{\text{U}(1)} \times \frac{\text{SO}(2, n_+)}{\text{SO}(2) \times \text{SO}(n_+)} . \quad (3.35)$$

We note that \mathcal{K} can also be expressed by the following integrals over the internal manifold:

$$\mathcal{K} = \ln \int K^1 \wedge *K^1 - 2 \ln \int e^{-\hat{\varphi}} J^3 \wedge *J^3 , \quad (3.36)$$

where we have used the expansion (3.16) of J^3 .

As required by $\mathcal{N} = 2$ supergravity $\mathcal{M}_{\text{S.K.}}$ is a special Kähler manifold. This is equivalent to the requirement that \mathcal{K} can be derived from a holomorphic function, the prepotential \mathcal{F} , according to the formula $\mathcal{K} = -\ln i[\bar{X}^I \mathcal{F}_I - X^I \bar{\mathcal{F}}_I]$, where $\mathcal{F}_I = \partial_I \mathcal{F}$. In the case of the Kähler potential (3.34), this relationship is satisfied for the prepotential

$$\mathcal{F} = -\frac{\tau \eta_{AB} z^A z^B}{4X^0} , \quad (3.37)$$

and a choice of special coordinates X^I

$$X^I = (X^0, X^\iota) = (1, \tau, z^A), \quad I = 0, \dots, n_+ + 1; \quad \iota = 1, \dots, n_+ + 1. \quad (3.38)$$

From the covariant derivatives (2.50) and the definition of the complex fields τ and z^A in (3.30), we find that the complex covariant derivatives are given by

$$\begin{aligned} D_\mu \tau &= \partial_\mu \tau + t G_\mu^1 \tau - t B_{2\mu} , \\ D_\mu z^A &= \partial_\mu z^A - (T_{1B}^A + \frac{1}{2} t \delta^A_B) G_\mu^1 z^B + (T_{1B}^A + \frac{1}{2} t \delta^A_B) C_\mu^B . \end{aligned} \quad (3.39)$$

We denote these covariant derivatives collectively as

$$D_\mu X^\iota = \partial_\mu X^\iota - V_\mu^I k_I^\iota , \quad (3.40)$$

where V^I labels $n_+ + 2$ vector fields in the theory

$$V^I = (V^0, V^1, V^A) = (G^1, B_2, C^A), \quad I = 0, \dots, n_+ + 1 , \quad (3.41)$$

and the k_I are a set of $n_+ + 2$ Killing vectors, describing the isometries of the target space (3.35) that are gauged. The k_I^ι are the components of the Killing vectors with respect to the coordinate basis $\partial/\partial X^\iota$ of the tangent space of $\mathcal{M}_{\text{S.K.}}$, $k_I = k_I^\iota \partial_{X^\iota}$. Thus, the k_I can be expressed as

$$\begin{aligned} k_0 &= -t\tau \partial_\tau + (T_{1B}^A + \frac{1}{2} t \delta^A_B) z^B \partial_{z^A} , \\ k_1 &= t \partial_\tau , \\ k_A &= -(T_{1A}^B + \frac{1}{2} t \delta^B_A) \partial_{z^B} , \end{aligned} \quad (3.42)$$

where, following the labeling of the vector fields introduced in (3.41), k_0 is the Killing vector associated to G^1 , k_1 is associated to B_2 , and the k_A correspond to the vector

fields C^A . In order to check that the gauge transformations induced by the covariant derivatives (3.39) are indeed isometries of the target manifold (3.35), we can use the property that Killing vectors k_I of a Kähler manifold should depend on a so-called Killing prepotential \mathcal{P}_I . The \mathcal{P}_I are real quantities that determine the isometries via the equation

$$k_I^\flat = iG^{i\bar{j}}\partial_{\bar{j}}\mathcal{P}_I, \quad (3.43)$$

where $G^{i\bar{j}}$ is the inverse of the Kähler metric obtained from (3.34). We can indeed find solutions for (3.43) and the k_I found in (3.42). We find the expressions

$$\begin{aligned} \mathcal{P}_0 &= i\frac{\tau + \bar{\tau}}{2(\tau - \bar{\tau})}t + i\frac{(z - \bar{z})_A(T_{1B}^A + \frac{1}{2}t\delta^A_B)(z + \bar{z})^B}{(z - \bar{z})^2}, \\ \mathcal{P}_1 &= i\frac{1}{\tau - \bar{\tau}}t, \\ \mathcal{P}_A &= -2i\frac{(z - \bar{z})_B(T_{1A}^B + \frac{1}{2}t\delta^B_A)}{(z - \bar{z})^2}. \end{aligned} \quad (3.44)$$

The formulation of the gauge transformations in terms of Killing vectors makes it easy to calculate the commutators, and we find the algebra

$$\begin{aligned} [k_0, k_1] &= tk_1, \\ [k_0, k_A] &= -(T_{1A}^B + \frac{1}{2}t\delta^B_A)k_B, \\ [k_1, k_A] &= [k_A, k_B] = 0. \end{aligned} \quad (3.45)$$

The algebra (3.45) is the semi-direct sum of two Abelian sub-algebras [40]: the Abelian algebra of the coordinate shifts k_1 and k_A , and the algebra consisting of the sole generator k_0 . As any semi-direct sum of Abelian algebras, it is solvable.³ Such solvable Lie algebras seem to be a general feature of the gaugings obtained in G -structure compactifications [40, 18, 38].

One can see that the non-Abelian field strengths in the effective action (3.28) have the correct form $\mathcal{D}A^I = dA^I + \frac{1}{2}f_{JK}^I A^J \wedge A^K$, with the structure constants f_{JK}^I defined by $[k_J, k_K] = f_{JK}^I k_I$. After the orientifold projection, the non-Abelian field strengths (2.51) are reduced to

$$\begin{aligned} \mathcal{D}G^1 &= dG^1, \\ \mathcal{D}B_2 &= dB_2 + tG^1 \wedge B_2, \\ \mathcal{D}C^A &= dC^A - (T_{1B}^A + \frac{1}{2}t\delta^B_A)G^1 \wedge C^B, \end{aligned} \quad (3.46)$$

from which one can read off the structure constants f_{01}^1 and f_{0B}^A corresponding to (3.45).

³A Lie algebra \mathfrak{g} is said to be solvable if its commutator series \mathfrak{g}^n , defined by $\mathfrak{g}^n = [\mathfrak{g}^{n-1}, \mathfrak{g}^{n-1}]$, vanishes for some n . Here, equation (3.45) tells us that $\mathfrak{g}^2 = [\mathfrak{g}, \mathfrak{g}]$ consists only of the commuting generators k_1 and k_A , so we have $\mathfrak{g}^3 = 0$.

The quadratic couplings of the vector field strengths $\mathcal{D}A^I$ are determined by the prepotential \mathcal{F} as well. In an $\mathcal{N} = 2$ supergravity theory, they should take the form

$$\frac{1}{2} \int \text{Re}(\mathcal{N})_{IJ} \mathcal{D}A^I \wedge \mathcal{D}A^J - \text{Im}(\mathcal{N})_{IJ} \mathcal{D}A^I \wedge * \mathcal{D}A^J, \quad (3.47)$$

where, up to electric/magnetic duality transformations, the complex matrix \mathcal{N} depends on the scalar fields in the vector multiplets as follows:

$$\mathcal{N}_{IJ} = \bar{\mathcal{F}}_{IJ} + 2i \frac{\text{Im}(\mathcal{F})_{IK} \text{Im}(\mathcal{F})_{JL} X^L X^K}{\text{Im}(\mathcal{F})_{MN} X^M X^N}. \quad (3.48)$$

The \mathcal{F}_{IJ} are the second derivatives of the prepotential \mathcal{F} with respect to the coordinates X^I . One can verify that the quadratic couplings found in the action (3.28), are indeed of the above form, and we give the explicit expressions for the components of \mathcal{N} in terms of the special coordinates (τ, z^A) in section B.1 of the appendix.

Apart from the quadratic couplings (3.47), the action also contains a Chern-Simons type term

$$-\frac{1}{4} \int T_{1B}^A dB_2 \wedge C_A \wedge C^B + t B_2 \wedge dC^A \wedge C_A. \quad (3.49)$$

As found in [53], terms of this type are necessary when certain transformations that do not leave the prepotential \mathcal{F} invariant, are gauged. One can allow for a gauge transformation $\delta X^i = \Lambda^I k_I^i$ which leads to a variation of the prepotential (3.37) of the form

$$\delta \mathcal{F} = \Lambda^I C_{I,JK} X^J X^K, \quad (3.50)$$

where the $C_{I,JK}$ are constant, real and symmetric in the last two indices. The real part of the matrix \mathcal{N} (3.47) changes under this variation, and in order for the total Lagrangian to be invariant, it must contain the extra topological term

$$\delta S = \frac{2}{3} \int C_{I,JK} A^I \wedge A^J \wedge (dA^K - \frac{3}{8} f_{LM}^K A^L \wedge A^M). \quad (3.51)$$

The Killing vectors k_1 and k_A give rise to a variation of the prepotential of the form (3.50), with constants

$$C_{1,AB} = -\frac{1}{4} t \eta_{AB}, \quad C_{A,1B} = C_{A,B1} = +\frac{1}{4} (T_{1A}^C + \frac{1}{2} t \delta^C_A) \eta_{CB}, \quad (3.52)$$

which, when substituted into equation (3.51) give precisely the contribution (3.49). For the constants (3.52), the terms in (3.51) containing the wedge product of four vector potentials cancel. A similar situation was encountered in [40] for compactifications to four dimensions of M-theory on manifolds with SU(3)-structure.

Hypermultiplets

The action (3.27) leads to the following action for the hypermultiplet scalars (3.31)

$$\begin{aligned}
S_{\text{hyper}} = & \int \frac{-1}{(T - \bar{T})^2} DT \wedge *D\bar{T} - \frac{1}{16} DH^P_Q \wedge *DH^P_Q \\
& + \frac{1}{4\phi^2} d\phi \wedge *d\phi + \frac{1}{2\phi} (\text{Im}\mathcal{M})_{PQ} Db^P \wedge *Db^Q \\
& + \frac{1}{2\phi} (\text{Im}\mathcal{M})^{-1 PQ} (Dc_{1P} + (\text{Re}\mathcal{M})_{PR} Db^R) \\
& \qquad \qquad \qquad \wedge * (DC_{1Q} + (\text{Re}\mathcal{M})_{QS} Db^S) \\
& + \frac{1}{4\phi^2} (D\tilde{\phi} + b^P Dc_{2P} - c_{2P} Db^P) \\
& \qquad \qquad \qquad \wedge * (D\tilde{\phi} + b^Q Dc_{2Q} - c_{2Q} Db^Q).
\end{aligned} \tag{3.53}$$

In the first line of (3.53) the T -dependent term contains the action for an $\text{SU}(1, 1)/\text{U}(1)$ coset, and the term containing H^P_Q gives the action for the first component of the projected metric moduli space (3.17), the $\text{SO}(2, n_- - 2)/\text{SO}(2) \times \text{SO}(n_- - 2)$ coset. Therefore, the first line of the action describes the following submanifold of the σ -model target space:

$$\frac{\text{SU}(1, 1)}{\text{U}(1)} \times \frac{\text{SO}(2, n_- - 2)}{\text{SO}(2) \times \text{SO}(n_-)}, \tag{3.54}$$

which is of the same type as (3.35), and is therefore also a special Kähler manifold. The complete action (3.53) is a sigma-model action for a quaternion-Kähler target space constructed out of the special Kähler submanifold (3.54) using the c-map [54, 55]. The c-map describes a class of quaternion-Kähler manifolds which can be constructed out of special Kähler manifolds. In the case of the manifold (3.54), the image of the c-map is the coset space

$$\mathcal{M}_{\text{Q.K.}} = \frac{\text{SO}(4, n_-)}{\text{SO}(4) \times \text{SO}(n_-)}, \tag{3.55}$$

which is known to be quaternion-Kähler. In appendix C we verify that (3.53) describes the $\text{SO}(4, n_-)$ coset (3.55) explicitly in terms of the underlying $\text{SO}(4, n_-)$ coset matrices parametrized by the moduli. This gives us an explicit view of the relation between the coset representatives of (3.55) and variables used in the c-map metric (3.53).

The covariant derivatives in the action (3.53) are

$$\begin{aligned}
D\xi^{iP} &= d\xi^{iP} - G^1 T_{1Q}^P \xi^{iQ}, \\
Db^P &= db^P - G^1 (T_{1Q}^P b^Q + \frac{1}{2} t \delta^P_Q) b^Q, \\
Dc_{2P} &= dc_{2P} + G^1 (T_{1P}^Q + \frac{1}{2} t \delta^Q_P) c_{2Q} - C^A \eta_{AB} T_{2P}^B, \\
DT &= dT + G^1 t T, \\
D\tilde{\phi} &= d\tilde{\phi} - C_A T_{2P}^A b^P.
\end{aligned} \tag{3.56}$$

In the first line of (3.56), we chose to express the covariant derivatives in terms of the variables ξ^{iP} related to H^P_Q as in equation (3.19). The isometries gauged by these covariant derivatives can be described in terms of the following Killing vectors on $\mathcal{M}_{Q.K.}$:

$$\begin{aligned}
k_0 &= T_{1Q}^P \xi^{iQ} \partial_{\xi^{iP}} + (T_{1Q}^P + \frac{1}{2} \delta^P_Q) b^Q \partial_{b^P} \\
&\quad - (T_{1P}^Q + \frac{1}{2} \delta^Q_P) c_{2Q} \partial_{c_{2P}} + t T \partial_T, \\
k_A &= \eta_{AB} T_{2P}^B (\partial_{c_{2P}} + \partial_{\tilde{\phi}}).
\end{aligned} \tag{3.57}$$

As is required for consistency, the gauged symmetries (3.57) on $\mathcal{M}_{Q.K.}$ have the same algebra as the gauged symmetries (3.42) on $\mathcal{M}_{S.K.}$. The only non-trivial commutator relation we need to verify is $[k_0, k_A]$, and we obtain

$$\begin{aligned}
&= \eta_{AB} T_{2P}^B (T_{1Q}^P + \frac{1}{2} t \delta^P_Q) b^Q \partial_{\tilde{\phi}} \\
&\quad + \eta_{AB} T_{2P}^B (T_{1Q}^P + \frac{1}{2} \delta^P_Q) \partial_{c_{2Q}} \\
&= \eta_{AB} (T_{1C}^B - \frac{1}{2} \delta^B_C) T_{2P}^C (\partial_{c_{2P}} - \partial_{\tilde{\phi}}) \\
&= -(T_{1A}^B + \frac{1}{2} \delta^B_A) \eta_{BC} T_{2P}^C (\partial_{c_{2P}} - \partial_{\tilde{\phi}}),
\end{aligned} \tag{3.58}$$

where we have used the constraints (2.32). In the last line of (3.58) we recognize the commutator $[k_0, k_A]$ from (3.45).

The Killing vectors (3.57) on $\mathcal{M}_{Q.K.}$ also depend on a set of Killing prepotentials. However, their calculation is more involved than in the case of the Killing prepotentials on $\mathcal{M}_{S.K.}$. Therefore, we have chosen to present it in section B.2 of the appendix. We also need to know the expression for the Killing prepotentials in order to verify that the potential obtained from the compactification is consistent with $\mathcal{N} = 2$ supergravity. We find agreement, but since the calculation is somewhat lengthy as well, we included it in section B.3 of the appendix.

This concludes our discussion of the effective action obtained from $O6$ orientifold compactifications. We have identified all the canonical quantities and structures which determine the action of gauged $\mathcal{N} = 2$ supergravity theories, and verified that the effective action (3.27)-(3.29) obtained from the compactification agrees is indeed of the required form.

3.3 $O4/O8$ orientifolds

We can now discuss the second type of orientifold projection, which leads to a vacuum with $O4$ and/or $O8$ orientifold planes. This section follows essentially the same line of reasoning as the previous one, with only the results differing. To avoid unnecessary repetition, we present a lot of the arguments more concisely.

Recalling equation (3.4), the action of \mathcal{O} on the internal spinors is now

$$\begin{aligned} S(\eta_{\pm}^1) &= \pm \eta_{\mp}^2, \\ S(\eta_{\pm}^2) &= \mp \eta_{\mp}^1. \end{aligned} \tag{3.59}$$

Using their decomposition (2.12) into bi-spinors, we find that S leaves the two-forms Ω and J invariant:

$$\begin{aligned} S(J) &= J, \\ S(\Omega) &= \Omega, \end{aligned} \tag{3.60}$$

whereas the complex one-form K again transforms as

$$S(K) = \bar{K}. \tag{3.61}$$

This result confirms the intuitive picture that fixed-point loci of S are either five- or nine- dimensional. Looking at the action of S locally around a fixed point, the $O4/O8$ orientifold projection flips the direction in $T_2\mathcal{Y}_6$ spanned by $\text{Im}(K)$, and either zero or all four directions along $T_4\mathcal{Y}_6$, leading to the transformation properties (3.60), (3.61) of differential forms on these tangent spaces. The above transformation properties will lead to a different projection of the Kaluza-Klein expansions for the fields, which we will now investigate.

The transformations (3.60) imply that perturbations of the internal metric may only be expanded with respect to two-forms which remain invariant under the transformation S . However, this does not imply that the set of two-forms $\Lambda^{2,\text{finite}}$ which defines the consistent Kaluza-Klein truncation, can not include other two-forms which transform as $S(\omega) = -\omega$, and which may appear in the expansion of the various form fields in the spectrum.

Therefore, we can again define eigenspaces of “even” two-forms $\omega^A \in H^{2,+}$ and “odd” two-forms $\omega^P \in H^{2,-}$ as in (3.12). The spaces $H^{2,\pm}$, however, have different properties when compared to their $O6$ counterparts, as becomes apparent when we look at the reduction of the intersection form η with respect to these subspaces. The $\text{SO}(3, n)$ metric defined by η again splits into a block-diagonal form

$$\eta^{\alpha\beta} = \begin{pmatrix} \eta^{AB} & 0 \\ 0 & \eta^{PQ} \end{pmatrix}, \tag{3.62}$$

but the metrics η_{AB} and η_{PQ} now have a different signature compared to the case of the $O6$ orientifold projection. Following the transformation property (3.60), the

two-forms J and Ω , which are positive-normed with respect to the metric η , must be contained in $H^{2,+}$. The signature of η_{AB} on $H^{2,+}$ is therefore $(3, n_+ - 3)$, whereas $H^{2,-}$ contains only negative-normed two-forms, i.e. η_{PQ} is negative-definite, with signature $(0, n_-)$.⁴ The exterior derivatives of the differential forms ω^A, ω^P, K^1 and K^2 take the same form as in the case of the $O6$ projection (3.15).

3.3.1 The spectrum

We can now discuss which modes of the effective theory survive the $O4/O8$ orientifold projection. As follows from equation (3.60), the fluctuations of the two-forms Ω and J should all be expanded with respect to even forms ω^A . Therefore, the counterpart of equation (3.16) is

$$J^x = e^{-\frac{\rho}{2}} \xi_A^x \omega^A, \quad A = 1, \dots, n_-; \quad x = 1, \dots, 3. \quad (3.63)$$

The ξ_A^i describe the coset space

$$\frac{\text{SO}(3, n_+ - 3)}{\text{SO}(3) \times \text{SO}(n_+ - 3)}, \quad (3.64)$$

and contain $(3n_+ - 9)$ degrees of freedom. The reduction of the parameter space (2.37) to (3.64) corresponds to the following projection of the matrix $H^\alpha{}_\beta$:

$$H^\alpha{}_\beta = \begin{pmatrix} H^A{}_B & 0 \\ 0 & -\delta^P{}_Q \end{pmatrix}. \quad (3.65)$$

The perturbations of the metric along the two-dimensional component $T_2\mathcal{Y}_6$ of the internal manifold and along the non-compact directions, are truncated in the same manner as in the case of the $O6$ projection. Therefore, we are again left with the four-dimensional metric $g_{\mu\nu}$, the scalars g_{11} and g_{22} , as well as the vector field G_μ^1 . Also the scalar fields ρ and φ are preserved as before.

Formally, the projection of the NS two-form \hat{B} is also the same as in section 3.2.1, again with the caveat that the set of forms ω^P is now of a different nature, as follows from the discussion surrounding (3.62). Thus we have the scalar fields $b_P, P = 1, \dots, n_-, b_{12}$, and the vector field B_2 .

Another difference arises in the truncation of the modes coming from the RR one- and three-form fields \hat{A} and \hat{C} . As explained in section 3.1, the $O4/O8$ orientifold projection only involves the map

$$\mathcal{O}_{O4/O8} = S\Omega_p, \quad (3.66)$$

⁴Irrespective of the different nature of the spaces $H^{2,\pm}$, of their elements and of the intersection metric, compared to section 3.2, we will use the same symbols for these quantities throughout this section.

	$j = 2$	$j = 1$	$j = 0$
\hat{g}	$g_{\mu\nu}$	G_μ^1	g_{11}, g_{22} $H^A_B [\times(3n_+ - 9)]$ ρ
$\hat{\varphi}$			φ
\hat{B}		$B_{2\mu}$	b_{12} b_P
\hat{A}		A_μ	a_1
\hat{C}		$C_{12\mu}/\tilde{C}$ $C_{P\mu}$	$c_{2\mu\nu}/\gamma_1$ c_{1P} c_{2A}

Table 3.2: Spectrum of the effective theory after O4/O8 orientifold projection. The matrix H^A_B now contains $(3n_+ - 9)$ scalar degrees of freedom.

and no factor of $(-1)^{F_L}$ is called for. As indicated in equation (3.9), this implies that the RR one-form \hat{A} should now be expanded with respect to even forms, and \hat{C} should be expanded with respect to odd forms. Therefore, the remaining components of \hat{A} are the scalar a_1 , as well as the four-dimensional vector field A_μ . The surviving components of \hat{C} are now scalar fields c_{1P} , c_{2A} and $c_{2\mu\nu}$ (which is dualized into a scalar γ_1), as well as vector fields $C_{12\mu}$ and $C_{P\mu}$.

An overview of the spectrum after the O4/O8 orientifold projection is given in table 3.2. We count a total of $(4 + n_-)$ vector fields, $(4n_+ + 2n_- - 2)$ scalar fields, and the metric. These are the bosonic degrees of freedom of an $\mathcal{N} = 2$ supergravity theory with one gravity multiplet, $(n_- + 3)$ vector multiplets, and $(n_+ - 2)$ hypermultiplets. As we demonstrate in the following sections, the scalar target space is of a similar form as the target space of the O6 theory. It turns out that the special Kähler space associated to the vector multiplets is now

$$\mathcal{M}_{\text{S.K.}} = \frac{\text{SU}(1,1)}{\text{U}(1)} \times \frac{\text{SO}(2, n_- + 2)}{\text{SO}(2) \times \text{SO}(n_- + 2)}, \quad (3.67)$$

which is $2(n_- + 3)$ -dimensional, and the quaternionic space describing the hypermultiplets is

$$\mathcal{M}_{\text{Q.K.}} = \frac{\text{SO}(4, n_+ - 2)}{\text{SO}(4) \times \text{SO}(n_+ - 2)}, \quad (3.68)$$

which is $4(n_+ - 2)$ -dimensional. The results (3.67) and (3.68) are to be expected, since we know the dimension of $\mathcal{M}_{\text{S.K.}}$ and $\mathcal{M}_{\text{Q.K.}}$ from the number of scalar fields in the $\mathcal{N} = 2$ vector and hypermultiplets, and since the total moduli space $\mathcal{M}_{\text{S.K.}} \times \mathcal{M}_{\text{Q.K.}}$ must be a subset of the $\mathcal{N} = 4$ moduli space (2.52). In the next section, we look at the effective action, and characterize the resulting $\mathcal{N} = 2$ supergravity theory in full detail.

3.3.2 The $\mathcal{N} = 2$ theory

We can now look at the effective action obtained by reducing the $\mathcal{N} = 4$ action (2.47)-(2.49) to those terms containing only fields which survive the $O4/O8$ projection. As a consequence, the scalar kinetic term (2.47) is reduced to

$$\begin{aligned}
S_{\text{scalar}} = & \int -\frac{1}{2} * R + d(\varphi + \frac{1}{2}\eta + \frac{1}{2}\rho) \wedge *d(\varphi + \frac{1}{2}\eta + \frac{1}{2}\rho) \\
& -\frac{1}{16} DH^A{}_B \wedge *DH^B{}_A + \frac{1}{4} e^{2\varphi} g^{11} da_1 \wedge *da_1 \\
& + \frac{1}{4} e^\rho \eta_{PQ} Db^P \wedge *Db^Q + \frac{1}{4} e^{2\eta} Db_{12} \wedge *Db_{12} \\
& + \frac{1}{8} (g^{ij} g^{kl} Dg_{ik} \wedge *Dg_{jl} + D\rho \wedge *D\rho) \\
& + \frac{1}{4} e^{2\varphi+2\rho} g^{11} (D\gamma_1 - b^P Dc_{1P}) \wedge *(D\gamma_1 - b^Q Dc_{1Q}) \\
& + \frac{1}{4} e^{2\varphi+\rho} (H_{AB} g^{22} Dc_2^A \wedge *Dc_2^B \\
& \quad + \eta_{PQ} g^{11} (Dc_1^P + a_1 Db^P) \wedge *D(c_1^Q + a_1 b^Q)),
\end{aligned} \tag{3.69}$$

the kinetic and topological terms (2.48) for the vector fields become

$$\begin{aligned}
S_{\text{vec}} = & \frac{1}{4} \int e^{-(2\varphi+\eta+\rho)} g_{11} dG^1 \wedge *dG^1 \\
& + e^{-(2\varphi+\eta+\rho)} g^{22} (\mathcal{D}B_2 - dG^1 b_{12}) \wedge *(\mathcal{D}B_2 - dG^1 b_{12}) \\
& + e^{\rho-\eta} (\mathcal{D}\tilde{C} - b_P \mathcal{D}C^P + \frac{1}{2} b^2 dA - (\gamma_1 - b^P c_{1P}) dG^1) \\
& \quad \wedge *(\mathcal{D}\tilde{C} - b_Q \mathcal{D}C^Q + \frac{1}{2} b^2 dA - (\gamma_1 - b^P c_{1P}) dG^1) \\
& + e^{-(\rho+\eta)} (dA - dG^1 a_1) \wedge *(dA - dG^1 a_1) \\
& - e^{-\eta} \eta_{PQ} (\mathcal{D}C^P - dAb^P - dG^1 c_1^P) \\
& \quad \wedge *(\mathcal{D}C^Q - dAb^Q - dG^1 c_1^Q) \\
& - b_{12} \eta_{PQ} \mathcal{D}C^P \wedge \mathcal{D}C^Q + 2b_{12} dA \wedge \mathcal{D}\tilde{C} \\
& - 2(\mathcal{D}B_2 - dG^1 b_{12}) \\
& \quad \wedge ((c_{1P} + a_1 b_P) \mathcal{D}C^P - a_1 \mathcal{D}\tilde{C} - (\gamma_1 + \frac{1}{2} a_1 b^2) dA \\
& \quad + (a_1 \gamma_1 - \frac{1}{2} c_{1P} c_1^P - c_{1P} a_1 b^P) dG^1) \\
& - (dB_2 \wedge C_P T_{1Q}^P C^Q - 2B_2 t \wedge dA \wedge \tilde{C} + B_2 t \wedge dC^P \wedge C_P),
\end{aligned} \tag{3.70}$$

and the scalar potential (2.49) reduces to the expression

$$\begin{aligned}
S_{\text{potential}} = & \int * \left(\frac{5}{8} e^{2\varphi+3\eta+\rho} g_{22}(t)^2 \right. \\
& - \frac{1}{16} e^{2\varphi+\eta+\rho} (g^{11} [H, T_1]_B^A [H, T_1]_A^B + 2g^{22} [H, T_2]_P^A [H, T_2]_A^P) \\
& + \frac{1}{4} e^{2\varphi+2\rho+\eta} b^R b^S \left(-g^{11} \eta_{PQ} (T_{1R}^P + \frac{1}{2} t \delta_R^P) (T_{1S}^Q + \frac{1}{2} t \delta_S^Q) \right. \\
& \quad \left. + g^{22} H_{AB} T_{2R}^A T_{2S}^B \right) \\
& + \frac{1}{4} e^{4\varphi+3\eta+2\rho} H_{AB} (T_{1C}^A c_2^C - \frac{1}{2} t c_2^A - T_{2P}^A (c_1^P + a_1 b^P)) \\
& \quad \cdot (T_{1D}^B c_2^D - \frac{1}{2} t c_2^B - T_{2Q}^B (c_1^Q + a_1 b^Q)) \Big). \tag{3.71}
\end{aligned}$$

We can identify the degrees of freedom in the vector multiplets by looking at the scalars which determine the gauge kinetic couplings in the action (3.70). One finds that the special Kähler space (3.67) is described by the following set of $3+n_-$ complex fields:

$$\begin{aligned}
\tau &= b_{12} + i e^{-\eta}, \\
u &= (\gamma_1 + \frac{1}{2} a_1 b_P b^P) - i e^{-\varphi} \sqrt{g_{11}} (e^{-\rho} - \frac{1}{2} b_P b^P), \\
v &= a_1 + i e^{-\varphi} \sqrt{g_{11}}, \\
w^P &= c_1^P + a_1 b^P + i e^{-\varphi} \sqrt{g_{11}} b^P. \tag{3.72}
\end{aligned}$$

The complex scalar τ describing the $SU(1,1)$ coset in the $\mathcal{N} = 4$ theory is again preserved by the projection. We refer to appendix C for the action of the orientifold on the $SO(6, n)$ coset component of the $\mathcal{N} = 4$ moduli space. Equation (C.17) provides an explicit picture of the separation of the scalar degrees of freedom into vector- and hypermultiplets, and was used to identify the complex variables (3.72). The complex scalars u, v, w^P span the $SO(2, n_+ + 2)$ component of the moduli space (3.67), and we denote them together as $z^a = (u, v, w^P)$, $a = 1, \dots, n_- + 2$.⁵

As will become clear from the structure of the effective action, the bosonic spectrum fits into the following $\mathcal{N} = 2$ multiplets:

- the gravity multiplet, containing the metric $g_{\mu\nu}$ and the graviphoton G_μ^1 ,
- $(n_- + 3)$ vector multiplets, each containing one vector and one complex scalar. These are the pairs $(B_{2\mu}, \tau)$, (\tilde{C}_μ, u) , (A_μ, v) and (C^P, w^P) .
- $(n_+ - 2)$ hypermultiplets, containing the $(4n_+ - 8)$ remaining scalar degrees of freedom from the fields

$$H^A_B, c_{2A}, e^{-(\varphi+\frac{\rho}{2})} \sqrt{g_{22}}. \tag{3.73}$$

As in the case of the O6 orientifold projection, we observe that both hyper- and vector multiplets carry a dilaton dependence.

⁵The complex variables z^a are not to be confused with the (real) coordinates z^a used in chapter 2 (footnote 3).

Vector multiplets

In terms of the complex variables τ and $z^a = (u, v, w^P)$ defined in (3.72), the action for the scalars in the vector multiplets becomes

$$S_{\text{vector}} = \int \frac{-1}{(\tau - \bar{\tau})^2} D\tau \wedge *D\tau + G_{a\bar{b}} D z^a \wedge *D\bar{z}^{\bar{b}}, \quad (3.74)$$

where the coupling $G_{a\bar{b}}$ is again a Kähler metric for the $\text{SO}(2, n_+ + 2)$ coset space. The total Kähler potential determining the couplings (3.74) is

$$\mathcal{K} = -\ln \frac{i}{4} (\tau - \bar{\tau}) \left(-2(u - \bar{u})(v - \bar{v}) + (w - \bar{w})^P \eta_{PQ} (w - \bar{w})^Q \right), \quad (3.75)$$

which may be written in terms of the $z^a = (u, v, w^P)$ as

$$\mathcal{K} = -\ln i(\bar{\tau} - \tau) - \ln \left[-\frac{1}{4} \eta_{ab} (z - \bar{z})^a (z - \bar{z})^b \right], \quad (3.76)$$

with a metric η_{ab} given by

$$\eta_{ab} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & \eta_{PQ} \end{pmatrix}. \quad (3.77)$$

Since η_{PQ} has the negative-definite signature $(0, n_+)$, it follows that the combined metric η_{ab} has signature $(1, n_+ + 1)$. This is a Kähler potential of the same form as the one found for the $O6$ orientifold, this time for the special Kähler space (3.67). It can be described in terms of the holomorphic prepotential

$$\mathcal{F} = -\frac{X^1(-2X^2X^3 + \eta_{PQ}X^PX^Q)}{4X^0}, \quad (3.78)$$

for a choice of special coordinates

$$X^I = (X^0, X^1, X^2, X^3, X^P) = (1, \tau, u, v, w^P), \quad I = 0, \dots, n_- + 3. \quad (3.79)$$

Evaluating the Kähler potential in terms of the original Kaluza-Klein field variables, we find the same result as in (3.34),

$$\mathcal{K} = \ln \left(\frac{1}{4} e^{2\varphi + \eta + \rho} g^{11} \right), \quad (3.80)$$

which may be expressed by the same geometrical formula (3.36).

From the covariant derivatives (2.50), we find that the covariant derivatives of the complex fields (3.72) are given by

$$\begin{aligned} D_\mu \tau &= \partial_\mu \tau + t G_\mu^1 \tau - t B_{2\mu}, \\ D_\mu u &= \partial_\mu u - t G_\mu^1 u + t \tilde{C}_\mu, \\ D_\mu v &= \partial_\mu v, \\ D_\mu w^P &= \partial_\mu w^P - (T_{1Q}^P + \frac{1}{2} t \delta^P_Q) w^Q G_\mu^1 + (T_{1Q}^P + \frac{1}{2} t \delta^P_Q) C_\mu^Q. \end{aligned} \quad (3.81)$$

We now label the vector fields in our theory V^I , where the index $I = 0, \dots, n_- + 3$ is the same index that labels the coordinates X^I in (3.79). The corresponding vector fields are given by

$$V^I = (V^0, V^1, V^2, V^3, V^P) = (G^1, B_2, \tilde{C}, A, C^P). \quad (3.82)$$

Then, the isometries of the moduli space $\mathcal{M}_{\text{S.K.}}$ which are gauged by the covariant derivatives (3.81), can be described by the following Killing vectors on $\mathcal{M}_{\text{S.K.}}$

$$\begin{aligned} k_0 &= tu\partial_u - t\tau\partial_\tau + (T_{1Q}^P + \frac{1}{2}t\delta^P_Q)w^Q\partial_{w^P}, \\ k_1 &= t\partial_\tau, \\ k_2 &= -t\partial_u, \\ k_P &= -(T_{1P}^Q + \frac{1}{2}t\delta^Q_P)\partial_{w^Q}. \end{aligned} \quad (3.83)$$

In order to show that the k_I indeed correspond to isometries of the special Kähler manifold (3.67), we can again show that they depend on a set of Killing prepotentials satisfying the equations (3.43). Solving these equations leads to the set of prepotentials

$$\begin{aligned} \mathcal{P}_0 &= i\frac{(w - \bar{w})_P(T_{1Q}^P + \frac{1}{2}t\delta^P_Q)(w + \bar{w})^Q}{(z - \bar{z})^2} - i\frac{(v - \bar{v})(u + \bar{u})}{(z - \bar{z})^2}t \\ &\quad + i\frac{\tau + \bar{\tau}}{2(\tau - \bar{\tau})}t, \\ \mathcal{P}_1 &= i\frac{1}{\tau - \bar{\tau}}t, \\ \mathcal{P}_2 &= 2i\frac{v - \bar{v}}{(z - \bar{z})^2}t, \\ \mathcal{P}_P &= -2i\frac{(w - \bar{w})_Q(T_{1P}^Q + \frac{1}{2}t\delta^Q_P)}{(z - \bar{z})^2}, \end{aligned} \quad (3.84)$$

where the shorthand $(z - \bar{z})^2$ now stands for the expression

$$\begin{aligned} (z - \bar{z})^2 &= (z - \bar{z})^a \eta_{ab} (z - \bar{z})^b \\ &= -2(u - \bar{u})(v - \bar{v}) + (w - \bar{w})^P \eta_{PQ} (w - \bar{w})^Q. \end{aligned} \quad (3.85)$$

Since no fields are charged with respect to the vector field $V^3 = A$, there are no corresponding Killing vector k_3 and Killing prepotential \mathcal{P}_3 in equations (3.83) and (3.84).

We can evaluate the commutators of the gauge transformations using the Killing vectors k_I , and we find the gauge algebra

$$\begin{aligned} [k_0, k_1] &= tk_1, \\ [k_0, k_2] &= -tk_2, \\ [k_0, k_P] &= -(T_{1P}^Q + \frac{1}{2}t\delta^P_Q)k_Q, \end{aligned} \quad (3.86)$$

with all other commutators vanishing. This algebra is very similar to the one found in (3.45). We find that it is again the semi-direct sum of two Abelian subalgebras: the coordinate shifts generated by k_1, k_2 and k_P , and the algebra containing k_0 as a single element. The non-Abelian field-strengths are also consistent with the structure constants obtained in (3.86), and read

$$\begin{aligned}
\mathcal{D}G^1 &= dG^1, \\
\mathcal{D}B_2 &= dB_2 + tG^1 \wedge B_2, \\
\mathcal{D}\tilde{C} &= d\tilde{C} - tG^1 \wedge \tilde{C}, \\
\mathcal{D}A &= dA, \\
\mathcal{D}C^P &= dC^P - (T_{1Q}^P + \frac{1}{2}t\delta^P_Q)G^1 \wedge C^Q.
\end{aligned} \tag{3.87}$$

The couplings of the vector fields can be derived from the holomorphic prepotential (3.78) and its transformation properties as before. The quadratic couplings in the Lagrangian (3.70) can be found using the general formula (3.47) given in the previous section. The matrix \mathcal{N}_{IJ} describing these couplings is given in section B.1 of the appendix. The Chern-Simons term

$$-\frac{1}{4} \int dB_2 \wedge C_P \wedge T_{1Q}^P C^Q + tB_2 \wedge dC^P \wedge C_P + 2tB_2 \wedge \tilde{C} \wedge dA \tag{3.88}$$

is again of the form (3.51) for the following set of constants $C_{I,JK}$

$$\begin{aligned}
C_{1,23} = C_{1,32} &= \frac{1}{4}t, & C_{1,PQ} &= -\frac{1}{4}t\eta_{PQ}, \\
C_{2,13} = C_{2,31} &= -\frac{1}{4}t, & C_{P,1Q} = C_{P,Q1} &= \frac{1}{4}(T_{1P}^R + \frac{1}{2}t\delta^R_P)\eta_{RQ},
\end{aligned} \tag{3.89}$$

which can be obtained from the variation of the prepotential (3.78) under the gauge transformations (3.83).

Hypermultiplets

The Lagrangian for the scalars in the hypermultiplets (3.73) now takes the form

$$\begin{aligned}
S &= \int \frac{1}{2}e^{2\varphi+\rho}g^{22}D(e^{-\varphi-\frac{\rho}{2}}\sqrt{g_{22}}) \wedge *D(e^{-\varphi-\frac{\rho}{2}}\sqrt{g_{22}}) \\
&\quad + \frac{1}{4}e^{2\varphi+\rho}g^{22}H_{AB}Dc_2^A \wedge *Dc_2^B \\
&\quad - \frac{1}{16}DH^A_B \wedge *DH^B_A,
\end{aligned} \tag{3.90}$$

with the covariant derivatives

$$\begin{aligned}
D_\mu(e^{-\varphi-\frac{\rho}{2}}\sqrt{g_{22}}) &= \partial_\mu(e^{-\varphi-\frac{\rho}{2}}\sqrt{g_{22}}) + \frac{1}{2}tG_\mu^1 e^{-\varphi-\frac{\rho}{2}}\sqrt{g_{22}}, \\
D_\mu c_2^A &= \partial_\mu c_2^A - G_\mu^1(T_{1B}^A - \frac{1}{2}t\delta^A_B)c_2^B + C^P T_{2P}^A, \\
D_\mu H^A_B &= \partial_\mu H^A_B - G_\mu^1([T, H]^A_B).
\end{aligned} \tag{3.91}$$

Compared to the $O6$ orientifold compactification, the hypermultiplet sector now looks much simpler. We want to show that the action (3.90) is that of a sigma model with the quaternion-Kähler target space

$$\mathcal{M}_{\text{Q.K.}} = \frac{\text{SO}(4, n_+ - 2)}{\text{SO}(4) \times \text{SO}(n_+ - 2)}. \quad (3.92)$$

This manifold is of the same type as the hypermultiplet moduli space in the $O6$ orientifold theory. There, we showed that the Lagrangian for the scalars in the hypermultiplets matched the explicit description of the c-map given in [55]. Of course the manifold (3.92) is also the image of a suitable special Kähler manifold via the c-map, but the structure of the Lagrangian (3.90) does not make this relationship explicit. Therefore, it is more natural to work directly with the explicit description of the manifold (3.92) in terms of $\text{SO}(4, n_+ - 2)$ coset matrices.

To this end, we assemble the scalars into the following $\text{SO}(4, n_+ - 2)$ matrix \mathcal{M}^u_v , $u, v = 1, \dots, (n_+ + 2)$:

$$\mathcal{M}^u_v = \begin{pmatrix} -\frac{1}{2}e^{2\varphi+\rho}g^{22}(c_2)^2 & e^{2\varphi+\rho}g^{22} & e^{2\varphi+\rho}g^{22}c_{2B} \\ \frac{1}{4}e^{2\varphi+\rho}g^{22}(c_2)^4 & & \\ +e^{-2\varphi-\rho}g_{22} & -\frac{1}{2}e^{2\varphi+\rho}g^{22}(c_2)^2 & -\frac{1}{2}e^{2\varphi+\rho}g^{22}(c_2)^2c_{2B} \\ +H_{CD}c_2^C c_2^D & & -c_{2C}H^C_B \\ -\frac{1}{2}e^{2\varphi+\rho}g^{22}(c_2)^2c_2^A & e^{2\varphi+\rho}g^{22}c_2^A & e^{2\varphi+\rho}g^{22}c_2^A c_{2B} \\ -c_{2C}H^{CA} & & +H^A_B \end{pmatrix}, \quad (3.93)$$

where $(c_2)^2$ is shorthand for the contraction $c_2^A c_2^B \eta_{AB}$, and $(c_2)^4 = ((c_2)^2)^2$. The corresponding metric of signature $(4, n_+ - 2)$ is given by⁶

$$\eta_{uv} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \eta_{AB} \end{pmatrix}. \quad (3.94)$$

Indeed, the action (3.90) is identical to the coset action

$$S = \int D\mathcal{M}^u_v \wedge *D\mathcal{M}^v_u, \quad (3.95)$$

where the covariant derivative of the matrix \mathcal{M} can be written as

$$D_\mu \mathcal{M} = \partial_\mu \mathcal{M} - G_\mu^1[t_0, \mathcal{M}] - C_\mu^P[t_P, \mathcal{M}], \quad (3.96)$$

for the following matrices t_0 and t_P :

$$(t_0)^u_v = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -t & 0 \\ 0 & 0 & T_{1B}^A - \frac{1}{2}t\delta^A_B \end{pmatrix}, \quad (t_P)^u_v = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -T_{2P}^C \eta_{CB} \\ -T_{2P}^A & 0 & 0 \end{pmatrix}. \quad (3.97)$$

⁶We recall from our discussion after equation (3.62) that η_{AB} has signature $(3, n_+ - 3)$, so the block matrix η_{uv} has signature $(4, n_+ - 2)$.

As a consistency check, we can verify that the gauge transformations contained in (3.96) also satisfy the algebra (3.86). In order to do this, we need to verify the non-trivial commutators, which, in our case, are the commutators $[t_0, t_P]$. Commuting the matrices given in (3.97) and using the identities (2.32), we find

$$[t_0, t_P] = (T_{1P}^Q + \frac{1}{2}t\delta_P^Q)t_P. \quad (3.98)$$

which is consistent with the algebra (3.86), the extra minus coming from the fact that (3.86) was computed in terms of the associated Killing vectors. Therefore, the gaugings in the hypermultiplet sector are compatible with those of the vector multiplet sector discussed in the previous section. The Killing prepotentials for gauged isometries of the quaternion-Kähler target space (3.92) are given in section B.2 of the appendix, where we also verify the consistency of the scalar potential.

This concludes our discussion of the $O4/O8$ orientifold compactifications. We have seen that the resulting low-energy effective action is a consistent $\mathcal{N} = 2$ supergravity theory.

Chapter 4

Conclusions

In this thesis, we have investigated the possible orientifold projections of type IIA string theory compactified on $SU(2)$ -structure manifolds. Imposing the orientifold projection on the spectrum reduces the amount of supersymmetry of the low-energy effective theory from $\mathcal{N} = 4$ to $\mathcal{N} = 2$. We have found two different ways in which the orientifold projection can act on the field content of the $\mathcal{N} = 4$ effective theory by looking at the action of the orientifold on the internal spinors that parametrize the low-energy supersymmetry transformations. The two projections correspond to vacua containing $O6$ orientifold planes and vacua containing $O4$ and $O8$ orientifold planes respectively.

We have applied the projections to the low-energy effective action obtained from compactification on $SU(2)$ -structure manifolds [37, 18, 20], and have shown that the two resulting actions can be written in the form of a standard $\mathcal{N} = 2$ supergravity. By identifying the complex scalar fields in the vector multiplet sector, we could verify that the various couplings in the effective Lagrangian are indeed those of a gauged $\mathcal{N} = 2$ supergravity theory. The total scalar field space after the orientifold projection is of the form

$$\begin{aligned}\mathcal{M}_{O6} &= \frac{SU(1,1)}{U(1)} \times \frac{SO(2, n_+)}{SO(2) \times SO(n_+)} \times \frac{SO(4, n_-)}{SO(4) \times SO(n_-)}, \\ \mathcal{M}_{O4/O8} &= \frac{SU(1,1)}{U(1)} \times \frac{SO(2, n_- + 2)}{SO(2) \times SO(n_- + 2)} \times \frac{SO(4, n_+ - 2)}{SO(4) \times SO(n_+ - 2)}.\end{aligned}\tag{4.1}$$

The number n_{\pm} is the number of two-forms in the Kaluza-Klein expansion which transforms with eigenvalue ± 1 under the involution S which is part of the orientifold map. The first two factors in (4.1) appear as the scalar target space of the vector multiplets, and are special Kähler manifolds. The third factor is a quaternion-Kähler manifold, and forms the scalar target space of the hypermultiplets. We observe that the two orientifold projections select a different subspace of the $SO(6, n)$ coset from the original $\mathcal{N} = 4$ moduli space, and project the vector fields as well, in such a way that the resulting theory is $\mathcal{N} = 2$ supersymmetric. To separate the scalar degrees of freedom into hyper- and vector multiplet sectors, we have performed various field

redefinitions. For both orientifold projections, these field redefinitions mixed the degrees of freedom from the Ramond and Neveu-Schwarz sectors. In particular, both target hyper- and vector multiplet spaces depend on the original dilaton, and are therefore expected to receive string loop corrections.

The presence of torsion, which leads one to use a Kaluza-Klein expansion with respect to differential forms which are not closed, leads to gauged symmetries in the effective theory. In the presence of suitable torsion components, which depend on the chosen compactification manifold, isometries on all components of the scalar target spaces (4.1) can become gauged. In both cases, the resulting gauge algebra is a semi-direct sum of two Abelian sub-algebras, similar to the algebras found in other G -structure compactifications [38, 40].

A natural extension of the present work would be to investigate whether the combination of both orientifold projections could give rise to a consistent, and non-trivial gauged $\mathcal{N} = 1$ supergravity theory. Alternatively, one could check whether non-geometric fluxes, such as those considered in [18], can be used to arrange for spontaneous $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ supersymmetry breaking along the lines of [56].

An application of these results could be to study the potential of the effective $\mathcal{N} = 2$ theories obtained in chapter 3, and investigate moduli stabilization, as was done for $SU(2)$ -structure compactifications of type IIB in [57].

Appendix A

Spinor conventions and projections

In this appendix we give a brief overview of the conventions used for the spinor representations in various dimensions, and discuss the transformation properties of those spinors under the orientifold map. This section is largely based upon [31], with some adaptations due to our slightly different conventions.

A.1 Representations

In agreement with the compactification ansatz, the ten-dimensional spinors transform in a representation of $\text{Spin}(1,3) \times \text{Spin}(6)$. The corresponding decomposition of the ten-dimensional gamma-matrices γ_M is given by

$$\Gamma_\mu = \gamma_\mu \otimes \mathbb{1}, \quad \Gamma_m = \gamma_5 \otimes \gamma_m, \quad (\text{A.1})$$

where the $\gamma_\mu, \mu = 0, \dots, 3$ and $\gamma_m, m = 1, \dots, 6$ are the four-dimensional, respectively six-dimensional gamma-matrices, and γ_5 is the four-dimensional chirality operator. The ten-dimensional chirality operator Γ_{11} is the tensor product of the four- and six-dimensional chirality operators γ_5 and γ_7

$$\Gamma_{11} = \gamma_5 \otimes \gamma_7. \quad (\text{A.2})$$

We work with four- and six-dimensional Weyl spinors, and use subscript \pm to indicate their chirality. Hermitian conjugation is denoted by the symbol \dagger . Complex conjugation changes the chirality, and we have the following Majorana conditions in four and six dimensions:

$$\zeta_\pm = B_{(4)} \zeta_\mp^*, \quad \eta_\pm = B_{(6)} \eta_\mp^*, \quad (\text{A.3})$$

where the following relations hold

$$B_{(4)}^{-1} \gamma_\mu B_{(4)} = \gamma_\mu^*, \quad (\text{A.4a})$$

$$B_{(6)}^{-1} \gamma_m B_{(6)} = -\gamma_m^*. \quad (\text{A.4b})$$

The ten-dimensional spinors are Majorana-Weyl, and satisfy the Majorana condition

$$\varepsilon = B_{(10)}\varepsilon^*, \quad (\text{A.5})$$

where $B_{(10)}$ is given by

$$B_{(10)} = \Gamma_{11} \cdot B_{(4)} \otimes B_{(6)}, \quad (\text{A.6})$$

and satisfies

$$B_{(10)}^{-1}\Gamma_M B_{(10)} = -\Gamma_M^*. \quad (\text{A.7})$$

A.2 Transformation properties

We will now discuss the action of the orientifold involution on spinors. Locally, the target space involution S is a combination of a number of reflections. Since we want to preserve all four-dimensional symmetry, these reflections will be along directions in the internal space \mathcal{Y}_6 . A reflection that preserves the ten-dimensional Majorana property and only acts on the internal component of a ten-dimensional spinor should act on the spinors with the transformation

$$R_m = i\Gamma_m\Gamma_{11} = i\mathbb{1} \otimes \gamma_m\gamma_7. \quad (\text{A.8})$$

For an orientifold with Op -planes, S consists of $l = 10 - (p + 1)$ reflections. Taking the square of $S = R_{m_1}\dots R_{m_l}$, we get the following action on spinors

$$S^2 = (-1)^{\frac{l(l-1)}{2}}\mathbb{1}. \quad (\text{A.9})$$

In the case of $O6$ planes, we have $S^2 = -\mathbb{1}$, so we need to add the extra factor $(-1)^{F_L}$ to the total orientifold action \mathcal{O} , in order to ensure that $\mathcal{O}^2 = \mathbb{1}$ also for fermionic states. One can also verify the following property of the action of S on an internal spinor η

$$B_{(6)}S(\eta)^* = (-1)^l S(B_{(6)}\eta^*), \quad (\text{A.10})$$

thus, for the type IIA orientifold involutions S , which contain an odd number l of reflections, six-dimensional Majorana conjugation anticommutes with the action of the involution.

If the orientifold projection is to preserve some of the supersymmetry, S must map between the ten-dimensional supersymmetry parameters

$$\begin{aligned} S(\varepsilon_{10}^{\text{I}}) &= \varepsilon_{10}^{\text{II}}, \\ S(\varepsilon_{10}^{\text{II}}) &= \pm \varepsilon_{10}^{\text{I}}, \end{aligned} \quad (\text{A.11})$$

where the minus sign applies in the case of $O6$ orientifolds, accounting for the fact that $S^2 = -1$.

Since S is an isometry of our chosen $SU(2)$ -structure background \mathcal{Y}_6 , it must preserve the space of global spinors on the internal manifold. This space is spanned

by the η^i , and therefore a (2×2) matrix U exists, such that $S(\eta_+^i) = U^i_j \eta_-^j$. Adding the action on η_- as well, we find

$$S \begin{pmatrix} \eta_+^i \\ \eta_-^i \end{pmatrix} = \begin{pmatrix} 0 & U^i_j \\ -\bar{U}^i_j & 0 \end{pmatrix} \begin{pmatrix} \eta_+^j \\ \eta_-^j \end{pmatrix}, \quad (\text{A.12})$$

where we have used that $S(\eta_-) = -B_{(6)} S(\eta_+)^* = -\bar{U}^i_j \eta_+^j$, due to (A.10). Since the action of S , defined in (A.8), is unitary, U must be a unitary matrix as well.

We will now see that there are only two possible choices for the matrix U , to which all other matrices can be reduced by the choice of an appropriate basis of spinors η^i . As was explained in section 2.2, the $SU(2)$ -structure on \mathcal{Y}_6 is determined by the spinors η^i , up to a rotation of these spinors by a unitary matrix V^i_j . Choosing such a new basis $\tilde{\eta}$ defined by $\tilde{\eta}_+^i = V^i_j \eta_+^j$, the matrix U transforms as¹

$$\tilde{U} = V U V^T. \quad (\text{A.13})$$

Therefore, to find all the different involutions S of our background, we need to find all the possible unitary (2×2) matrices U , which are not related by a transformation (A.13).

O6 orientifolds

For an involution with seven-dimensional fixed-point loci, or a reflection along $l = 3$ directions, equation (A.9) tells us that $S^2 = -\mathbb{1}$, and therefore $U\bar{U} = \mathbb{1}$. Since U is also unitary, we find $U = U^T$. A symmetric unitary (2×2) matrix U can be written as

$$U = e^{i\alpha} \begin{pmatrix} e^{i\gamma} \cos(\beta) & i \sin(\beta) \\ i \sin(\beta) & e^{-i\gamma} \cos(\beta) \end{pmatrix} \sim \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (\text{A.14})$$

which may be brought into diagonal form $\tilde{U}^i_j = \delta^i_j$ by an appropriate transformation (A.13). With respect to a suitable basis of spinors η^i , the orientifold action therefore takes the form

$$S(\eta_{\pm}^i) = \pm \eta_{\mp}^i. \quad (\text{A.15})$$

Looking at the decomposition (2.4) of the ten-dimensional supersymmetry parameters, and using the transformation property (A.15), we see that imposing (A.11) (taking the minus sign in the second line) forces

$$\varepsilon_i^{\text{I}} = \varepsilon_i^{\text{II}}, \quad (\text{A.16})$$

reducing the available four-dimensional supersymmetry.

¹We note the appearance of V^T instead of V^\dagger in the transformation (A.13), which is due to the fact that the complex conjugate spinor η_-^i transforms as $\tilde{\eta}_-^i = \bar{V}^i_j \eta_-^j$.

O4/O8 orientifolds

For an involution S with five- or nine-dimensional fixed-point loci, or reflections along 1 or 5 internal directions, equation (A.9) tells us that $S^2 = \mathbb{1}$, which implies that $U\bar{U} = -\mathbb{1}$. Therefore U is now a skew-symmetric unitary matrix

$$U = \begin{pmatrix} 0 & e^{i\alpha} \\ -e^{i\alpha} & 0 \end{pmatrix} \sim \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad (\text{A.17})$$

which takes the form $U^i_j = \epsilon^{ij}$ in a suitable basis. Thus, we can work in a basis of spinors in which S acts on the η^i as

$$\begin{aligned} S(\eta^1_{\pm}) &= \pm \eta^2_{\mp}, \\ S(\eta^2_{\pm}) &= \mp \eta^1_{\mp}. \end{aligned} \quad (\text{A.18})$$

In the case of an *O4/O8* orientifold, the ten-dimensional supersymmetry parameters are related as in equation (A.11), now without the minus sign. Using (A.18) in the decomposition (2.4), we see that the four-dimensional supersymmetry transformations must satisfy

$$\begin{aligned} \varepsilon^I_1 &= \varepsilon^{\text{II}}_2, \\ \varepsilon^I_2 &= -\varepsilon^{\text{II}}_1. \end{aligned} \quad (\text{A.19})$$

We see that the antisymmetry of the matrix U forces a mixing between the two internal spinors in the case of an *O4/O8* orientifold projection. Therefore, the presence of an extra internal spinor, i.e. $\text{SU}(2)$ -structure, is necessary to define the (supersymmetric) *O4/O8* orientifold projection [31],² and this option is absent in the case of orientifolds of $\text{SU}(3)$ -structure compactifications [22, 21, 28].

²When studying so-called “dynamical $\text{SU}(3) \times \text{SU}(3)$ -structures”, this is a local requirement at the location of the O-planes [31].

Appendix B

$\mathcal{N} = 2$ supergravity couplings

This appendix contains some calculations that were used in chapter 3. We verify that the kinetic couplings of the vector fields in the $\mathcal{N} = 2$ theories obtained from the orientifold projections have the required form (3.47). We also calculate the Killing prepotentials describing the isometries on the quaternion-Kähler spaces and verify the consistency of the potential for the effective actions (3.27)-(3.29) and (3.69)-(3.71).

B.1 Gauge kinetic couplings

The quadratic couplings of the vector field strengths in the Lagrangian take the form

$$\frac{1}{2}\text{Re}(\mathcal{N}_{IJ})\mathcal{D}V^I \wedge \mathcal{D}V^J - \frac{1}{2}\text{Im}(\mathcal{N}_{IJ})\mathcal{D}V^I \wedge *\mathcal{D}V^J, \quad (\text{B.1})$$

where the matrix \mathcal{N} depends on the scalar fields in the vector multiplets. Up to possible electric/magnetic duality rotations, the matrix \mathcal{N} in an $\mathcal{N} = 2$ supergravity theory must be of the following form:

$$\mathcal{N}_{IJ} = \bar{\mathcal{F}}_{IJ} + 2i \frac{\text{Im}(\mathcal{F})_{IK}X^K \text{Im}(\mathcal{F})_{JL}X^L}{\text{Im}(\mathcal{F})_{MN}X^M X^N}, \quad (\text{B.2})$$

where the \mathcal{F}_{IJ} are the second derivatives of the prepotential \mathcal{F} with respect to the special coordinates X^I . We will now show that the quadratic couplings in the effective actions (3.28) and (3.70) are of the form (B.2) (without the need to perform additional electric/magnetic duality transformations).

Due to the high similarity of the $\mathcal{N} = 2$ theories obtained from the $O6$ and $O4/O8$ projections, the calculation is essentially the same in both cases. The target spaces described by the scalars in the vector multiplets are both cosets of the form

$$\frac{\text{SU}(1,1)}{\text{U}(1)} \times \frac{\text{SO}(2,n)}{\text{SO}(2) \times \text{SO}(n)}, \quad (\text{B.3})$$

the only difference being the complex dimension n of the second factor. Consequently, the prepotentials for both $\mathcal{N} = 2$ theories can be written as

$$\mathcal{F} = \frac{X^1 X^p \eta_{pq} X^q}{4X^0}, \quad (\text{B.4})$$

where the indices p and q take values $p, q = 1, \dots, n$, and the metric η_{pq} has signature $(1, n-1)$. We will use this notation to evaluate the formula (B.2) for general spaces of the form (B.3). In order to obtain the specific couplings for the $O6$ and $O4/O8$ orientifold, we just need to substitute the X^p by the special coordinates X^A defined in (3.38) or X^a , defined in (3.79), and η_{pq} by the corresponding metric η_{AB} or η_{ab} in the result.

Deriving \mathcal{F} given in (B.4) with respect to the $X^I = (X^0, X^1, X^p)$ twice, we find

$$\mathcal{F}_{IJ} = \frac{1}{2} \begin{pmatrix} -\tau X^2 & \frac{1}{2} X^2 & \tau X_q \\ \frac{1}{2} X^2 & 0 & -X_q \\ \tau X_p & -X_p & \tau \eta_{pq} \end{pmatrix}, \quad (\text{B.5})$$

where $X^2 = X^p \eta_{pq} X^q$ and $X_p = \eta_{pq} X^q$. We have already set $X^0 = 1$ and $X^1 = \tau$, since this holds in both cases. Furthermore we find

$$\text{Im}(F_{IJ})X^J = \frac{1}{4}i \begin{pmatrix} -\bar{\tau}\bar{X}^2 - i\tau\text{Im}(X^2) + \bar{\tau}\eta_{pq}X^p\bar{X}^q \\ \frac{1}{2}(X - \bar{X})^2 \\ (\tau - \bar{\tau})(X - \bar{X})_p \end{pmatrix}, \quad (\text{B.6})$$

and

$$\text{Im}(F_{IJ})X^I X^J = \frac{1}{4}i(\tau - \bar{\tau})(X - \bar{X})^p \eta_{pq} (X - \bar{X})^q = e^{-\mathcal{K}}. \quad (\text{B.7})$$

We can now use the definitions (3.38) and (3.79) of the special coordinates X^p for the $O6$ and $O4/O8$ orientifold projections to evaluate equations (B.5)-(B.7), and calculate the matrix \mathcal{N}_{IJ} by inserting the results into the formula (B.2) for both cases. For the $O6$ projection, we find the matrix

$$\begin{aligned} \mathcal{N}_{O6} = & \frac{1}{2} \begin{pmatrix} -b_{12}c_1^C c_{1C} & \frac{1}{2}c_1^C c_{1C} & b_{12}c_{1B} \\ \frac{1}{2}c_1^C c_{1C} & 0 & -c_{1B} \\ b_{12}c_{1A} & -c_{1A} & -b_{12}\eta_{AB} \end{pmatrix} \\ & + \frac{i}{2}e^{-2\varphi-\rho-\eta} \begin{pmatrix} -g_{11} - g^{22}(b_{12})^2 & b_{12}g^{22} & e^{2\varphi+\rho}c_1^C H_{CB} \\ -e^{2\varphi+\rho}H_{ABC_1^A}c_{1B} & b_{12}g^{22} & 0 \\ e^{2\varphi+\rho}H_{ACC_1^C} & -g^{22} & -e^{2\varphi+\rho}H_{AB} \end{pmatrix}, \end{aligned} \quad (\text{B.8})$$

which agrees with the couplings in the effective action (3.28) for the vector fields $(V^0, V^1, V^A) = (G^1, B_2, C^A)$. The couplings of the $O4/O8$ theory are given in equation (B.9) (page 57), and these agree with the effective action (3.70) for the set of vector fields $(V^0, V^1, V^2, V^3, V^P) = (G^1, B_2, \tilde{C}, A, C^P)$.

$$\mathcal{N}_{O4/O8} = \frac{1}{2} \begin{pmatrix} 2b_{12}(a_1\tilde{\gamma} - \frac{1}{2}c_1^R c_{1R}) & -a_1\gamma_1 + c_1^R c_{1R} & -b_{12}a_1 & -b_{12}(\gamma_1 + \frac{1}{2}a_1b^2) & b_{112}(c_{1Q} + a_1b_Q) \\ -a_1\gamma_1 + c_1^R c_{1R} + a_1b^R c_{1R} & 0 & a_1 & \gamma_1 + \frac{1}{2}a_1b^2 & -(c_{1Q} + a_1b_Q) \\ -b_{12}a_1 & a_1 & 0 & b_{12} & 0 \\ -b_{12}(\gamma_1 + \frac{1}{2}a_1b^2) & \gamma_1 + \frac{1}{2}a_1b^2 & b_{12} & 0 & 0 \\ b_{12}(c_{1P} + a_1b_P) & -(c_{1P} + a_1b_P) & 0 & 0 & -b_{12}\eta_{PQ} \end{pmatrix} \\
+ \frac{i}{2} \begin{pmatrix} -e^{-2\varphi-\rho-\eta}(g_{11} + g^{22}(b_{12})^2) & e^{-\rho-\eta}\tilde{\gamma}b_Q & e^{-\rho-\eta}a_1 + e^{-\eta}b^R c_{1R} & -e^{-\rho-\eta}\tilde{\gamma}b_Q & \\ +e^{-\eta}c_1^R c_{1R} - e^{-\rho-\eta}(a_1)^2 & +\frac{1}{2}e^{-\rho-\eta}\tilde{\gamma}b^2 & b_{12}e^{-2\varphi-\rho-\eta}g^{22} & \tilde{\gamma}e^{\rho-\eta} & -e^{-\eta}c_{1Q} \\ -e^{-\rho-\eta}(\tilde{\gamma})^2 & & & & \\ b_{12}e^{-2\varphi-\rho-\eta}g^{22} & 0 & -e^{-2\varphi-\rho-\eta}g^{22} & 0 & 0 \\ e^{\rho-\eta}\tilde{\gamma} & 0 & -e^{\rho-\eta} & -\frac{1}{2}e^{\rho-\eta}b^2 & e^{\rho-\eta}b_Q \\ e^{-\rho-\eta}a_1 + e^{-\eta}b^R c_{1R} & 0 & -\frac{1}{2}e^{\rho-\eta} + e^{-\eta}b^2 & -e^{-\rho-\eta} + e^{-\eta}b^2 & \frac{1}{2}e^{\rho-\eta}b^2b_Q \\ +\frac{1}{2}e^{\rho-\eta}\tilde{\gamma}b^2 & 0 & -\frac{1}{2}e^{\rho-\eta}b^2 & -\frac{1}{4}e^{\rho-\eta}(b^2)^2 & -e^{-\eta}b_Q \\ -e^{-\eta}c_{1P} - e^{\rho-\eta}b_P\tilde{\gamma} & 0 & e^{\rho-\eta}b_P & \frac{1}{2}e^{\rho-\eta}b^2b_P - e^{-\eta}b_P & e^{-\eta}(\eta_{PQ} - e^{\rho}b_Pb_Q) \end{pmatrix}, \\
b^2 = b^P b_P, \quad \tilde{\gamma} = \gamma_1 - b^P c_{1P}.$$

(B.9): The kinetic couplings of the gauge fields in the O4/O8 orientifold theory.

B.2 Killing prepotentials

Like isometries of Kähler manifolds, isometries of quaternion-Kähler manifolds can be derived from Killing prepotentials as well. However, in contrast to the Kähler case, the prepotentials \mathcal{P}_I are no longer scalar functions of the moduli space. Instead, there is now an $SU(2)$ triplet $\mathcal{P}_I^x, x = 1, 2, 3$ for each isometry. A triplet \mathcal{P}_I^x determines an isometry by the equation [41]

$$-k_I^u K_{uv}^x = \partial_v \mathcal{P}_I^x + \epsilon^{xyz} \omega_v^y \mathcal{P}_I^z, \quad (\text{B.10})$$

where ω^x is the $SU(2)$ connection, and K^x is the $SU(2)$ curvature form

$$K^x = d\omega^x + \frac{1}{2} \epsilon^{xyz} \omega^y \wedge \omega^z. \quad (\text{B.11})$$

In other words, the $SU(2)$ covariant derivative acting on the triplet \mathcal{P}_I^x is equal to $(-)$ the insertion of the Killing vector k_I^u into the $SU(2)$ curvature form K_{uv}^x . We can now calculate these quantities for the quaternion-Kähler target spaces (3.55) and (3.68) obtained from the orientifold projections. This serves as a check that the gauge transformations indeed correspond to isometries of the hypermultiplet moduli space. Furthermore, the scalar potential in an $\mathcal{N} = 2$ supergravity theory also depends on these Killing prepotentials, and therefore we need to calculate them in order to check the consistency of the potential, which we do in section B.3. The following approach is based on the one outlined in [41].

Both quaternion-Kähler target spaces are scalar cosets of the form

$$\mathcal{M}_{\text{Q.K.}} = \frac{SO(4, n)}{SO(4) \times SO(n)}, \quad (\text{B.12})$$

for different dimensions m . As is discussed in appendix C, these cosets can be represented by a $(4+n) \times 4$ matrix Z_{pa} whose columns represent four (pseudo)-orthogonal $\mathbb{R}^{4, n}$ vectors.

$$Z_{pa} : Z_{pa} \eta^{pq} Z_{qb} = \delta_{ab}, \quad (\text{B.13})$$

where p, q are $SO(4, n)$ indices, a, b are $SO(4)$ indices, and η^{pq} is a metric of signature $(4, n)$. From the matrix Z the $SO(4)$ component θ_{ab} of the connection on $\mathcal{M}_{\text{Q.K.}}$ can be obtained

$$\theta_{ab} = Z_{pa} \eta^{pq} dZ_{qb}, \quad (\text{B.14})$$

from which one can then extract the $SU(2)$ connection by decomposing with respect to the three self-dual 't Hooft matrices Σ^{x+} given in [41]:

$$\omega^x = -\frac{1}{2} \text{tr}(\theta \Sigma^{x+}), \quad x = 1, 2, 3. \quad (\text{B.15})$$

Explicitly, the components ω^x in (B.15) are

$$\begin{aligned} \omega^1 &= \theta_{12} + \theta_{34}, \\ \omega^2 &= \theta_{24} + \theta_{31}, \\ \omega^3 &= \theta_{23} + \theta_{14}. \end{aligned} \quad (\text{B.16})$$

Using these formulas, we will now solve equation (B.10) for the two cosets (3.55) and (3.68).

B.2.1 $O6$ orientifold

The matrix Z_{pa} describing the $SO(4, n_-)$ coset obtained from the $O6$ projection is given in section C.2. Extracting the relevant columns from the $SO(6, n)$ coset in equation (C.13), we can obtain the explicit form of the matrix Z

$$Z_{O6} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-\varphi - \frac{\rho}{2}} \sqrt{g_{22}} + e^{\varphi + \frac{\rho}{2}} \sqrt{g^{22}} & e^{\frac{\rho}{2}} (\gamma - b^R c_{2R}) & -\xi^{iR} c_{2R} 0 \\ \cdot (a_2 \gamma - \frac{1}{2} (c_2)^2 - c_{2R} a_2 b^R) & -a_2 e^{-\frac{\rho}{2}} & \\ e^{\varphi + \frac{\rho}{2}} \sqrt{g^{22}} & 0 & 0 \\ -e^{\varphi + \frac{\rho}{2}} \sqrt{g^{22}} (\gamma + \frac{1}{2} a_2 b^2) & e^{-\frac{\rho}{2}} - \frac{1}{2} e^{\frac{\rho}{2}} b^2 & -\xi^{iP} b_P \\ -e^{\varphi + \frac{\rho}{2}} \sqrt{g^{22}} a_2 & -e^{\frac{\rho}{2}} & 0 \\ e^{\varphi + \frac{\rho}{2}} \sqrt{g^{22}} (c_{2P} + a_2 b_P) & e^{\frac{\rho}{2}} b_P & \xi^i_P \end{pmatrix}, \quad (\text{B.17})$$

where the index i in the last column takes the values $i = 1, 2$. The matrix Z satisfies (B.13) for the following metric η :

$$\eta = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \eta_{PQ} \end{pmatrix}, \quad (\text{B.18})$$

which is obtained by similarly reducing the complete $\mathbb{R}^{6, n}$ metric η_{IJ} from (C.11) to the subspace in which the columns of (B.17) live.

Using (B.14) and (B.16), we find the $SU(2)$ connection

$$\begin{aligned} \omega^1 &= -\frac{1}{2} (e^\varphi \sqrt{g^{22}} da_2 + e^{\varphi + \rho} \sqrt{g^{22}} (b^P dc_{2P} - d\gamma)) \\ &\quad + \frac{1}{4} (\xi^{1P} d\xi_P^2 - \xi^{2P} d\xi_P^1), \\ \omega^2 &= \frac{1}{2} (e^{\varphi + \frac{\rho}{2}} \sqrt{g^{22}} \xi^{1P} (dc_{2P} + a_2 db_P) - e^{\frac{\rho}{2}} \xi^{2P} db_P), \\ \omega^3 &= -\frac{1}{2} (e^{\varphi + \frac{\rho}{2}} \sqrt{g^{22}} \xi^{2P} (dc_{2P} + a_2 db_P) + e^{\frac{\rho}{2}} \xi^{1P} db_P). \end{aligned} \quad (\text{B.19})$$

Now we need to solve equation (B.10) for the \mathcal{P}_I^x . It is convenient to express the Killing vectors on the quaternionic side in terms of the variables in (B.19) as follows:

$$\begin{aligned} k_0 &= T_{1Q}^P \xi^{xQ} \partial_{\xi^{xP}} - t a_2 \partial_{a_2} - c_{2Q} (T_{1P}^Q + \frac{1}{2} \delta^Q_P) \partial_{c_{2P}} - b_Q (T_{1P}^Q - \frac{1}{2} \delta^Q_P) \partial_{b_P}, \\ k_A &= \eta_{AB} T_{2P}^B \partial_{c_{2P}}. \end{aligned} \quad (\text{B.20})$$

Inserting the SU(2) connection (B.19) and the Killing vectors (B.20) into equation (B.10), we find the solutions

$$\begin{aligned}
\mathcal{P}_0^1 &= \frac{1}{2}(te^\varphi\sqrt{g^{22}}a_2 + e^{\varphi+\rho}\sqrt{g^{22}}c_{2Q}(T_{1P}^Q + \frac{1}{2}t\delta^Q_P)b^P - \xi_P^1 T_{1Q}^P \xi^{2Q}), \\
\mathcal{P}_0^2 &= \frac{1}{2}e^\rho \xi_P^2 (T_{1Q}^P + \frac{1}{2}t\delta^P_Q)b^Q \\
&\quad - \frac{1}{2}e^{\varphi+\frac{\rho}{2}}\sqrt{g^{22}}\xi_P^1 \left(T_{1Q}^P (c_2^Q + a_2 b^Q) - \frac{1}{2}t(c_2^P - a_2 b^P) \right), \\
\mathcal{P}_0^3 &= \frac{1}{2}e^\rho \xi_P^1 (T_{1Q}^P + \frac{1}{2}t\delta^P_Q)b^Q \\
&\quad + \frac{1}{2}e^{\varphi+\frac{\rho}{2}}\sqrt{g^{22}}\xi_P^2 \left(T_{1Q}^P (c_2^Q + a_2 b^Q) - \frac{1}{2}t(c_2^P - a_2 b^P) \right),
\end{aligned} \tag{B.21}$$

for the isometries gauged by G^1 . The isometries gauged by the vector fields C^A have the prepotentials

$$\begin{aligned}
\mathcal{P}_A^1 &= -\frac{1}{2}e^{\varphi+\rho}\sqrt{g^{22}}b_P T_{2A}^P, \\
\mathcal{P}_A^2 &= \frac{1}{2}e^{\varphi+\frac{\rho}{2}}\sqrt{g^{22}}\xi_P^1 T_{2A}^P, \\
\mathcal{P}_A^3 &= \frac{1}{2}e^{\varphi+\frac{\rho}{2}}\sqrt{g^{22}}\xi_P^2 T_{2A}^P.
\end{aligned} \tag{B.22}$$

The Killing prepotentials (B.21) may also be expressed by the following integral formulas over the internal manifold \mathcal{Y}_6 :

$$\begin{aligned}
\mathcal{P}_0^1 &= \frac{1}{2}e^{\varphi+\rho}\sqrt{g^{22}} \int_{\mathcal{Y}_6} (\hat{C}_- \wedge d\hat{B} + J^1 \wedge dJ^1 \wedge \hat{A}) + \frac{1}{2}e^\rho \int_{\mathcal{Y}_6} J^1 \wedge dJ^2 \wedge K^2, \\
\mathcal{P}_0^2 &= \frac{1}{2}e^{\varphi+\rho}\sqrt{g^{22}} \int_{\mathcal{Y}_6} (d\hat{B} \wedge \hat{A} \wedge J^1 - dJ^1 \wedge \hat{C}_-) - \frac{1}{2}e^{\frac{3\rho}{2}} \int_{\mathcal{Y}_6} d\hat{B} \wedge K^2 \wedge J^2, \\
\mathcal{P}_0^3 &= -\frac{1}{2}e^{\varphi+\rho}\sqrt{g^{22}} \int_{\mathcal{Y}_6} (d\hat{B} \wedge \hat{A} \wedge J^2 - dJ^2 \wedge \hat{C}_-) - \frac{1}{2}e^{\frac{3\rho}{2}} \int_{\mathcal{Y}_6} d\hat{B} \wedge K^2 \wedge J^1.
\end{aligned} \tag{B.23}$$

\hat{C}_- contains those components of the three-form field \hat{C} which have one leg along the direction K^2 , which is orthogonal to the orientifold plane. In the integrals (B.23), this leaves us with only the components $c_{2P}K^2 \wedge \omega^P$ (we recall that $c_{2A}K^2 \wedge \omega^A$ is projected out by the orientifold projection). For the prepotentials (B.22) we find

$$\begin{aligned}
\mathcal{P}_A^1 &= \frac{1}{2}e^{\varphi+\rho}\sqrt{g^{22}}\eta_{AB} \int_{\mathcal{Y}_6} d\hat{B} \wedge \omega^B \wedge K^1, \\
\mathcal{P}_A^2 &= -\frac{1}{2}e^{\varphi+\rho}\sqrt{g^{22}}\eta_{AB} \int_{\mathcal{Y}_6} dJ^1 \wedge \omega^B \wedge K^1, \\
\mathcal{P}_A^3 &= \frac{1}{2}e^{\varphi+\rho}\sqrt{g^{22}}\eta_{AB} \int_{\mathcal{Y}_6} dJ^2 \wedge \omega^B \wedge K^1.
\end{aligned} \tag{B.24}$$

B.2.2 $O4/O8$ orientifold

For the $O4/O8$ orientifold theory, we look at equation (C.17) to obtain the matrix Z describing the $SO(4, n_+ - 2)$ coset.

$$Z_{O4/O8} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\frac{1}{2}e^{\varphi+\frac{\rho}{2}}\sqrt{g^{22}}c_{2B}c_2^B & -\xi^{xB}c_{2B} \\ +e^{-\varphi-\frac{\rho}{2}}\sqrt{g^{22}} & \\ e^{\varphi+\frac{\rho}{2}}\sqrt{g^{22}} & 0 \\ e^{\varphi+\frac{\rho}{2}}\sqrt{g^{22}}c_{2A} & \xi_A^x \end{pmatrix}, \quad (\text{B.25})$$

where we recall that the index x in the second column now takes the values $x = 1, 2, 3$. The corresponding metric of signature $(4, n_+ - 2)$ was already given in (3.94).

Proceeding as outlined in the previous sections, we find the $SU(2)$ connection¹

$$\omega^x = (-1)^x \frac{1}{2} e^{\varphi+\frac{\rho}{2}} \sqrt{g^{22}} (\xi^{xA} dc_{2A} - \frac{1}{2} \epsilon^{xyz} \xi^{yA} d\xi_A^z). \quad (\text{B.26})$$

The gauged isometries of the hypermultiplet moduli space are described by the Killing vectors k_0 and k_P . In terms of the variables used in (B.26), they read

$$\begin{aligned} k_0 &= T_{1B}^A \xi^{xB} \partial_{\xi^{xA}} - c_{2A} (T_{1B}^A + \frac{1}{2} t \delta^A_B) \partial_{c_{2B}} + \frac{1}{2} t \chi \partial_\chi, \\ k_P &= -T_{2P}^A \partial_{c_{2A}}, \end{aligned} \quad (\text{B.27})$$

where we have used the shorthand $\chi = e^{\varphi+\frac{\rho}{2}} \sqrt{g^{22}}$. Solving the equation (B.10) now leads to the prepotentials

$$\mathcal{P}_0^x = (-1)^{(x+1)} \frac{1}{2} (\xi_A^x (T_{1B}^A - \frac{1}{2} t \delta^A_B) c_2^B - \frac{1}{2} \epsilon^{xyz} \xi_A^y T_{1B}^A \xi^{zB}), \quad (\text{B.28})$$

$$\mathcal{P}_P^x = (-1)^x \frac{1}{2} e^{\varphi+\frac{\rho}{2}} \sqrt{g^{22}} \xi_A^x T_{2P}^A, \quad (\text{B.29})$$

which can also be described by the following integral formulas

$$\mathcal{P}_0^x = (-1)^{(x+1)} \frac{1}{2} (e^{\varphi+\rho} \sqrt{g^{22}} \int_{\mathcal{Y}_6} dJ^1 \wedge \hat{C}_- - \epsilon^{xyz} e^\rho \int_{\mathcal{Y}_6} dJ^y \wedge J^z \wedge K^2), \quad (\text{B.30})$$

$$\mathcal{P}_P^x = (-1)^{(x+1)} \frac{1}{2} e^{\varphi+\rho} \sqrt{g^{22}} \eta_{PQ} \int dJ^x \wedge \omega^Q \wedge K^1, \quad (\text{B.31})$$

where C_- now stands for the components $c_{2A} K^2 \wedge \omega^A$, since these are the ones preserved by the orientifold projection.

B.3 The potential

In a four-dimensional $\mathcal{N} = 2$ supergravity theory, the target space of the scalar moduli, together with a choice of gaugings, fixes the theory. In particular the scalar

¹The indices y, z on the right-hand side of equation (B.26) are summed over, the index x is of course free.

potential can be expressed as [41]

$$\mathcal{V} = e^{\mathcal{K}} X^I \bar{X}^J (g_{\bar{i}j} k_I^{\bar{i}} k_J^j + 4h_{uv} k_I^u k_J^v) - \left(\frac{1}{2}(\text{Im}\mathcal{N})^{-1}{}^{IJ} + 4e^{\mathcal{K}} X^I \bar{X}^J\right) \mathcal{P}_I^x \mathcal{P}_J^x. \quad (\text{B.32})$$

\mathcal{K} is again the Kähler potential which describes the couplings $g_{\bar{i}j}$ of the vector multiplet moduli z^i , the X^I are the special coordinates and the k_I are the Killing vectors associated to the gauge symmetries. h_{uv} contains the couplings in the kinetic term describing the quaternion-Kähler target space of the hypermultiplets. We computed the matrix \mathcal{N}_{IJ} and the prepotentials \mathcal{P}_I^x in the previous sections. As a final consistency check on the $\mathcal{N} = 2$ effective theories we have obtained, we can now verify that the potential obtained from compactification also satisfy this equation.

B.3.1 O6 orientifold

Using the results from section 3.2.2, it is straightforward to compute

$$\begin{aligned} e^{\mathcal{K}} X^I \bar{X}^J g_{\bar{i}j} k_I^{\bar{i}} k_J^j &= \frac{3}{8} e^{2\varphi+\rho+3\eta} g_{22} t^2 + \frac{1}{4} e^{2\varphi+\eta+\rho} g^{11} \xi_A^3 T_{1B}^A T_{1C}^B \xi^{3C} \\ &= \frac{3}{8} e^{2\varphi+\eta+\rho} g^{11} t^2 - \frac{1}{16} e^{2\varphi+\eta+\rho} g^{11} [H, T_1]_B^A [H, T_1]_A^B. \end{aligned} \quad (\text{B.33})$$

The last equality follows from the decomposition (3.19), the tracelessness of the T_{1B}^A and the normalization of the ξ^{xA} . To compute the contribution from the hypermultiplet sector, essentially one only needs to plug the Killing vectors $k_I^u X^I$ into the the differentials dq^u in the effective action (3.53). We obtain the result

$$\begin{aligned} e^{\mathcal{K}} X^I \bar{X}^J h_{uv} k_I^u k_J^v &= -\frac{1}{16} e^{2\varphi+\rho+\eta} g^{11} [H, T_1]_Q^P [H, T_1]_P^Q + \frac{1}{4} e^{2\varphi+\rho+3\eta} g_{22} (t)^2 \\ &\quad + \frac{1}{4} e^{4\phi+\rho+3\eta} (ta_2)^2 \\ &\quad + \frac{1}{4} e^{2\varphi+2\rho+\eta} g^{11} H_{PQ} (T_{1R}^P + \frac{1}{2} t \delta^P_R) b^R (T_{1S}^Q + \frac{1}{2} t \delta^Q_S) b^S \\ &\quad + \frac{1}{4} e^{4\varphi+3\rho+3\eta} \left(b^P (c_{2Q} (T_{1P}^Q + \frac{1}{2} t \delta^Q_P) - c_{1A} T_{2P}^A) \right)^2 \\ &\quad + \frac{1}{4} e^{2\varphi+2\rho+\eta} g^{22} (\xi_A^3 T_{2P}^A b^P)^2 \\ &\quad + \frac{1}{4} e^{4\varphi+2\rho+3\eta} H_{PQ} \\ &\quad \quad \left(T_{1R}^P (c_2^R + a_2 b^R) - T_{2A}^P c_1^A - \frac{1}{2} t (c_2^P - a_2 b^P) \right) \\ &\quad \quad \cdot \left(T_{1S}^Q (c_2^S + a_2 b^S) - T_{2B}^Q c_1^B - \frac{1}{2} t (c_2^Q - a_2 b^Q) \right) \\ &\quad + \frac{1}{4} e^{2\varphi+\rho+\eta} g^{22} H_{PQ} T_{2A}^P \xi^{3A} T_{2B}^Q \xi^{3B}. \end{aligned} \quad (\text{B.34})$$

When computing the last term in equation (B.32), it turns out that the prefactor multiplying the prepotentials \mathcal{P}_0^x cancels:

$$\left(\frac{1}{2}(\text{Im}\mathcal{N})^{-1}{}^{IJ} + 4e^{\mathcal{K}} X^I \bar{X}^J\right) \mathcal{P}_I^x \mathcal{P}_J^x = e^\eta \eta^{AB} \mathcal{P}_A^x \mathcal{P}_B^x. \quad (\text{B.35})$$

The remaining contribution to the potential is then

$$\begin{aligned} -e^\eta \eta^{AB} \mathcal{P}_A^x \mathcal{P}_B^x &= -\frac{1}{4} e^{2\varphi+2\rho+\eta} g^{22} \eta_{AB} T_{2P}^A b^P T_{2Q}^B b^Q \\ &\quad -\frac{1}{4} e^{2\varphi+\rho+\eta} g^{22} \eta_{AB} T_{2P}^A \xi^{iP} T_{2Q}^B \xi^{iQ}. \end{aligned} \quad (\text{B.36})$$

Combining the last lines of equations (B.34) and (B.36), we obtain

$$\begin{aligned} \frac{1}{4} e^{2\varphi+\rho+\eta} g^{22} (H_{PQ} T_{2A}^P \xi^{3A} T_{2B}^Q \xi^{3B} - \eta_{AB} T_{2P}^A \xi^{iP} T_{2Q}^B \xi^{iQ}) \\ = -\frac{1}{8} e^{2\varphi+\rho+\eta} g^{22} [H, T]_A^P [H, T]_P^A, \end{aligned} \quad (\text{B.37})$$

again by using equation (3.19), and the orthonormality properties of the ξ . We can now see that the sum of the three terms (B.33), (B.34) and (B.36) gives precisely the potential obtained in (3.29).

B.3.2 $O4/O8$ orientifold

We now compute the potential for the $O4/O8$ orientifold projection. The contributions from the first two terms in (B.32) are

$$\begin{aligned} e^\mathcal{K} X^I \bar{X}^J g_{ij} k_i^{\bar{j}} k_j^{\bar{I}} &= \frac{1}{2} e^{2\varphi+\rho+3\eta} g_{22}(t)^2 \\ &\quad -\frac{1}{4} e^{2\varphi+2\rho+\eta} g^{11} \eta_{PQ} (T_{1R}^P + \frac{1}{2} t \delta^P_R) b^R (T_{1S}^Q + \frac{1}{2} t \delta^Q_S) b^S, \end{aligned} \quad (\text{B.38})$$

$$\begin{aligned} e^\mathcal{K} X^I \bar{X}^J h_{uv} k_I^u k_J^v &= -\frac{1}{16} e^{2\varphi+\rho+\eta} g^{11} [H, T_1]_B^A [H, T_1]_A^B \\ &\quad + \frac{1}{4} e^{4\varphi+2\rho+3\eta} H_{AB} (T_{1C}^A c_2^C - \frac{1}{2} t c_2^A - T_{2P}^A (c_1^P + a_1 b^P)) \\ &\quad \cdot (T_{1D}^B c_2^D - \frac{1}{2} t c_2^B - T_{2Q}^B (c_1^Q + a_1 b^Q)) \\ &\quad + \frac{1}{4} e^{2\varphi+2\rho+\eta} g^{22} H_{AB} T_{2P}^A b^P T_{2Q}^B b^Q \\ &\quad + \frac{1}{8} e^{2\varphi+\rho+3\eta} g_{22}(t)^2. \end{aligned} \quad (\text{B.39})$$

In the third term, the prepotentials \mathcal{P}_0^x drop out again. We find

$$\begin{aligned} -\left(\frac{1}{2}(\text{Im}\mathcal{N})^{-1}{}^{IJ} + 4e^\mathcal{K} X^I \bar{X}^J\right) \mathcal{P}_I^x \mathcal{P}_J^x &= -e^\eta \eta^{PQ} \mathcal{P}_P^x \mathcal{P}_Q^x \\ &= -\frac{1}{4} e^{2\varphi+\rho+\eta} g^{22} \eta^{PQ} \xi_A^3 T_{2P}^A \xi_B^3 T_{2Q}^B \\ &= -\frac{1}{8} e^{2\varphi+\rho+\eta} [H, T]_A^P [H, T]_P^A. \end{aligned} \quad (\text{B.40})$$

The last equality can be deduced in the same way as equation (B.37). The potential obtained from compactification, as given in the effective action (3.71), is equal to the sum of the contributions (B.38), (B.39) and (B.40).

Appendix C

SO(m, n) coset spaces

Symmetric spaces of the form

$$\mathcal{M} = \frac{\text{SO}(m, n)}{\text{SO}(m) \times \text{SO}(n)} \quad (\text{C.1})$$

appear as part of the moduli spaces in the effective theories discussed in the previous chapters. A point in \mathcal{M} can be thought of as representing an m -dimensional subspace of positive-normed vectors in $\mathbb{R}^{m, n}$. Indeed, a point in $\text{SO}(m, n)$ is a pseudo-orthonormal basis in $\mathbb{R}^{m, n}$, and we can divide out an $\text{SO}(m) \times \text{SO}(n)$ subgroup by considering the equivalence class all orthonormal bases related by an $\text{SO}(m)$ rotation of the m positive-normed basis vectors, and an $\text{SO}(n)$ rotation of the n negative-normed vectors in its orthogonal complement. Consequently, a point in \mathcal{M} can be identified with the space spanned by the m positive-normed basis vectors of an $\text{SO}(m, n)$ matrix (or, equivalently, with its complement, the space spanned by the n negative-normed basis vectors).

A representation for these coset spaces is given by [58]

$$L(X)^{\Lambda\Sigma} = \begin{pmatrix} (\mathbb{1} + XX^T)^{\frac{1}{2}} & X \\ X^T & (\mathbb{1} + X^T X)^{\frac{1}{2}} \end{pmatrix}, \quad (\text{C.2})$$

where X is a real $(m \times n)$ matrix of coordinates, and $\Lambda, \Sigma = 1, \dots, m + n$ are indices in $\mathbb{R}^{m, n}$. One can see that L is an $\text{SO}(m, n)$ matrix with respect to the $\text{SO}(m, n)$ metric $\eta_{\Lambda\Sigma} = \text{diag}(\mathbb{1}_m, -\mathbb{1}_n)$. The first m columns of L form m orthogonal vectors $V^{\bar{\alpha}}$, $\bar{\alpha} = 1, \dots, m$:

$$V^{\Lambda\bar{\alpha}} = \begin{pmatrix} (\mathbb{1} + XX^T)^{\frac{1}{2}} \\ X^T \end{pmatrix}. \quad (\text{C.3})$$

A matrix which is invariant under $\text{SO}(m)$ rotations in the space spanned by the $V^{\bar{\alpha}}$, and $\text{SO}(n)$ rotations of its orthogonal complement, is then given by

$$M^{\Lambda\Sigma} = (LL^T)^{\Lambda\Sigma}, \quad (\text{C.4})$$

and the canonical action for a sigma-model with target space \mathcal{M} is given by

$$S_{\mathcal{M}} = \int -dM_{\Sigma}^{\Lambda} \wedge *dM_{\Lambda}^{\Sigma}. \quad (\text{C.5})$$

An equivalent formula for $M^{\Lambda\Sigma}$ is given by

$$M^{\Lambda\Sigma} = 2V^{\bar{a}\Lambda}V^{\bar{a}\Sigma} - \eta^{\Lambda\Sigma}. \quad (\text{C.6})$$

The general formula (C.6) can be recognized in equations (2.23), (2.55) and (3.19). We now give explicit formulas for the $V^{\bar{a}}$, $\bar{a} = 1, \dots, 6$ describing the moduli space of the original $\mathcal{N} = 4$ supergravity, and for the orientifold action on the $V^{\bar{a}}$, giving us an explicit description of the moduli spaces of the orientifolded theories.

C.1 $SO(6, n)$ coset

As it was found in [37], the action (2.47) for the scalar sector of the $\mathcal{N} = 4$ supergravity theory is of the form (C.5). Recalling (2.56), the full scalar kinetic term is given by

$$S_{\text{scalar}} = \int \frac{1}{4 \text{Im}(\tau)^2} D\tau \wedge *D\bar{\tau} - \frac{1}{16} DM^I{}_J \wedge *DM^J{}_I, \quad (\text{C.7})$$

where $\tau = b_{12} + ie^{-\eta}$, and the D are appropriate covariant derivatives. The matrix M_{IJ} written in [37] is repeated in terms of our conventions in equation (C.8) (page 67).

Six orthonormal vectors $V^{\bar{a}}$ that determine the matrix M_{IJ} can be written explicitly in terms of the moduli as follows:

$$V_I{}^{\bar{a}} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-\varphi - \frac{\rho}{2}} e_i^{\bar{j}} + e^{\varphi + \frac{\rho}{2}} e^{k\bar{j}} c_{ki} & -a_i e^{-\frac{\rho}{2}} + e^{\frac{\rho}{2}} (\gamma_i - b^\gamma c_{i\gamma}) & -\xi^{x\gamma} c_{i\gamma} \\ e^{\varphi + \frac{\rho}{2}} \delta_{ik} e^{k\bar{j}} & 0 & 0 \\ -e^{\varphi + \frac{\rho}{2}} e^{k\bar{j}} (\gamma_k + \frac{1}{2} a_k b^2) & e^{-\frac{\rho}{2}} - \frac{1}{2} e^{\frac{\rho}{2}} b^2 & -\xi^{x\beta} b_\beta \\ -e^{\varphi + \frac{\rho}{2}} e^{k\bar{j}} a_k & -e^{\frac{\rho}{2}} & 0 \\ e^{\varphi + \frac{1}{2}\rho} e^{k\bar{j}} (c_{k\alpha} + a_k b_\alpha) & e^{\frac{\rho}{2}} b_\alpha & \xi^x{}_\alpha \end{pmatrix}, \quad (\text{C.9})$$

where we have split the multi-index $\bar{a} = (\bar{i}, 3, x)$, $\bar{i} = 1, 2$; $x = 1, 2, 3$. The matrix $e_i^{\bar{j}}$ is a vielbein relating the two-dimensional metric g_{ij} and a flat metric $\delta_{\bar{i}\bar{j}}$

$$g_{ij} = e_i^{\bar{i}} e_j^{\bar{j}} \delta_{\bar{i}\bar{j}}, \quad (\text{C.10})$$

and the shorthand expressions b^2 and c_{ij} are explained in (C.8). The $V^{\bar{a}}$ are orthonormal with respect to the $SO(6, n)$ metric

$$\eta_{IJ} = \begin{pmatrix} 0 & \delta_{i\bar{j}} & 0 & 0 & 0 \\ \delta_{i\bar{j}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \eta_{\alpha\beta} \end{pmatrix} \quad (\text{C.11})$$

$$M_{IJ} = \begin{pmatrix}
e^{-2\varphi-\rho} g_{ij} + H^{\alpha\beta} c_{i\alpha} c_{j\beta} & & & & & & & \\
+e^\rho (\gamma_i - b^\alpha c_{i\alpha}) (\gamma_j - b^\beta c_{j\beta}) & \dots & & & & & & \dots \\
+e^{2\varphi+\rho} g^{kl} c_{ki} c_{lj} + e^{-\rho} a_i a_j & & & & & & & \dots \\
e^{2\varphi+\rho} \delta_{ii} g^{jk} c_{kj} & e^{2\varphi+\rho} \delta_{ii} \delta_{jj} g^{ij} & & & & & & \dots \\
-e^{-\rho} a_j + H^{\alpha\beta} b_\alpha c_{j\beta} & & & & & & & \\
-e^{2\varphi+\rho} g^{kl} (\gamma_k + \frac{1}{2} a_k b^2) c_{lj} & -e^{2\varphi+\rho} \delta_{jj} g^{jk} & e^{-\rho} + H^{\alpha\beta} b_\alpha b_\beta + \frac{1}{4} e^\rho (b^2)^2 & & & & & \dots \\
-\frac{1}{2} e^\rho b^2 (\gamma_j - b^\alpha c_{j\alpha}) & \cdot (\gamma_k + \frac{1}{2} a_k b^2) & \cdot (\gamma_l + \frac{1}{2} a_l b^2) & & & & & \dots \\
-e^\rho (\gamma_j - b^\alpha c_{j\alpha}) & -e^{2\varphi+\rho} g^{kl} (\gamma_k + \frac{1}{2} a_k b^2) a_l & e^{2\varphi+\rho} g^{kl} a_k a_l & & & & & \dots \\
-e^{2\varphi+\rho} g^{kl} a_k c_{lj} & + \frac{1}{2} e^\rho b^2 & + e^\rho & & & & & \dots \\
e^\rho b_\alpha (\gamma_j - b^\gamma c_{j\gamma}) - H^\gamma_\alpha c_{j\gamma} & -H^\gamma_\alpha b_\gamma - \frac{1}{2} e^\rho b^2 b_\alpha & H_{\alpha\beta} + e^\rho b_\alpha b_\beta & & & & & \\
+e^{2\varphi+\rho} g^{kl} (c_{k\alpha} + a_k b_\alpha) c_{lj} & e^{2\varphi+\rho} \delta_{jj} g^{jk} & -e^{2\varphi+\rho} g^{kl} a_k & + e^{2\varphi+\rho} g^{kl} (c_{k\alpha} + a_k b_\alpha) & & & & \\
& \cdot (c_{k\alpha} + a_k b_\alpha) & \cdot (\gamma_l + \frac{1}{2} a_l b^2) & \cdot (c_{l\alpha} + a_l b_\alpha) & & & & \dots \\
& & & & & & & \dots \\
& & & & & & & \dots
\end{pmatrix}$$

$$\begin{aligned}
c_{ij} &= a_i \gamma_j - \frac{1}{2} \eta^{\alpha\beta} (c_{i\alpha} c_{j\beta} + c_{i\alpha} a_j b_\beta + a_i b_\alpha c_{j\beta}) + \epsilon_{ij} \beta, \\
b^2 &= b^\alpha b_\alpha.
\end{aligned}$$

(C.8): The full $SO(6, n)$ coset matrix. The multi-index I splits as $I = (i, \hat{i}, 5, 6, \alpha)$, with $i, \hat{i} = 1, 2$ and $\alpha = 1, \dots, n$, numbering the two-forms in the Kaluza-Klein expansion. Since M is symmetric, the elements of the upper triangle are left in ellipsis in order to improve readability.

We can now look at the action of the orientifold projections from chapter 3 on the vectors $V^{\bar{a}I}$. It follows from the discussion of the moduli spaces for the orientifold theories (see the discussion around equations (3.25), (3.26), (3.67) and (3.68)), that the projection should reduce the $\mathcal{N} = 4$ coset space into a product

$$\frac{\text{SO}(6, n)}{\text{SO}(6) \times \text{SO}(n)} \xrightarrow{\mathcal{O}} \frac{\text{SO}(2, n_1)}{\text{SO}(2) \times \text{SO}(n_1)} \times \frac{\text{SO}(4, n - n_1)}{\text{SO}(4) \times \text{SO}(n - n_1)}, \quad (\text{C.12})$$

where the number n_1 depends on the projection. This is realized by projecting the $V^{\bar{a}}$ onto two orthogonal subspaces, one of which is four-dimensional, the other two-dimensional.

C.2 $O6$ orientifold

Table 3.1 lists the scalar fields that survive the $O6$ orientifold projection. The degrees of freedom of the metric g_{ij} on the torus are reduced to the diagonal components g_{11} , g_{22} , correspondingly, the vielbein $e_i^{\bar{i}}$ is reduced to $\text{diag}(\sqrt{g_{11}}, \sqrt{g_{22}})$. As discussed at the beginning of section 3.2.1, the moduli of the internal metric, encoded in the $\xi^{x\alpha}$ are reduced to ξ^{3A} , containing $n_+ - 1$ moduli, and ξ^{iP} , containing $2n_- - 2$ degrees of freedom. For the remaining scalar fields in the matrix $V_I^{\bar{a}}$, table 3.1 tells us whether they survive the projection. The result is the projected matrix $\tilde{V}_I^{\bar{a}}$ given in equation (C.13) (page 69).

The matrix (C.13) is reduced to two orthonormal vectors in \mathbb{R}^{2, n_+} , the left and right columns of (C.13), and four vectors in \mathbb{R}^{4, n_-} , the three center columns of (C.13). As one can see, the outer columns depend only on the scalar fields that determine the complex coordinates z^A (3.30) in the vector multiplets. These scalars span the component

$$\frac{\text{SO}(2, n_+)}{\text{SO}(2) \times \text{SO}(n_+)} \quad (\text{C.14})$$

of the special Kähler space (3.35). Indeed, constructing an $\text{SO}(2, n_+)$ matrix out of them according to the general formula (C.6), one can reproduce the scalar kinetic term (3.32) from the formula (C.5).

The inner columns of (C.13) contain all scalar fields in the hypermultiplets (3.31). They make up four orthonormal vectors in \mathbb{R}^{4, n_-} , which describe the quaternion-Kähler component (3.55) of the moduli space,

$$\frac{\text{SO}(4, n_-)}{\text{SO}(4) \times \text{SO}(n_-)}. \quad (\text{C.15})$$

Again, we can obtain the kinetic term (3.53) using the general formulas (C.6) and (C.5). This way, we can explicitly see that the c-map metric given by (3.53) corresponds to the scalar coset (C.15).

$$\tilde{V}_I^{\bar{a}} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\frac{1}{2}e^{\varphi+\frac{\rho}{2}}c_{1A}c_1^A + e^{-\varphi-\frac{\rho}{2}}\sqrt{g_{11}} & 0 & 0 & 0 & -\xi^{3A}c_{1A} \\ 0 & e^{\varphi+\frac{\rho}{2}}\sqrt{g^{22}}(a_2\gamma - \frac{1}{2}(c_{2P}c_2^P + 2c_{2P}a_2b^P)) + e^{-\varphi-\frac{\rho}{2}}\sqrt{g_{22}} & e^{\frac{\rho}{2}}(\gamma - b^P c_{2P}) & -\xi^{iP}c_{2P} & 0 \\ e^{\varphi+\frac{\rho}{2}}\sqrt{g^{11}} & 0 & 0 & 0 & 0 \\ 0 & e^{\varphi+\frac{\rho}{2}}\sqrt{g^{22}} & 0 & 0 & 0 \\ 0 & -e^{\varphi+\frac{\rho}{2}}\sqrt{g^{22}}(\gamma + \frac{1}{2}a_2b^2) & e^{-\frac{\rho}{2}} - \frac{1}{2}e^{\frac{\rho}{2}}b^2 & -\xi^{iP}b_P & 0 \\ 0 & -e^{\varphi+\frac{\rho}{2}}\sqrt{g^{22}}a_2 & -e^{\frac{\rho}{2}} & 0 & 0 \\ e^{\varphi+\frac{\rho}{2}}\sqrt{g^{11}}c_{1A} & 0 & 0 & 0 & \xi^3_A \\ 0 & e^{\varphi+\frac{\rho}{2}}\sqrt{g^{22}}(c_{2P} + a_2b_P) & e^{\frac{\rho}{2}}b_P & \xi^i_P & 0 \end{pmatrix}$$

(C.13): The projected coset representative for the $O6$ projection. With respect to the $SO(6, n)$ metric η_{IJ} given in (C.11), replacing $\eta_{\alpha\beta}$ by the block-diagonal form given in (3.14), the two outer columns are orthogonal to the three inner columns (which represent four $SO(6, n)$ vectors, since the fourth column still carries an index $i = 1, 2$).

C.3 O_4/O_8 orientifold

The projection of the $SO(6, n)$ coset to the subspace

$$\frac{SO(2, n_- + 2)}{SO(2) \times SO(n_- + 2)} \times \frac{SO(4, n_+ - 2)}{SO(4) \times SO(n_+ - 2)}, \quad (\text{C.16})$$

can be seen explicitly in equation (C.17) (page 71). To arrive at this result, we proceed as in the last section, this time using the spectrum of the O_4/O_8 theory, listed in table 3.2.

$$\tilde{V}_I^{\bar{a}} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{\frac{\rho}{2}} \sqrt{g^{11}} (a_1 \gamma_1 - \frac{1}{2} (c_{1P} c_1^P + 2c_{1P} a_1 b^P)) & 0 & e^{\frac{\rho}{2}} (\gamma_1 - b^P c_{1P}) & 0 \\ 0 & -\frac{1}{2} e^{\frac{\rho}{2}} \sqrt{g^{22}} c_{2A} c_2^A & -a_1 e^{-\frac{\rho}{2}} & -\xi^{xA} c_{2A} \\ e^{\frac{\rho}{2}} \sqrt{g^{11}} & +e^{-\frac{\rho}{2}} \sqrt{g^{22}} & 0 & 0 \\ 0 & e^{\frac{\rho}{2}} \sqrt{g^{22}} & 0 & 0 \\ -e^{\frac{\rho}{2}} \sqrt{g^{11}} (\gamma_1 + \frac{1}{2} a_1 b^2) & 0 & e^{-\frac{\rho}{2}} - \frac{1}{2} e^{\frac{\rho}{2}} b^2 & 0 \\ -e^{\frac{\rho}{2}} \sqrt{g^{11}} a_1 & 0 & -e^{\frac{\rho}{2}} & 0 \\ 0 & e^{\frac{\rho}{2}} \sqrt{g^{22}} c_{2A} & 0 & \xi^x_A \\ e^{\frac{\rho}{2}} \sqrt{g^{11}} (c_{1P} + a_1 b_P) & 0 & e^{\frac{\rho}{2}} b_P & 0 \end{pmatrix}$$

(C.17): The projected coset for $O4/O8$ orientifold projections. As in (C.13), the columns $V^{\bar{a}}$ are projected onto two orthogonal subspaces. Columns one and three contain the scalar fields in the vector multiplets, and live in an $\mathbb{R}^{2, n-+2}$ subspace of $\mathbb{R}^{6, n}$. Columns two and four contain the scalars in the hypermultiplets and live in the orthogonal $\mathbb{R}^{4, n-+2}$ subspace (note that the fourth column still carries an index $x = 1, 2, 3$).

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