Orientifolds and R^4-couplings on generalized geometries

Orientifolds und R^4-Kopplungen auf verallgemeinerten Geometrien

Diplomarbeit vorgelegt von

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Abstract:

This diploma thesis is concerned with two aspects of string theory compactifications. First, the effect of orientifold projections on the particle spectrum of type IIA string theory compactified on a six-dimensional manifold will be considered. A convenient description of the effect of the projection, already known for type IIB theory, will be transfered to type IIA theory. Furthermore, perturbative corrections to the four-dimensional effective action will analyzed. The form of these correction are already known for compactifications on Calabi-Yau manifolds. These terms will be calculated again for the more general case of compactifications on a generic Riemannian manifold.

Zusammenfassung:

In dieser Diplomarbeit werden zwei Aspekte von String Theorie Kompaktifizierungen untersucht. Als erstes wird die Wirkung von Orientifold Projektionen auf das Teilchen Spektrum der Typ IIA String Theorie, kompaktifiziert auf einer sechsdimensionalen Mannigfaltigkeit, betrachtet. Eine für Typ IIB String Theorie bekannte vereinfachte Beschreibung dieser Projektion wird auf diesen Fall angewandt. Darüberhinaus werden Korrekturen zur vier-dimensionalen effektiven Wirkung von Typ II String Theorien betrachtet. Diese Korrekturen sind für Kompaktifizierungen auf Calabi-Yau Mannigfaltigkeiten bekannt. Hier werden diese für den allgemeineren Fall einer Kompaktifizierung auf einer beliebigen Riemann'schen Mannigfaltigkeit berechnet.

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1 Introduction

The Standard Model of particle physics has proven to be very successful in describing the physics of elementary particles (see e.g. [1]). Kinematics and interactions governed by electroweak and strong interactions have been implemented with great success in the framework of quantum field theory and the predictions of the Standard Model have been validated with a high degree of precision. Nevertheless, it contains a number of unsolved issues. The most important ones are the lack of a consistent description of gravity within quantum field theory and the high number of 19 free parameters in the Standard Model. This indicates, that the Standard Model cannot serve as a fundamental theory, but rather as a low-energy effective version of a more general theory.

String theory together with supersymmetry forms an extension of the Standard Model that possibly offers a way out of these issues. Supersymmetry (see e.g. [2, 3, 4]) extends the Poincaré group by a symmetry that exchanges bosons and fermions. In the minimal supersymmetric Standard Model (MSSM) the particle spectrum is doubled, as no particle of the Standard Model can transform into another Standard Model particle and thus all Standard Model particles have transform in particles not included in the Standard Model. One of the most outstanding predictions of the MSSM is the unification of the three gauge couplings at a scale of order 10^{16} GeV, which supports the idea of an underlying unified theory that governs the physics beyond the Standard Model scale. Moreover, the lightest supersymmetry partner of a Standard Model particle may be a candidate for dark matter.

In the Standard Model all elementary particles are considered to be point particles. String theory (see e.g. [5, 6, 7, 8]) on the contrary is build from regarding one-dimensional extended objects, the strings, as the fundamental objects. Strings can be either closed or open. Each vibrational mode of a string is interpreted as a different particle. It was shown, that string theory only yields a spectrum containing bosons and fermions, if it is extended to a supersymmetric theory. Supersymmetric string theories may serve as a more fundamental theory, containing both the Standard Model and gravity and being less arbitrary than the Standard Model. Five consistent superstring theories can be constructed: Type I, IIA, IIB theory and heterotic string theory with gauge group $SO(32)$ or $E_8 \times E_8$. In the low energy limit, that is the limit of letting the string length shrink to zero, these yield effective supergravity field theories. This thesis will be concerned with aspects of this effective theories.

A major drawback of all string theories is on the other hand, that consistency imposes strong restrictions on the background they are constructed on. In the weak-coupling limit, a large class of consistent backgrounds can be viewed as a ten-dimensional space-times. In order to build realistic models, string theories have to be "compactified" using the Kaluza-Klein mechanism, that is assuming ten-dimensional space-time to be a product of four-dimensional space-time with a compact six-dimensional "internal" manifold. In this thesis, compactifications will be considered, that lead to $N = 2$ supersymmetric theories in four dimensions. In [9] it was argued, that manifolds with $SU(3)$-structure meet this requirement.[1] Calabi-Yau manifolds are a subclass of these manifolds, that are additionally Ricci-flat. The discussion in this thesis will begin with compactifications on these manifolds, as their discussion is less complex. The more general case of compactifications on manifolds with $SU(3)$-manifolds will be treated afterwards.

The research on open strings has revealed the importance of objects extended in more then one dimensions, called D-branes, defined as submanifolds of space-time on which open stings can end. D-branes are of outstanding relevance for the construction of realistic string theory models, as they can be used to build theories with non-Abelian gauge groups, required in order to reproduce the verified predictions of the Standard Model. However, string theory with D-branes can only be consistently compactified to four dimensions, when further extended objects, called orientifold planes, are present.

Theories with orientifolds are constructed by modding out states by a discrete symmetry transformation of the theory, implemented by the orientifold projection operator. This truncation additionally projects out half of the supersymmetry generators. The orientifold compactifications discussed in this thesis will be based on type IIA theory. The truncation of the light particle spectrum have been calculated explicitly in [10] for Calabi-Yau and in [11] for $SU(3)$-structure manifold compactifications. The truncated spectra were deduced by determining the fields of the low-energy actions that are invariant under action of the orientifold projection operator. As both compactifications yield $N = 2$ supersymmetry in four dimensions, the orientifold projection reduces supersymmetry to $N = 1$.

The light spectra contain a number of $N = 2$ hypermultiplets, which are truncated to $N = 1$ chiral multiplets by the orientifold projection. However, the fields constituting the $N = 1$ multiplets can not be identified straightforwardly [11, 10]. In [12] an alternative description of the truncation of the hypermultiplets was developed and demonstrated to coincide with the results of [13] for type IIB Calabi-Yau orientifolds. The authors of [12] use the fact, that the hypermultiplet action can be given in terms of a superconformal action, i.e. an action being invariant under

[1] The most general manifolds that lead to $N = 2$ supersymmetry after compactification are manifolds with two $SU(3)$-structures [9]. However, for simplicity the discussion in this thesis will be restricted to the case where both $SU(3)$-structures coincide, i.e. to manifolds with $SU(3)$-structure.

supersymmetry as well as conformal transformations. In the superconformal action the $N = 1$ supermultiplets that remain in the spectrum after applying the orientifold projection can easily be identified. In this thesis, it will be shown, that the same method can be applied to type IIA theory. Furthermore, it will be argued, that the same procedure can be adapted to type IIA theory compactified on manifolds with $SU(3)$-structure.

Beside the aspects of orientifold compactifications a second feature of type II string theories will be discussed in this thesis. It is known, that tree-level and one-loop corrections lead to additional terms in the low-energy effective action. In this thesis, the attention will be drawn to corrections to the graviton action in ten-dimensions. These are proportional to a sum of different contractions of four Riemann tensors and thus referred to as R^4-term corrections. When compactified on a Calabi-Yau manifold, these terms yield corrections to all fields that originate from compactifying the Einstein term to four dimensions [14, 15]. It is known how these corrections descend to the four-dimensional action when compactified on a Calabi-Yau manifold. In this thesis, these corrections will be compactified on a generic six-dimensional manifold and their contributions to the Einstein term and the kinetic term of the volume of the internal manifold will be determined. Particularly, the calculation will not be restricted Ricci-flat manifolds and thus the results also apply for compactifications on manifolds with $SU(3)$-structure.

This thesis divides into four sections:

- To start with, in section 2 the main aspects of type II string theories and their compactifications will be reviewed. The low-energy effective action and the particle spectrum obtained after compactifying the theory will be given.
- Second, action for a number of $N = 2$ hypermultiplets will be discussed in section 3. In particular, the connection between a superconformal hypermultiplet action and an action, invariant only under the super Poincaré group will be reviewed. Furthermore, the application of this connection to string theory will be discussed.
- The description of the hypermultiplet sector of type IIA theory on a Calabi-Yau manifold or a manifold with $SU(3)$-structure in term of superconformal quantities given in section 3 will be used in section 4 to present a convenient formulation of the truncation of the spectrum induced by the orientifold projection. This possibility has been pointed out in [12] for type IIB string theory. Here, it will be shown, that their calculations can be easily transferred to type IIA theory.
- R^4-term corrections will be discussed in section 5. Their contributions to the action in four-dimensions will be calculated for type IIA and IIB theory

compactified on an arbitrary Riemannian manifold. It will be shown, that only the corrections to the Einstein term assumes the same form as in the Calabi-Yau case. The corrections to the volume kinetic term differ from the Calabi-Yau case, but can be absorbed by a redefinition of the ten-dimensional metric.

Additionally, the supersymmetry algebra and its representations used in the main text are reviewed in appendix A and appendix B summarizes the fundamental definitions of complex geometry. Furthermore, appendix C contains additional details on type IIA compactifications and in appendix D supplemental informations on the calculations in section 5 are given.

2 Type II string theories

The goal of this section is to outline the results of string theory that will be necessary to present the calculations and their results in section 4 and 5. In particular, the low-energy limit of string theory, i.e. the limit of letting the length of the strings shrink to zero, will be subject of the discussion. General references for this subject are [8, 6].

Five consistent superstring theories can be constructed: Type I, IIA, IIB theory and heterotic string theory with gauge group $SO(32)$ or $E_8 \times E_8$. Here, only type IIA/B theories will be treated. Both are $N = 2$ supersymmetric theories. They impose strong constraints on the background constructed on. In the weak-coupling limit, both predict a flat ten-dimensional space-time. Thus, in the following, the background will be assumed to be a ten-dimensional manifold. This obviously forms an obstacle for building realistic theories. The solution to this issue will be discussed separately below.

2.1 Low-energy effective actions

In the low energy limit, both type II theories can be approximated by an effective field theory. Their bosonic part is build from the following fields:

- The metric $\hat{g}$,
- a real scalar field ϕ, the dilaton,
- and a 2-form field $\hat{B}_2$,

which are common to type IIA and IIB. In addition, both contain a number of form fields

- type IIA theory: $\hat{C}_1$, $\hat{C}_3$ and
- type IIB theory: $\hat{C}_0$, $\hat{C}_2$, $\hat{C}_4$,

where $\hat{C}_p$ denotes p-form fields. The fermionic fields will not be treated here for simplicity, but as the bosonic fields listed above are components of some supermultiplet, the couplings of the fermionic fields are determined via supersymmetry. The low energy effective action for the bosonic fields is given by [8]

$$\begin{aligned} S^{\text{IIA}} = & \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{|G|} e^{-2\phi} \left[R + 4(\partial_M \phi)(\partial^M \phi) - \tfrac{1}{2}(|H_3|^2 + |F_2|^2 + |\tilde{F}_4|^2) \right] \\ & - \frac{1}{4\kappa_{10}^2} \int B_2 \wedge F_4 \wedge F_4 \, , \end{aligned} \tag{2.1}$$

$$S^{\mathrm{IIB}} = \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{|G|}e^{-2\phi}\Big[R + 4(\partial_M\phi)(\partial^M\phi) \\ -\tfrac{1}{2}(|H_3|^2 + |F_1|^2 + |\tilde{F}_3|^2 + \tfrac{1}{2}|\tilde{F}_5|^2)\Big] \\ - \frac{1}{4\kappa_{10}^2}\int C_4 \wedge H_3 \wedge F_3 \ , \tag{2.2}$$

where $M = 0, \ldots, 9$, κ_{10} denotes the ten-dimensional gravitational coupling constant and

$$\begin{aligned} &F_{p+1} = d\hat{C}_p \quad \text{for } p = 1, \ldots, 4 \ , && H_3 = dB_2 \ , \\ &\tilde{F}_3 = F_3 - \hat{C}_0 H_3 \ , && \tilde{F}_4 = F_3 + \hat{C}_1 \wedge H_3 \ , \\ &\tilde{F}_5 = F_5 - \tfrac{1}{2}\hat{C}_2 \wedge H_3 + \tfrac{1}{2}\hat{B}_2 \wedge F_3 && \end{aligned} \tag{2.3}$$

are field strengths of the fields $\hat{C}_i$, $\hat{B}_2$ and their norm is given by $|F_p| = \frac{1}{i!}(F_p)^{M_1 \ldots M_p}(F_p)_{M_1 \ldots M_p}$ and analogously for H_3.

2.2 Compactifications

As the background is assumed to be ten-dimensional space-time, it is not straightforward how one can construct realistic string theory models, which require a four-dimensional space-time background. In order to construct theories in four dimensions, the Kaluza-Klein mechanism will be applied to the effective actions (2.1) and (2.2). That is, the ten-dimensional space-time will be assumed to be a fibre bundle over ordinary four-dimensional space time $M_{1,3}$. Realistic four-dimensional theories can be obtained in this way only if the fibre Y is compact. Here all calculations will be restricted to the simpler case, where the ten-dimensional metric becomes block diagonal

$$G_{MN} = \begin{pmatrix} g_{\mu\nu}(x) & 0 \\ 0 & \gamma_{ij}(x,y) \end{pmatrix} \ , \qquad \begin{aligned} &\mu, \nu = 0, \ldots, 3 \\ &i, j = 4, \ldots, 9 \end{aligned} \tag{2.4}$$

where x^μ are coordinates on the four-dimensional space-time and y^i in turn coordinates on the "internal" manifold Y.

2.2.1 Calabi-Yau compactifications

The relevance for considering compactifications on Calabi-Yau manifolds (see appendix B) is to obtain an effective theory in four dimensions with $N = 2$ supersymmetry. Furthermore, the metric of the internal manifold should solve the Einstein equation. This imposes restrictions on the geometry of Y, namely it has to be a Ricci-flat Kähler manifold or equivalently a manifold with $SU(3)$ holonomy (for a general reference see for example [6]). This class of manifolds are called Calabi-Yau manifolds.

In order to obtain an effective four-dimensional action one starts by separating the modes of the fields in parts depending solely on internal or four-dimensional space-time coordinates. In the following the discussion will be restricted to the modes, which are massless with respect to the internal wave operator. These are given by the harmonic forms on Y. Higher excitations are assumed to be too massive to contribute to the low energy effective theory.

The spaces of harmonic p-forms will be denoted by $H^p(Y)$ and analogously the space of harmonic (p,q)-forms by $H^{(p,q)}$. For Calabi-Yau manifolds the dimensions $h^{(p,q)}$ of $H^{(p,q)}$, the Hodge numbers, are strongly restricted. Namely for a Calabi-Yau manifold of complex dimension three (see, for example, [16])

$$h^{(p,q)} = h^{(q,p)} = h^{(3-p,3-q)} \ , \quad h^{(1,0)} = h^{(2,0)} = 0 \ , \quad h^{(0,0)} = h^{(3,0)} = 1 \ . \tag{2.5}$$

Thus, there are no harmonic one- or five-forms on a Calabi-Yau manifold and the only independent parameters are $h^{(2,1)}$ and $h^{(1,1)}$.

In the following the basis elements of the spaces of harmonic forms will be denoted by:

- $H^{(1,1)} = \text{span}\{\omega_{\hat{a}}\} \quad (\hat{a} = 1, \dots, h^{(1,1)})$
- $H^{(2,2)} = \text{span}\{\tilde{\omega}_{\hat{a}}\} \quad (\hat{a} = 1, \dots, h^{(2,2)} = h^{(1,1)})$
- $H^{(1,2)} = \text{span}\{\chi_K\} \quad (K = 1, ..., h^{2,1})$
- $H^3 = \text{span}\{\alpha_\Lambda, \beta^\Lambda\} \quad (\Lambda = 0, \dots, h^{(2,1)})$, as $\dim H^3 = h^3 = 2h^{(2,1)} + 2$.
  The basis of H^3 is chosen to be symplectic, i.e.

  $$\int \alpha_\Lambda \wedge \beta^\Sigma = \delta_\Lambda{}^\Sigma \ , \tag{2.6}$$

  with all other intersections vanishing.

Expanding the fields of type IIA/IIB theory in terms of harmonic forms on Y gives [17]

$$\begin{aligned} \hat{C}_1 &= A^0(x) \ , \qquad \hat{B}_2 = B_2(x) + b^{\hat{a}}(x)\omega_{\hat{a}} \ , \\ \hat{C}_3 &= A^{\hat{a}}(x) \wedge \omega_{\hat{a}} + \xi^\Lambda(x)\alpha_\Lambda - \tilde{\xi}_\Lambda(x)\beta^\Lambda \ , \end{aligned} \tag{2.7a}$$

and analogously for type IIB [18]

$$\begin{aligned} \hat{C}_2 &= C_2(x) + c^{\hat{a}}(x)\omega_{\hat{a}} \ , \qquad \hat{B}_2 = B_2(x) + b^{\hat{a}}\omega_{\hat{a}} \ , \\ \hat{C}_4 &= D_2^{\hat{a}}(x)\omega_A + V^\Lambda(x) \wedge \alpha_\Lambda - U_\Lambda(x) \wedge \beta^\Lambda + \rho_{\hat{a}}(x)\tilde{\omega}^{\hat{a}} \ . \end{aligned} \tag{2.7b}$$

multiplet	**type IIA**		**type IIB**	
gravity multiplet	1	$(g_{\mu\nu}, A^0)$	1	$(g_{\mu\nu}, V_0)$
vector multiplets	$h^{(1,1)}$	$(A^{\hat{a}}, v^{\hat{a}}, b^{\hat{a}})$	$h^{(2,1)}$	(V^K, z^K)
hypermultiplets	$h^{(2,1)}$	$(z^K, \xi^K, \tilde{\xi}_K)$	$h^{(1,1)}$	$(v^{\hat{a}}, b^{\hat{a}}, c^{\hat{a}}, \rho_{\hat{a}})$
tensor multiplet	1	$(B_2, \phi, \xi_0, \tilde{\xi}_0)$		
double tensor multiplet			1	(B_2, C_2, ϕ, C_0)

Table 2.1: Number of $N = 2$ multiplets for type IIA/B string theory compactified on a Calabi-Yau manifold and their bosonic components [8].

Furthermore, the kinetic terms of the internal metric has to be translated into fields in four dimensions [8]:

$$\gamma_{i\bar{j}} = iv^{\hat{a}}(x)\,(\omega_{\hat{a}})_{i\bar{j}}\,, \tag{2.8a}$$

$$\gamma_{ij} = i\bar{z}^K(x)\left(\frac{(\bar{\chi}_K)_{i\bar{m}\bar{n}}\Omega^{\bar{m}\bar{n}j}}{||\Omega||^2}\right)\,, \qquad ||\Omega||^2 = \tfrac{1}{3!}\Omega_{ijk}\bar{\Omega}^{ijk}\,, \tag{2.8b}$$

where Ω is the holomorphic $(3,0)$-form on Y. It can be shown [8], that varying $v^{\hat{a}}$ is equivalent to changing the Kähler form on Y and varying z^K results in a modification of the complex structure of Y. Thus, the fields $v^{\hat{a}}$ and z^K will be referred to as deformations of the Kähler form and complex structure, respectively.

Under supersymmetry transformations, the fields in (2.7) form a number of different multiplets. The complete spectra is given (without proof) in table 2.1. Finally, to end up with a four-dimensional action, the integral in the ten-dimensional action over the internal manifold will be performed, resulting in effective actions in four dimensions. Here, only the effective action for type IIA theory will be treated, as the four-dimensional effective action of type IIB theory (see [19, 18, 20]) is not required for the discussion in the following sections.

The four-dimensional action for type IIA theory compactified on a Calabi-Yau manifold is

Type IIA theory [17]

$$\begin{aligned} S^{(4)}_{\mathrm{IIA}} = &\int d^4x\sqrt{|g|}(-\tfrac{1}{2}\mathcal{R} + \mathcal{L}^{HM}_{IIA} - G^J_{\hat{a}\hat{b}}(\partial^\mu t^{\hat{a}})(\partial_\mu \bar{t}^{\hat{a}})) \\ &+ \int \left[\tfrac{1}{4}\mathrm{Im}\mathcal{N}_{\hat{a}\hat{b}}F^{\hat{a}} \wedge *F^{\hat{b}} + \tfrac{1}{4}\mathrm{Re}\mathcal{N}_{\hat{a}\hat{b}}F^{\hat{a}} \wedge *F^{\hat{b}}\right]\,, \end{aligned} \tag{2.9}$$

with

$$\begin{aligned}\mathcal{L}_{IIA}^{HM} = &-(\partial_\mu D)^2 - G^{cs}_{K\bar{L}}\partial_\mu z^K \partial^\mu \bar{z}^L - \tfrac{1}{4}e^{4D}(\partial_\mu a - (\tilde{\xi}_\Lambda \partial_\mu \xi^\Lambda - \xi^\Lambda \partial_\mu \tilde{\xi}_\Lambda))^2 \\ &+ \tfrac{1}{2}e^{2D}(\mathrm{Im}\mathcal{M})^{-1\,\Lambda\Sigma}(\partial_\mu \tilde{\xi}_\Lambda - \mathcal{M}_{\Lambda\Pi}\partial_\mu \xi^\Pi)(\partial^\mu \tilde{\xi}_\Sigma - \bar{\mathcal{M}}_{\Sigma\Gamma}\partial^\mu \xi^\Gamma)\ .\end{aligned} \tag{2.10}$$

Here $\mathcal{R}$ is the Ricci scalar on four-dimensional space-time and the field strengths F^Λ are defined analogously to their ancestors in ten-dimensions $F^{\hat{a}} = dA^{\hat{a}}$. The vector multiplet scalars have been redefined as

$$t^{\hat{a}} := b^{\hat{a}} + iv^{\hat{a}} \tag{2.11}$$

and their couplings are encoded in the matrices $\mathcal{N}$ and G^J. These are given explicitly in appendix C.

The hypermultiplet sector of the Lagrangian is summarized in the term $\mathcal{L}_{IIA}^{HM}$. The scalar a arises after dualising the tensor B_2 and therewith the tensor-multiplet to a further hypermultiplet (see appendix A). Furthermore, the dilaton has been redefined as

$$e^D = e^\phi\,(\mathrm{vol}(Y))^{-\frac{1}{2}}\ . \tag{2.12}$$

The kinetics terms of the scalars z^K are determined by

$$G^{cs}_{K\bar{L}} = \partial_{z^K}\partial_{\bar{z}^L}K^{cs}\ , \quad K^{cs} = -\ln i\left[\bar{Z}^\Lambda \mathcal{F}_\Lambda - Z^\Lambda \bar{\mathcal{F}}_\Lambda\right]\ , \tag{2.13}$$

where the quantities Z^Λ and $\mathcal{F}_\Lambda$ are related to the geometry of Y via

$$Z^\Lambda(z) = \int_Y \Omega(z)\wedge\beta^\Lambda\ , \qquad \mathcal{F}_\Lambda(z) = \int_Y \Omega(z)\wedge\alpha_\Lambda\ . \tag{2.14}$$

Furthermore, it can be shown, that $\mathcal{F}_\Lambda$ is the partial derivative of a function $\mathcal{F}$ with respect to Z^Λ and $\mathcal{F}(Z)$ has to be holomorphic and homogeneous of degree two. Thus,

$$\mathcal{F} = 2\mathcal{F}_\Lambda Z^\Lambda\ , \qquad \mathcal{F}_\Lambda = \partial_{Z^\Lambda}\mathcal{F} = Z^\Sigma(\partial_{Z^\Sigma}\partial_{Z^\Lambda}\mathcal{F})\ . \tag{2.15}$$

The matrix $\mathcal{M}$ is given in terms of Z^Λ and $\mathcal{F}_{\Lambda\Sigma} := \partial_\Lambda\partial_\Sigma\mathcal{F}$ by

$$\mathcal{M}_{\Lambda\Sigma} = \bar{\mathcal{F}}_{\Lambda\Sigma} + 2i\frac{(\mathrm{Im}\mathcal{F})_{\Lambda\Pi}Z^\Pi\,(\mathrm{Im}\mathcal{F})_{\Sigma\Xi}Z^\Xi}{Z^\Pi(\mathrm{Im}\mathcal{F})_{\Pi\Xi}Z^\Xi}\ . \tag{2.16}$$

2.2.2 Generalized Compactifications

A more general framework for string theory compactifications is provided by manifolds with $SU(3)$-structure [9]. Calabi-Yau compactifications discussed in the last section provide the most general background, that leads to $N = 2$ supersymmetry in fours dimensions and obeys the equation of motion for the metric. Manifolds with $SU(3)$-structure provide a background for string theory compactifications, that equally leads to $N = 2$ supersymmetry in four dimensions, but in general their metrics do not solve the equations of motion.

Manifolds with $SU(3)$-structure can be defined as manifolds, on which a globally non-vanishing spinor η can be defined. This spinor can be used to decompose the supersymmetry parameters in ten dimensions as

$$\begin{aligned} \epsilon_1 &= \varepsilon_+^1 \otimes \eta_- + \varepsilon_-^1 \otimes \eta_+ \, , \\ \epsilon_2 &= \varepsilon_+^2 \otimes \eta_\pm + \varepsilon_-^2 \otimes \eta_\mp \, . \end{aligned} \tag{2.17}$$

The upper sign in the second equation holds for type IIA theory, whereas the lower sign applies to type IIB theory and the following decomposition of the ten-dimensional gamma matrices $\Gamma^A = (\Gamma^\mu, \Gamma^m)$ have been used:

$$\Gamma^\mu = \gamma^\mu \otimes \mathbf{1} \, , \quad \mu = 0,1,2,3 \, , \quad \Gamma^m = \gamma^5 \otimes \gamma^m \, , \quad m = 1, \ldots, 6 \, , \tag{2.18}$$

with $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. This decomposition of the supersymmetry parameters selects eight of the 32 supercharges of the $N = 2$ theory in ten dimensions, which descend to the effective theory in four dimensions, leading to $N = 2$ supersymmetry. Calabi-Yau manifolds are incorporated in this more general class of compactifications and correspond a manifold with $SU(3)$-structure, whose two spinors coincide everywhere and are covariantly constant.

The compactification on manifolds with $SU(3)$-structure resembles the process described for Calabi-Yau manifolds. The procedure is in principle the same, but in contrast to the Calabi-Yau case the light fields generally can not be identified with the harmonic forms on Y [9]. Thus, it will be assumed, that there is a finite-dimensional subspace $\Lambda^p_{\text{finite}}$ of the space of p-forms, $\Lambda^p TY$, that is in one-to-one correspondence with the light modes of the four-dimensional theory. The fields of the ten-dimensional theory will be expanded in elements of this subspaces. The basis for these spaces will be labelled:

- $\Lambda^2 TY_{\text{finite}} = \text{span}\{\omega_{\hat{a}}\} \quad (\hat{a} = 1, \ldots, b_J)$
- $\Lambda^3 TY_{\text{finite}} = \text{span}\{\alpha_\Lambda, \beta^\Lambda\} \quad (\Lambda = 0, \ldots, b_\rho)$.
- $\Lambda^4 TY_{\text{finite}} = \text{span}\{\tilde{\omega}_{\hat{a}}\} \quad (\hat{a} = 1, \ldots, b_J)$

Furthermore, it will be assumed in the following, that no one- and five-forms on Y contribute to the light modes, i.e. $\Lambda^1 TY_{\text{finite}} = \Lambda^5 TY_{\text{finite}} = \emptyset$.

The expressions for the fields $\hat{C}_p$ and $\hat{B}_2$ are altogether the same as in the Calabi-Yau case, (2.7a) and (2.7b), except for the substitution of $H^p(Y)$ by $\Lambda^P_{\text{finite}}$. The only effect on the spectrum is that the number of multiplets is no longer given by the hodge numbers of the internal manifold, but by the dimensions of $\Lambda^p_{\text{finite}}$.

The main difference to Calabi-Yau compactifications is the parameterization of the deformations of the internal metric: As manifolds with $SU(3)$-structure are generally not complex manifolds, the decomposition of the deformations of its metric as in (2.8) cannot be simply adopted. Instead, the deformations of the internal metric will be parameterized by means of the spinor η: At first these can be used to define a two- and a three form on Y by

$$J^{mn} = \mp 2i\bar{\eta}_\pm \gamma^{[m}\gamma^{n]}\eta_\pm \,, \tag{2.19}$$

$$\Omega^{mnp}_\eta = -2i\bar{\eta}_- \gamma^{[m}\gamma^n\gamma^{p]}\eta_+ \,, \quad \bar{\Omega}^{mnp}_\eta = -2i\bar{\eta}_+ \gamma^{[m}\gamma^n\gamma^{p]}\eta_- \,. \tag{2.20}$$

It can be shown, that the deformations of the metric can be completely specified in terms of variations of J and Ω. These in turn will be parameterized by

$$iJ = -iv^{\hat{a}}\omega_{\hat{a}} \,, \quad \Omega = Z^\Lambda \alpha_\Lambda - \mathcal{F}_\Lambda \beta^\Lambda \,. \tag{2.21}$$

The names of the parameters $v^{\hat{a}}$, Z^Λ and $\mathcal{F}_\Lambda$ are chosen in the style of the fields appearing in Calabi-Yau compactifications, as they enter the effective action in the same way. These complete the spectrum of type IIA theory compactified on a manifold with $SU(3)$-structure. The resulting spectrum is summarized in table 2.2. The kinetic terms of the effective action in four-dimensions assumes the same form as in the Calabi-Yau case, (2.9) for type IIA theory.

multiplet	**type IIA**		**type IIB**	
gravity multiplet	1	$(g_{\mu\nu}, A^0)$	1	$(g_{\mu\nu}, V_0)$
vector multiplets	b_J	(A^A, v^A, b^A)	b_ρ	(V^Λ, z^Λ)
hypermultiplets	b_ρ	$(z^\Lambda, \xi^\Lambda, \tilde{\xi}_\Lambda)$	b_J	(v^A, b^A, c^A, ρ_A)
tensor multiplet	1	$(B_2, \phi, \xi_0, \tilde{\xi}_0)$		
double tensor multiplet			1	(B_2, C_2, ϕ, C_0)

Table 2.2: Number of $N = 2$ multiplets for type IIA/B string theory compactified on a manifold with $SU(3)$-structure and their bosonic components

One of the most important differences between Calabi-Yau compactifications and compactifications on manifolds with $SU(3)$-structure is, that the effective action of the latter contains additional couplins encoded in a superpotential term [9]. These couplings emerge as a result of the non-closeness of the elements of Λ^p_{finite}. But as the discussion in this thesis will only be concerned with the kinetic terms, the superpotential will not be discussed here.

3 Superconformal Quotient

In this section the connection between superconformal and supersymmetric invariant actions for a number of self-interacting supermultiplets will be reviewed. The discussion will cover tensor and hypermultiplets in four dimensions, as only for these the superconformal formulation will be relevant here. To be precise, it will be shown, that the action for the hypermultiplets of type IIA theory compactified on a Calabi-Yau manifold or manifold with $SU(3)$-structure can be constructed as a superconformal action for tensor multiplets. This relation will be essential for the description of the orientifold projection in the following chapter. This chapter closely follows [21, 22].

3.1 Superconformal actions

The bosonic components of the $N = 2$ tensor multiplet are a complex scalar v, a real scalar x and an anti-symmetric tensor $t_{\mu\nu}$ (see appendix A). The following discussion will again mostly be restricted to the bosonic fields. The bosonic part of a generic Lagrangian for $n + 1$ superconformal tensor multiplets reads

$$\begin{aligned} \mathcal{L}^{TM} &= \mathcal{L}_{x^I x^J}\left(\tfrac{1}{4}(\partial_\mu x^I \partial^\mu x^J - H^J_\mu H^{\mu J}) + \partial_\mu v^I \partial^\mu v^J\right) \\ &\quad + \frac{1}{2} i(\mathcal{L}_{v^I x^J}\partial_\mu v^I - \mathcal{L}_{\bar{v}^I x^J}\partial_\mu \bar{v}^I)H^{\mu J} \qquad (I = \flat, 0, \ldots, n-1)\,, \end{aligned} \tag{3.1}$$

with $H^{\mu I} = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\partial_\nu t^I_{\rho\sigma}$ being the field strengths of $t^I_{\mu\nu}$ and $\mathcal{L}_{x^I x^J} = \partial_{x^I}\partial_{x^J}\mathcal{L}$ (and similarly $\mathcal{L}_{v^I x^J}$) denoting partial derivatives of a function $\mathcal{L}(x^I, v^I, \bar{v}^I)$. The labelling of the fields is chosen in a way that will be convenient in following. The action is completely specified by the function $\mathcal{L}$. However it cannot be chosen arbitrary, in order to yield an action invariant under superconformal transformations. It can be shown, that any function that gives rise to a superconformal action can be written by a contour integral

$$\mathcal{L} = \mathrm{Im} \oint_C \frac{d\zeta}{2\pi i \zeta} H\left(\eta^I(x^\mu, \theta, \bar{\theta}, \zeta)|_{\theta=0}\right)\,, \tag{3.2}$$

where H has to be homogeneous of first degree. $\eta^I(\zeta)$ are projective superfields, defined as

$$\eta^I(x^\mu, \theta, \bar{\theta}, \zeta) = \frac{V^I(x^\mu, \theta, \bar{\theta})}{\zeta} + X^I(x^\mu, \theta, \bar{\theta}) - \zeta \bar{V}^I(x^\mu, \theta, \bar{\theta})\,, \tag{3.3}$$

where V^I are linear and X^I chiral $N = 1$ superfields. Thus,

$$\eta^I(x^\mu, \theta, \bar{\theta}, \zeta)|_{\theta=0} = \frac{v^I}{\zeta} + x^I - \zeta \bar{v}^I\,, \tag{3.4}$$

with

$$x^I = X^I|_{\theta=0} \quad \text{and} \quad v^I = V^I|_{\theta=0} \, . \tag{3.5}$$

In order to establish the connection to a hypermultiplet action, the tensors $t^I_{\mu\nu}$ have to be exchanged by scalar fields y_I by means of the duality between tensor and hypermultiplets in four dimensions (see appendix A.3). After the duality transformation the scalars of the hypermultiplets are v^I and

$$\omega_I = \frac{1}{2}(\mathcal{L}_{x^I} + iy_I) \, . \tag{3.6}$$

The resulting hypermultiplet Lagrangian can be described by a single function $\chi(v, \bar{v}, \omega, \bar{\omega})$

$$\begin{aligned} \mathcal{L} = \quad & \chi_{v^I \bar{v}^J}(\partial_\mu v^I)(\partial^\mu \bar{v}^J) + \chi_{\omega_I \bar{v}^J}(\partial_\mu \omega_I)(\partial^\mu \bar{v}^J) \\ & + \chi_{v^I \bar{\omega}_J}(\partial_\mu v^I)(\partial^\mu \bar{\omega}_J) + \chi_{\omega_I \bar{\omega}_J}(\partial_\mu \omega_I)(\partial^\mu \bar{\omega}_J) \, , \end{aligned} \tag{3.7}$$

where the subscripts again denote partial derivatives. The function χ is given by the Legendre transform of $\mathcal{L}$ with respect to all x^I

$$\chi(v, \bar{v}, x(\omega, \bar{\omega})) = \mathcal{L}(v, \bar{v}, x) - (\omega_I + \bar{\omega}_I)x^I \, . \tag{3.8}$$

3.2 Descending form superconformal to super-Pointcaré symmetry

By extending super-Poincaré to superconformal symmetry, the action becomes invariant under further transformation. A general discussion of the superconformal group can be found for example in [2]. The additional transformations, under which the bosonic component fields change non-trivially, are the dilation transformation acting on the space-time coordinates x^μ as

$$x^\mu \to \lambda \, x^\mu \, , \text{ with } \lambda \in \mathbb{R} \tag{3.9}$$

and an $SU(2)$ transformation of the fields parameterized by $(\varepsilon^3, \varepsilon^\pm = \varepsilon^1 \pm i\varepsilon^2)$ and generated by

$$\delta v^I = i\varepsilon^3 v^I + \varepsilon^+ x^I \, , \; \delta \bar{v}^I = i\varepsilon^3 \bar{v}^I + \varepsilon^- x^I \, , \; \delta\omega_I = \varepsilon^+ \mathcal{L}_{v^I} \, , \; \delta\bar{\omega}_I = \varepsilon^- \mathcal{L}_{\bar{v}^I} \, . \tag{3.10}$$

The fields (v^I, ω_I) can be shown to transform under the dilation transformation with weight 2 and zero respectively

$$v^I \to \lambda^2 v^I \, , \quad \omega_I \to \omega_I \, . \tag{3.11}$$

In order to descend to super-Pointcaré theory, these additional symmetries have to be removed. For this purpose, scale and $SU(2)$ invariant fields will be defined and afterwards both symmetries will be gauged. At this point the field $\eta^\flat$ becomes a compensator, that is, it will only be used to gauge the conformal symmetry by setting $\omega^\flat$ and $v^\flat$ to constant values and defining fields, that are invariant under (3.11)

$$t^\Lambda := \frac{v^\Lambda}{v^\flat} . \tag{3.12}$$

Eventually, this results in an action of the form [21]

$$\begin{aligned} S = \int d^4x \sqrt{|g|} \Big(h_{t^\Lambda \bar{t}^\Sigma} (\partial^\mu t^\Lambda)(\partial_\mu \bar{t}^\Sigma) - \tfrac{1}{2} \mathrm{Re} \left[(\partial_{t^\Lambda} \partial_{\bar{\omega}_\Sigma} K) \, (\partial^\mu t^\Lambda)(\partial_\mu \bar{\omega}_J) \right] \\ - \tfrac{1}{4} \, (\partial_{\omega_\Lambda} \partial_{\bar{\omega}_\Sigma} K) \, (\partial^\mu \omega_\Lambda)(\partial_\mu \bar{\omega}_\Sigma) \Big) \ , \end{aligned} \tag{3.13}$$

with $\Lambda, \Sigma = 0, \ldots, n$ and

$$h_{\omega_\Lambda \bar{\omega}_\Sigma} = -\tfrac{1}{4} \partial_{\omega_\Lambda} \partial_{\bar{\omega}_\Sigma} K + \chi^{-2} |v^\flat|^2 t^\Lambda \bar{t}^\Sigma \ , \tag{3.14}$$

$$K = \ln \left(\frac{\chi(x^I, v^I, \bar{v}^I)}{\sqrt{v^\flat \bar{v}^\flat}} \right) \tag{3.15}$$

and $\omega_\flat$ set to zero in all equations.

3.3 Connection to type IIA string theory compactifications

In [23, 21] it has been shown, that a hypermultiplet Lagrangian of the form (2.10),

$$\begin{aligned} \mathcal{L}^{HM}_{IIA} = & - (\partial_\mu D)^2 - G^{cs}_{K\bar{L}} \partial_\mu z^K \partial^\mu \bar{z}^L - \tfrac{1}{4} e^{4D} (\partial_\mu a - (\tilde{\xi}_\Lambda \partial_\mu \xi^\Lambda - \xi^\Lambda \partial_\mu \tilde{\xi}_\Lambda))^2 \\ & + \tfrac{1}{2} e^{2D} (\mathrm{Im}\mathcal{M})^{-1\,\Lambda\Sigma} (\partial_\mu \tilde{\xi}_\Lambda - \mathcal{M}_{\Lambda\Pi} \partial_\mu \xi^\Pi)(\partial^\mu \tilde{\xi}_\Sigma - \bar{\mathcal{M}}_{\Sigma\Gamma} \partial^\mu \xi^\Gamma) \end{aligned} \tag{3.16}$$

is equal to (3.13) if the the function H is chosen to be

$$H(\eta(\zeta)) = \frac{\mathcal{F}(\eta)}{\eta^\flat} \ , \tag{3.17}$$

where $\mathcal{F}$ is given in (2.15). The contour integral over H in (3.2) can be easily evaluated by examining the poles of the integrand in (3.2). These are located at the roots $\zeta_\pm$ of $\zeta\eta^\flat$

$$\zeta_\pm = \frac{x^\flat \pm |\vec{r}^\flat|}{2\bar{v}^\flat} \ , \tag{3.18}$$

where the vectors $\vec{r}^I$ are defined as

$$\vec{r}^I = (x^I, 2\mathrm{Re}(v^I), 2\mathrm{Im}(v^I)) \ . \tag{3.19}$$

The value of η^Λ $(\Lambda = 1, \dots, n)$ at these roots will be important below and are given by

$$\eta^\Lambda_\pm := \eta^\Lambda(\zeta_\pm) = x^\Lambda - \frac{x^\flat}{2}\left(\frac{v^\Lambda}{v^\flat} + \frac{\bar{v}^\Lambda}{\bar{v}^\flat}\right) \pm \frac{|\vec{r}^\flat|}{2}\left(-\frac{v^\Lambda}{v^\flat} + \frac{\bar{v}^\Lambda}{\bar{v}^\flat}\right) . \tag{3.20}$$

In order to obtain the action (2.10), the contour C has to enclose only the pole ζ_+. Performing the integration in (3.2) using the Residue theorem yields

$$\mathcal{L} = \mathrm{Im}\oint_C \frac{d\zeta}{2\pi i\zeta} H(\eta^I(\zeta)) = \frac{1}{|\vec{r}^\flat|}\mathrm{Im}(\mathcal{F}(\eta^\Lambda_+)) . \tag{3.21}$$

Calculating the Legendre transform of $\mathcal{L}$ gives [22]

$$\chi = -\frac{2v^\flat\bar{v}^\flat}{|r^\flat|^3}\,\eta^\Lambda_+\,(\mathrm{Im}\mathcal{F}_{\Lambda\Sigma})\,\eta^\Sigma_- . \tag{3.22}$$

Additionally, the fields appearing in the Lagrangian (2.10) have to be given in terms of the fields $(v^\Lambda, \omega_\Lambda)$ [22]:

$$e^{-2D} = \frac{\chi}{4r^\flat} , \tag{3.23a}$$

$$z^K = \eta^K_+/\eta^0_+ , \tag{3.23b}$$

$$\xi^\Lambda = \frac{\vec{r}^\flat \cdot \vec{r}^\Lambda}{2(r^\flat)^2} , \tag{3.23c}$$

$$\tilde{\xi}_\Lambda = -\mathrm{Im}\, w_\Lambda + \frac{x^\flat}{2(r^\flat)^2}\,\mathrm{Re}\,\mathcal{F}_\Lambda(\eta_+) , \tag{3.23d}$$

$$a = -\mathrm{Im}\,\omega_0 - \mathrm{Im}\,\frac{v^\Lambda w_\Lambda}{2v^\flat} - \frac{x^\flat}{4(r^\flat)^2}\,\mathrm{Re}\left(\eta^\Lambda_+\tilde{\xi}_\Lambda - \mathcal{F}_\Lambda(\eta_+)\xi^\Lambda\right) , \tag{3.23e}$$

where x^I has to be understood as a function of v^I and w^I:

$$x^I(v,w) = \frac{\partial\chi}{\partial\chi_I} \quad , \quad \chi_I = w_I + \bar{w}_I . \tag{3.24}$$

4 Orientifold projection of type IIA theory compactifications

In the discussion of open strings, higher dimensional extended objects, D-branes [8], appear, on which open string can end. These also carry physical degrees of freedom. A compactification of theory containing strings as well as D-branes can not be performed consistently without introducing further extended objects into the theory, the orientifold planes. These will be subject of this section.

4.1 Definition of the projection operator

Orientifold planes (see e.g. [24]) are defined as fixed points of an involutive isometry σ of the internal space Y and are labelled by their dimension p as $O(p-1)$-planes. They truncate the spectrum to states, which are invariant under the orientifold projection operator $\mathcal{O}$, defined by the composition of σ with the world-sheet parity operator Ω_p, that is the operator interchanging left and right moving modes on the string. For type IIA theory, only $O6$-planes can be introduced consistently. The orientifold projection operator for these planes is defined as

$$\mathcal{O} = \Omega_p(-1)^{F_L}\,\sigma\ , \text{ with } \quad \sigma^* J = -J\ , \tag{4.1}$$

where F_L is the number of left-moving modes on the string. For Calabi-Yau compactifications J is the Kähler form of the internal manifold and for compactifications on $SU(3)$-structure manifolds the two-form defined in (2.19). Additionally, applying the orientifold projection reduces the amount of supersymmetry. In order to not completely break supersymmetry, but reduce supersymmetry only to $N = 1$ the restriction

$$\sigma^*\Omega = e^{2i\theta}\bar{\Omega}\ , \tag{4.2}$$

has to be imposed, where in the Calabi-Yau case Ω is the holomorphic three-form and in the case of a $SU(3)$-structure manifold the tree-form defined in (2.19) and $e^{2i\theta}$ is a constant phase. In the following it will be assumed, that θ is set to zero by a redefinition of Ω.

4.2 $N = 1$ spectrum and effective action

In this section the orientifold projection of type IIA theory compactified on a Calabi-Yau or a manifold with $SU(3)$-structure will be described using the formalism of superconformal projective superfields. In [12] it was shown, that the effect of the orientifold projection on the hypermultiplet sector of type IIB theory compactified on a Calabi-Yau manifold can easily be described using projective superfields. Here it will be shown, that the same also applies for type IIA theory on Calabi-Yau manifolds as well as $SU(3)$-structure manifolds.

As shown in the previous section, a description of the hypermultiplet part of the Lagrangian (2.9) of type IIA theory compactified on a Calabi-Yau or manifold with $SU(3)$-structure (2.10) can be given in terms of superconformal tensor multiplets. The effect of the orientifold projection on the spectrum and the associated action have been computed in [10, 11] by examining the behavior of each field under the action of the orientifold projection operator. Here, it will be shown, that the same spectrum and action can be obtained by requiring the projective superfields $\eta^I(\zeta)$ to be either parity-even or parity-odd under $\eta(\zeta) \to \eta(-\zeta)$.

As the hypermultiplet scalars arise from expanding the three forms C_3 and Ω, the spectrum of the orientifolded theory depends substantially on the way the orientifold projection effects the three-forms on the internal manifold. In order to discuss the orientifold projection of both, compactifications on Calabi-Yau and manifolds with $SU(3)$-structure, on the same footing, from now on for Calabi-Yau compactifications the notation of the spaces of three-forms corresponding to light modes will be adapted to the $SU(3)$-structure manifold case, i.e. $\Lambda^3 TY_{\text{finite}} \equiv H^3(Y)$ for Y being Calabi-Yau.

The orientifold projection splits $\Lambda^3 TY_{\text{finite}}$ into two subspaces of different parity $\Lambda^3 TY_{\text{finite}} = \Lambda^3_+ TY_{\text{finite}} \oplus \Lambda^3_- TY_{\text{finite}}$. Both subspaces have the same dimension [10, 11]. To see this, one first notes that $J \wedge J \wedge J$ is proportional to the volume form. As $\sigma^* J = -J$, for both compactifications, the volume form is odd under the orientifold projection. Equally, $\alpha_\Lambda \wedge \beta^\Lambda$ (no summation) is proportional to the volume form, and therefore

$$\alpha_\Lambda \in \Lambda^3_\pm TY_{\text{finite}} \Rightarrow \beta^\Lambda \in \Lambda^3_\mp TY_{\text{finite}} \, . \tag{4.3}$$

Thus, $\dim(\Lambda^3_+ TY_{\text{finite}}) = \dim(\Lambda^3_- TY_{\text{finite}}) = \frac{1}{2}\dim(\Lambda^3 TY_{\text{finite}})$. In the following, the basis of $\Lambda^3 TY_{\text{finite}}$ is assumed to split into

$$\begin{aligned} &(a_k, b^\lambda) \in \Lambda^3_+ TY_{\text{finite}} \, , & \quad & \lambda = 0, \ldots, \hat\Lambda \, , \\ &(a_\lambda, b^k) \in \Lambda^3_- TY_{\text{finite}} \, , & \text{with} \quad & k = \hat\Lambda + 1, \ldots, \dim(\Lambda^3 TY_{\text{finite}}) - 1 \, , \end{aligned} \tag{4.4}$$

and $0 \leqslant \hat\Lambda \leqslant n_H - 1$.

The goal of the following calculations will be to show, that the effect of the orientifold projection on the spectrum and Kähler potential of the theory can be equally obtained by demanding the projective superfields describing the hypermultiplet sector to have the parities

$$\eta^\flat(\zeta) = -\eta^\flat(-\zeta) \, , \tag{4.5a}$$

$$\eta^\lambda(\zeta) = -\eta^\lambda(-\zeta)\,, \tag{4.5b}$$

$$\eta^k(\zeta) = \quad \eta^k(-\zeta)\,. \tag{4.5c}$$

These conditions are equivalent to setting

$$x^\flat = w_\flat = 0\,, \quad x^\lambda = w_\lambda = 0 \quad \text{and} \quad v^k = 0\,. \tag{4.6}$$

The fact that $x^\flat$, x^λ and v^k have to vanish is obvious, due to (3.5). Furthermore, from (3.3) and (4.5a) one can see, that the entire $N = 1$ linear superfield $X^\flat$ and thus the corresponding $N = 1$ tensor multiplet is projected out. Therefore, the $N = 1$ chiral multiplet dual to this $N = 1$ tensor multiplet is projected out equally and the complex scalar of this $N = 1$ chiral multiplet $\omega_\flat$ has to vanish as well. The same argument can be applied to the scalars ω_λ.

With help of (3.23) one can identify the $N = 2$ component fields that are projected out:

$$\mathrm{Im}\,\eta^\lambda_+ = \frac{2\,\mathrm{Im}(\bar{v}^\lambda v^\flat)}{\sqrt{v^\flat \bar{v}^\flat}}\,, \qquad \mathrm{Re}\,\eta^\lambda_+ = 0\,, \tag{4.7a}$$

$$\mathrm{Re}\,z^\lambda = \frac{\mathrm{Im}\,(\bar{v}^\lambda v^\flat)}{\mathrm{Im}\,(\bar{v}^0 v^\flat)}\,, \qquad \mathrm{Im}\,z^\lambda = 0\,, \tag{4.7b}$$

$$\mathrm{Im}\,\eta^k_+ = 0\,, \qquad \mathrm{Re}\,\eta^k_+ = x^k\,, \tag{4.7c}$$

$$\mathrm{Re}\,z^k = 0\,, \qquad \mathrm{Im}\,z^k = x^k \frac{\sqrt{v^\flat \bar{v}^\flat}}{\mathrm{Im}(v^0 \bar{v}^\flat)}\,, \tag{4.7d}$$

$$\xi^\lambda = \mathrm{Re}\left(\frac{v^\lambda}{v^\flat}\right)\,, \qquad \xi^k = 0\,, \tag{4.7e}$$

$$\tilde{\xi}_\lambda = 0\,, \qquad \tilde{\xi}_k = -2\mathrm{Im}\,w_k\,. \tag{4.7f}$$

$$a = 0 \tag{4.7g}$$

Thus, fields $(\mathrm{Im}\,z^\lambda), (\mathrm{Re}\,z^k), \xi^k, \tilde{\xi}_\lambda, a$ are removed from the spectrum.

This is the spectrum of the orientifolded theory as calculated in [10, 11] and reduces the $N = 2$ hypermultiplets to $N = 1$ chiral multiplets. Inserting (4.7) into (2.10) yields the action

$$\begin{aligned} \mathcal{L}^{CM} &= (\partial_\mu D)^2 + \tilde{G}_{ab}\,\partial_\mu q^a \partial^\mu q^b - \tfrac{1}{2} e^{2D} (\mathrm{Im}\,M)_{\kappa\lambda} \partial_\mu \xi^\kappa \partial^\mu \xi^\lambda \\ &\quad - \tfrac{1}{2} (\mathrm{Im}\,M)^{-1\,kl} (\partial_\mu \tilde{\xi}_k - \mathrm{Re}\,M_{k\lambda} \partial_\mu \xi^l)(\partial^\mu \tilde{\xi}_l - \mathrm{Re}\,M_{l\kappa} \partial^\mu \xi^\kappa)\,, \end{aligned} \tag{4.8}$$

with the real fields

$$q^{\hat\lambda} = \mathrm{Re}\,z^{\hat\lambda}\,, \quad q^k = \mathrm{Im}\,z^k \quad (\hat\lambda = 1, \ldots, \hat\Lambda) \tag{4.9}$$

and

$$\tilde{G}_{ab} = \begin{pmatrix} G_{\hat{\kappa}\hat{\lambda}} & iG_{\hat{\kappa}l} \\ iG_{k\hat{\lambda}} & G_{kl} \end{pmatrix} \tag{4.10}$$

The kinetic terms for $N = 1$ chiral multiplets can be specified by a single function, the Kähler potential K (see appendix A). The main advantage of describing the orientifold projection using projective superfields, is that the Kähler potential associated with (4.8) can easily be derived. For that purpose it is useful not to switch from the fields (v^I, ω_I) to the set $(D, q^a, \xi^\lambda, \tilde{\xi}_k)$. The action for the hypermultiplets in terms of the fields (v^I, ω_I), before applying any projection, reads (see section 3)

$$\begin{aligned} S = \int d^4x\sqrt{|g|}\, \Big(h_{t^\Lambda \bar{t}^\Sigma}(\partial^\mu t^\Lambda)(\partial_\mu \bar{t}^\Sigma) - \tfrac{1}{2}\mathrm{Re}\left[(\partial_{t^\Lambda}\partial_{\bar{\omega}_\Sigma}K)\,(\partial^\mu t^\Lambda)(\partial_\mu \bar{\omega}_J)\right] \\ -\tfrac{1}{4}\,(\partial_{\omega_\Lambda}\partial_{\bar{\omega}_\Sigma}K)\,(\partial^\mu \omega_\Lambda)(\partial_\mu \bar{\omega}_\Sigma)\Big)\ , \end{aligned} \tag{4.11}$$

with

$$h_{\omega_\Lambda \bar{\omega}_\Sigma} = -\tfrac{1}{4}\partial_{\omega_\Lambda}\partial_{\bar{\omega}_\Sigma}K + \chi^{-2}|v^\flat|^2 t^\Lambda \bar{t}^\Sigma\ . \tag{4.12}$$

By inserting (4.6) the second term in (4.12) vanishes, as the index ranges of t^Λ and ω_Λ reduce and become disjoint. Thus, the action for the orientifolded hypermultiplet sector is given by

$$\begin{aligned} S = -\tfrac{1}{4}\int d^4x\sqrt{|g|}(\partial_{t^\lambda}\partial_{\bar{t}^\kappa}K)(\partial^\mu t^\lambda)(\partial_\mu \bar{t}^\kappa) + (\partial_{t^\lambda}\partial_{\bar{\omega}_k}K)(\partial^\mu t^\lambda)(\partial_\mu \bar{\omega}_k) + \\ (\partial_{\omega_k}\partial_{\bar{t}^\lambda}K)(\partial^\mu \omega_k)(\partial_\mu \bar{t}^\lambda) + (\partial_{\omega_l}\partial_{\bar{\omega}_l}K)(\partial^\mu \omega_k)(\partial_\mu \bar{\omega}_l)\ . \end{aligned} \tag{4.13}$$

Obviously,

$$-\tfrac{1}{4}K = -\tfrac{1}{4}\ln\left[|v^\flat|^{-1}\,\chi\right] \tag{4.14}$$

serves as a Kähler potential for the chiral multiplets. Using (3.22) it can be rewritten as

$$-\tfrac{1}{4}K = -\tfrac{1}{4}\ln\left[\mathrm{Im}(t^\lambda)\mathrm{Im}(\mathcal{F}_{\lambda\kappa})\,\mathrm{Im}(t^\kappa) - \frac{1}{4v^\flat \bar{v}^\flat}x^k\,\mathrm{Im}(\mathcal{F}_{kl})\,x^l\right]\ . \tag{4.15}$$

Using (4.6) and (3.23a) the Kähler potential (3.15) can be cast into a form comparable with the result of [10, 11]:

$$-\tfrac{1}{4}K = -\tfrac{1}{4}\ln\left(16\,e^{-2D}\right) = -\ln(e^{-8D}) - \tfrac{1}{4}\ln 16 \tag{4.16}$$

The coordinates used in [10, 11] are

$$N^\lambda = \tfrac{1}{2}\,\xi^\lambda + i\,\tfrac{1}{4|v^\flat|}\,\mathrm{Im}(\eta_+^\lambda)\,, \quad T_k = \tfrac{|v^\flat|}{4}\mathrm{Im}(\mathcal{F}_k) + i\,\tilde{\xi}_k\,. \tag{4.17}$$

To connect these to the fields (v^λ, ω_k) one can easily show using (4.7a) and (4.7e)

$$N^\lambda = \tfrac{1}{2}\bar{t}^\lambda \tag{4.18}$$

Furthermore, (3.6), (3.21) and (4.7f) can be used to show

$$T_k = \frac{1}{2}\bar{\omega}_k \tag{4.19}$$

Thus, the change of variables from (v_λ, ω_k) to (N^λ, T_k) is anti-holomorphically and the action (4.13) can equally be formulated for the fields N^λ and T^k:

$$S = -\int d^4x\sqrt{|g|}(\partial_{N^\lambda}\partial_{\bar{N}^\kappa}K)(\partial^\mu N^\lambda)(\partial_\mu \bar{N}^\kappa) + (\partial_{N^\lambda}\partial_{\bar{T}_k}K)(\partial^\mu N^\lambda)(\partial_\mu \bar{T}_k) + \tag{4.20}$$

$$(\partial_{T_k}\partial_{\bar{N}^\lambda}K)(\partial^\mu T_k)(\partial_\mu \bar{N}^\lambda) + (\partial_{T_l}\partial_{\bar{T}_l}K)(\partial^\mu N_k)(\partial_\mu \bar{T}_l)\,, \tag{4.21}$$

with the Kähler potential

$$-K = -\ln\left(e^{-2D}\right) - \ln 16\,. \tag{4.22}$$

Thus, the orientifold projection of type IIA theory can be equally implemented in the formalism of superconformal tensor calculus as described in this section.

5 R^4-term corrections in type II theories

In this section, one-loop and tree-level corrections to the four-dimensional effective action will be discussed. It can be shown, that these corrections lead to additional couplings in the effective action, that are build of contractions of four Riemann tensors. R^4-terms provide corrections to the four-dimensional Einstein term as well as kinetic terms of moduli of the internal manifold. In the following the corrections to the Einstein term as well as the kinetic term of the volume of the internal manifold will be calculated. These correction are already known for Calabi-Yau compactifications [14, 15]. In this section, these terms will be calculated for a compactification on a generic Riemannian manifold. Particularly, the restriction of Ricci-flatness will not be imposed. Thus, the results obtained in the following will also be valid for compactifications on manifolds with $SU(3)$-structure discussed in the previous sections.

5.1 R^4-term corrections in ten dimensions

As the calculations below strongly rely on the Riemann tensor, its definition and basic properties will be reviewed first: The Riemann tensor is a measure for the curvature of a manifold and defined using the metric connection [25]

$$\Gamma^M_{NP} = \tfrac{1}{2} G^{MK} (\partial_N G_{PK} + \partial_P G_{NK} - \partial_K G_{NP}) \,, \tag{5.1}$$

where G is the metric of the manifold. The Riemann curvature tensor is given by

$$\begin{aligned} R_{MNPQ} = {} & \tfrac{1}{2}(\partial^2_{MQ} G_{NP} + \partial^2_{NP} G_{MQ} - \partial^2_{MP} G_{NQ} - \partial^2_{NQ} G_{MP}) && (5.2) \\ + \quad & G_{KL}(\Gamma^K_{MQ}\Gamma^L_{NP} - \Gamma^K_{MP}\Gamma^L_{NQ}) \,. && (5.3) \end{aligned}$$

The Riemann tensors obviously exhibits the symmetries

$$R_{MNPQ} = -R_{MNQP} \,, \quad R_{MNPQ} = R_{PQMN} \,. \tag{5.4}$$

Furthermore, it obeys the first Bianchi identity

$$R_{MNPQ} + R_{MPQN} + R_{MQNP} = 0 \,. \tag{5.5}$$

These equations will be crucial for the calculations below.

The starting point for considering R^4-term corrections are the ten-dimensional low-energy effective actions for type IIA/B theory

$$\begin{aligned} S^{\mathrm{IIA}} = {} & \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{|G|} e^{-2\phi} \Big[R + 4(\partial_\mu \phi)(\partial^\mu \phi) - \tfrac{1}{2}(|H_3|^2 + |F_2|^2 + |\tilde{F}_4|^2) \Big] \\ & - \frac{1}{4\kappa_{10}^2} \int B_2 \wedge F_4 \wedge F_4 + \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{|G|}\, \mathcal{L}^-_{R^4} \,, \end{aligned} \tag{5.6}$$

$$
\begin{aligned}
S^{\mathrm{IIB}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{|G|} e^{-2\phi} \Big[R + 4(\partial_\mu \phi)(\partial^\mu \phi) \\
- \tfrac{1}{2}(|H_3|^2 + |F_1|^2 + |\tilde{F}_3|^2 + \tfrac{1}{2}|\tilde{F}_5|^2) \Big] \\
- \frac{1}{4\kappa_{10}^2} \int C_4 \wedge H_3 \wedge F_3 + \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{|G|}\, \mathcal{L}^{+}_{R^4} ,
\end{aligned} \tag{5.7}
$$

The the R^4-term corrections added are given by [14, 26]

$$
\begin{aligned}
\mathcal{L}^{\pm}_{R^4} &= \alpha'^3 \left[e^{-2\phi} \frac{\zeta(3)}{3 \cdot 2^{11}} \left(t_8 t_8 R^4 + \tfrac{1}{4} E_8 \right) + \frac{\pi^2}{9 \cdot 2^{11}} \left(t_8 t_8 R^4 \pm \tfrac{1}{4} E_8 \right) \right] \\
&= r^{+}\, t_8 t_8 R^4 + \tfrac{1}{4}\, r^{\pm}\, E_8 ,
\end{aligned} \tag{5.8}
$$

with

$$
r^{\pm} = \alpha'^3 \left(e^{-2\phi} \frac{\zeta(3)}{3 \cdot 2^{11}} \pm \frac{\pi^2}{9 \cdot 2^{11}} \right) . \tag{5.9}
$$

In the first line in (5.8), the corrections are separated into one-loop (first term) and tree-level (second term) corrections. The one-loop term is proportional to the Riemann zeta function $\zeta(3)$ [14]. In order to calculate the corrections induced by these terms in the four-dimensional action, it is more convenient to use the shortened form in the second line. The term $t_8 t_8 R^4$ is an abbreviation for the following contraction of the Riemann tensor [14]

$$
t_8 t_8 R^4 := t_8^{\mu_1 \cdots \mu_8} t_8^{\nu_1 \cdots \nu_8} R_{\mu_1 \mu_2 \nu_1 \nu_2} R_{\mu_3 \mu_4 \nu_3 \nu_4} R_{\mu_5 \mu_6 \nu_5 \nu_6} R_{\mu_7 \mu_8 \nu_7 \nu_8} , \tag{5.10}
$$

where t_8 is defined to act on any set of anti-symmetric matrices $M^{i=1,\dots,4}$ as [7]

$$
\begin{aligned}
& t_8^{\mu_1 \nu_1 \mu_2 \nu_2 \mu_3 \nu_3 \mu_4 \nu_4} M^1_{\mu_1 \nu_1} \dots M^4_{\mu_4 \nu_4} := \\
&\quad - 2 \Big[\mathrm{tr}(M^1 M^2) \mathrm{tr}(M^3 M^4) + \mathrm{tr}(M^2 M^3) \mathrm{tr}(M^1 M^4) + \mathrm{tr}(M^1 M^3) \mathrm{tr}(M^2 M^4) \Big] \\
&\quad + 8 \Big[\mathrm{tr}(M^1 M^2 M^3 M^3) + \mathrm{tr}(M^1 M^3 M^2 M^4) + \mathrm{tr}(M^1 M^3 M^4 M^2) \Big]
\end{aligned} \tag{5.11}
$$

and E_8 is defined as

$$
E_8 := \delta^{[\mu_1}_{\nu_1} \cdots \delta^{\mu_8]}_{\nu_8}\, R_{\mu_1 \mu_2}{}^{\nu_1 \nu_2} R_{\mu_3 \mu_4}{}^{\nu_3 \nu_4} R_{\mu_5 \mu_6}{}^{\nu_5 \nu_6} R_{\mu_7 \mu_8}{}^{\nu_7 \nu_8} . \tag{5.12}
$$

The bracket $[\cdots]$ denotes anti-symmetrization of the enclosed indices, i.e. $A^{[M} B^{N]} \equiv A^M B^N - A^N B^M$.

Due to the identities (5.4) and (5.5) a lot of different contractions of the Riemann tensor occurring in the definition of $t_8 t_8 R^4$ and E_8 are identical. Thus, it can be shown, that, as a result of (5.4) and (5.5), one can only construct 26 linearly independent contractions of the Riemann tensor on a manifold, whose dimension is equal to or higher than six [27]. Therefore, $t_8 t_8 R^4$ as well as E_8 can be expanded in a basis of scalars build by contracting four Riemann tensors. The basis chosen for the the calculations in this section is:

$$
\begin{aligned}
&A_{4,a} = R\, A_{3,a} \quad (a = 1, \dots, 8) && \\
&A_{4,9} = R_{NP} R^{N}{}_{R} R^{P}{}_{T} R^{RT} \,, && A_{4,10} = \left(R_{NP} R^{NP}\right)^2 \,, \\
&A_{4,11} = -R_{NP} R^{Q}{}_{T} R^{RT} R^{N}{}_{Q}{}^{P}{}_{R} \,, && A_{4,12} = -R_{NP} R^{N}{}_{R} R^{P}{}_{STU} R^{RSTU} \,, \\
&A_{4,13} = R_{NP} R^{QT} R^{N}{}_{QRS} R^{P}{}_{T}{}^{RS} \,, && A_{4,14} = R_{NP} R^{ST} R^{N}{}_{Q}{}^{P}{}_{R} R^{Q}{}_{S}{}^{R}{}_{T} \,, \\
&A_{4,15} = R_{NP} R^{ST} R^{N}{}_{QRS} R^{PQR}{}_{T} \,, && A_{4,16} = R_{NP} R^{NP} R_{MNPQ} R^{MNPQ} \,, \\
&A_{4,17} = R_{NP} R^{N}{}_{Q}{}^{P}{}_{R} R^{Q}{}_{STU} R^{RSTU} \,, && A_{4,18} = R_{NP} R^{N}{}_{QRS} R^{PQ}{}_{TU} R^{RSTU} \,, \\
&A_{4,19} = R_{NP} R^{N}{}_{QRS} R^{P}{}_{T}{}^{R}{}_{U} R^{QTSU} \,, && A_{4,20} = \left(R_{MNPQ} R^{MNPQ}\right)^2 \,, \\
&A_{4,21} = R_{MNPQ} R^{MNP}{}_{R} R^{Q}{}_{STU} R^{RSTU} \,, && A_{4,22} = R_{MNPQ} R^{MN}{}_{RS} R^{PQ}{}_{TU} R^{RSTU} \,, \\
&A_{4,23} = R_{MNPQ} R^{MN}{}_{RS} R^{PR}{}_{TU} R^{QSTU} \,, && A_{4,24} = R_{MNPQ} R^{MN}{}_{RS} R^{P}{}_{T}{}^{R}{}_{U} R^{QTSU} \,, \\
&A_{4,25} = R_{MNPQ} R^{M}{}_{R}{}^{P}{}_{S} R^{N}{}_{T}{}^{Q}{}_{U} R^{RTSU} \,, && A_{4,26} = R_{MNPQ} R^{M}{}_{R}{}^{P}{}_{S} R^{N}{}_{T}{}^{R}{}_{U} R^{QTSU} \,,
\end{aligned}
\tag{5.13}
$$

where $A_{3,a}$ constitute in turn a basis of scalars build of three Riemann tensors:

$$
\begin{aligned}
&A_{3,1} = R^3 \,, && A_{3,2} = R R_{NP} R^{NP} \,, \\
&A_{3,3} = R R_{MNPQ} R^{MNPQ} \,, && A_{3,4} = -R_{NP} R^{N}{}_{R} R^{PR} \,, \\
&A_{3,5} = R_{NP} R^{QR} R^{N}{}_{Q}{}^{P}{}_{R} \,, && A_{3,6} = R_{NP} R^{N}{}_{QRS} R^{PQRS} \,, \\
&A_{3,7} = R_{MNPQ} R^{MN}{}_{RS} R^{PQRS} \,, && A_{3,8} = R_{MNPQ} R^{M}{}_{R}{}^{P}{}_{S} R^{NRQS}
\end{aligned}
\tag{5.14}
$$

and it was again shown in [27], that there are eight linear independent contractions.

Using the definitions of t_8 and E_8 and repeated use of the symmetries of the Riemann tensor gives[2]

$$
t_8 t_8 R^4 = 12 A_{4,20} - 192 A_{4,21} + 24 A_{4,22} - 384 A_{4,24} + 192 A_{4,25} + 384 A_{4,26} \,, \tag{5.15}
$$

$$
\begin{aligned}
\tfrac{1}{4} E_8 = {} & 4A_{4,1} - 96A_{4,2} + 24A_{4,3} - 256A_{4,4} + 384A_{4,5} - 384A_{4,6} \\
& + 64A_{4,7} - 128A_{4,8} - 384A_{4,9} + 192A_{4,10} + 1536A_{4,11} - 768A_{4,12} \\
& + 384A_{4,13} - 768A_{4,14} + 768A_{4,15} - 96A_{4,16} + 768A_{4,17} - 768A_{4,18} \\
& + 1536A_{4,19} + 12A_{4,20} - 192A_{4,21} + 24A_{4,22} + 192A_{4,23} - 768A_{4,24} \\
& + 192A_{4,25} - 384A_{4,26} \,.
\end{aligned}
\tag{5.16}
$$

[2] The coefficients in (5.15) and (5.16) has been checked using the Invar tensor package for Maple [28].

These expressions were already calculated in [15] for Calabi-Yau manifolds. By setting all terms in (5.15) and (5.16) containing a Ricci tensor to zero, these expressions reduce to the ones calculated in [15]. Inserting the expansions (5.15) and (5.16) into (5.8) gives

$$
\begin{aligned}
\mathcal{L}^{\pm}_{R^4} = r^{\pm}\,[&4A_{4,1} - 96A_{4,2} + 24A_{4,3} - 256A_{4,4} + 384A_{4,5} - 384A_{4,6} \\
&+ 64A_{4,7} - 128A_{4,8} - 384A_{4,9} + 192A_{4,10} + 1536A_{4,11} \\
&- 768A_{4,12} + 384A_{4,13} - 768A_{4,14} + 768A_{4,15} - 96A_{4,16} + 768A_{4,17} \\
&-768A_{4,18} + 1536A_{4,19} + 192A_{4,23}] \\
&+ (r^{+} + r^{\pm})\,[12A_{4,20} - 192A_{4,21} + 24A_{4,22} + 192A_{4,25}] \\
&- 384(r^{+} + 2r^{\pm})A_{4,24} + 384(r^{+} - r^{\pm})A_{4,26}\,.
\end{aligned}
\tag{5.17}
$$

5.2 Descending to four dimensions

As the corrections to the four-dimensional action shall be determined, the R^4-terms have to be compactified just as the rest of the Lagrangian. The background considered for the compactification below is a product manifold $M_{1,3} \times Y$ with a metric of the form

$$
G_{MN} = \begin{pmatrix} g_{\mu\nu}(x) & 0 \\ 0 & v_6^{\frac{1}{3}}(x)\,\gamma_{ij}(y) \end{pmatrix}\,, \quad \text{with} \quad \int_Y d^6y\sqrt{|\det\gamma|} = 1\,, \tag{5.18}
$$

with $v_6(x)$ being the volume of Y. The internal manifold Y can be any Riemannian manifold.

In the following the range of the indices is $M, N, P, \dots \in \{0, \dots, 9\}$, used on tensors and vectors defined on the complete ten-dimensional space-time, and $i, j, k, \dots \in \{4, \dots, 9\}$, $\mu, \nu, \rho, \dots \in \{0, \dots, 3\}$ for quantities defined on the internal manifold and four-dimensional space-time, respectively.

The Riemann tensor on the ten-dimensional space-time can be calculated in terms of the Riemann tensors $\mathcal{R}_{\mu\nu\rho\sigma}$ and $\hat{R}_{ijkl}$ constructed using the metrics $g_{\mu\nu}$ and γ_{ik}, respectively, by inserting the metric (5.18) into the definition of the Riemann tensor (5.2). This gives [15]:

$$
R_{ijkl} = v_6^{\frac{1}{3}}\,\hat{R}_{ijkl} - \frac{1}{36} v_6^{\frac{2}{3}} (\partial_\mu \ln v_6)^2\,(\gamma_{ik}\gamma_{jl} - \gamma_{il}\gamma_{jk})\,, \tag{5.19a}
$$

$$
R_{\mu i \nu j} = -\frac{1}{36}\gamma_{ij}\,v_6^{\frac{1}{3}}\,(6\nabla_\mu\nabla_\nu \ln v_6 + (\partial_\mu \ln v_6)(\partial_\nu \ln v_6))\,, \tag{5.19b}
$$

$$
R_{\mu\nu\rho\sigma} = \mathcal{R}_{\mu\nu\rho\sigma}\,. \tag{5.19c}
$$

All other components ($R_{ijk\mu}$, $R_{\mu\nu\rho i}$ and $R_{ij\mu\nu}$) vanish. Moreover, it will be useful to calculate the Ricci tensor $R_{MN} := R^K{}_{MKN}$ and scalar $R := R^M{}_M$:

$$R_{ij} = \hat{R}_{ij} - \frac{1}{6} v_6^{\frac{1}{3}} \left((\partial_\mu \ln v_6)^2 + \nabla^\mu \nabla_\mu \ln v_6 \right) \gamma_{ij} \; , \tag{5.20a}$$

$$R_{\mu\nu} = \mathcal{R}_{\mu\nu} - \frac{1}{6} (\partial_\mu \ln v_6)(\partial_\nu \ln v_6) - \nabla_\mu \nabla_\nu \ln v_6 \; , \tag{5.20b}$$

$$R_{i\mu} = 0 \; , \tag{5.20c}$$

$$R = \mathcal{R} + v_6^{-\frac{1}{3}} \hat{R} - \frac{7}{6} (\partial^\mu \ln v_6)(\partial_\mu \ln v_6) - 2\, \nabla^\mu \nabla_\mu \ln v_6 \; . \tag{5.20d}$$

Here, also the Ricci tensors $\mathcal{R}_{\mu\nu}$, $\hat{R}_{ij}$ and Ricci scalars $\mathcal{R}$, $\hat{R}$ of the four-dimensional space-time and internal manifold respectively appear.

In the following corrections to the Einstein-term and the volume kinetic terms will be calculated. The relevant uncorrected part of the action is obtained from the ten-dimensional Einstein term, which is the same for type IIA and IIB theory. Using (5.20d) and integrating the last term in (5.20d) by parts one obtains [15]:

$$S_{\mathcal{R},v_6} = \int d^4x \sqrt{|g|} v_6 e^{-2\phi} \left(\mathcal{R} + \tfrac{5}{6} (\partial_\mu \ln v_6)(\partial^\mu \ln v_6) \right) \; . \tag{5.21}$$

Corrections to this action terms arise from terms, that are a product of either $\mathcal{R}$ or $(\partial_\mu \ln v_6)^2$ and a contraction of three Riemann tensors of the internal manifold only. Terms which depend on contractions of more than one Riemann tensors on four-dimensional space-time, e.g. $R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} R_{ijkl} R^{ijkl}$, or which are of higher order in $(\partial_\mu \ln v_6)^2$ will be omitted, as they do not contribute to one of the kinetic terms, but merely to higher couplings.

5.2.1 Corrections to the Einstein term

The corrections to the kinetic term of the four-dimensional space-time metric are induced by terms proportional to the ten-dimensional Ricci-scalar, as due to (5.19) and (5.20) only these contain a Riemann tensor on the four-dimensional space-time $\mathcal{R}$. Thus, the relevant terms are $A_{4,1}, \ldots, A_{4,8}$, which are defined as $RA_{3,1}, \ldots, RA_{3,8}$. Each of these contribute a correction proportional to $v_6^{-1} \mathcal{R} \hat{A}_{3,a}$:

$$RA_{3,a} \to \begin{cases} 4\, v_6^{-1}\, \mathcal{R}\, \hat{A}_{3,a} & \text{for } a = 1 \; , \\ 2\, v_6^{-1}\, \mathcal{R}\, \hat{A}_{3,a} & \text{for } a = 2, 3 \; , \\ \, v_6^{-1}\, \mathcal{R}\, \hat{A}_{3,3} & \text{for } a = 4, ..., 8 \; , \end{cases} \tag{5.22}$$

with $\hat{A}_{3,a}$ being defined analogous to $A_{3,a}$ as in (5.14), but with R_{MNKL} replaced by $\hat{R}_{ijkl}$, the Riemann tensor on the internal manifold Y. All contractions defining $A_{3,a}$

in (5.14) remain linear independent after the substitution, as this was proven for any manifold of minimum dimension six and Y having dimension six [27]. Restricting to corrections of this form gives

$$\mathcal{L}^{\pm}_{\mathcal{R}} = -2r^{\pm}\mathcal{R}\, v_6^{-1}\hat{E}_6 + \cdots \,, \tag{5.23}$$

where E_6 is defined analogous to E_8:

$$\hat{E}_6 := \delta^{[i_1}_{j_1}\cdots\delta^{i_6]}_{j_6}\hat{R}_{i_1i_2}{}^{j_1j_2}\hat{R}_{i_3i_4}{}^{j_3j_4}\hat{R}_{i_5i_6}{}^{j_5j_6} \tag{5.24}$$

and can be equally expanded in the basis $\{\hat{A}_{3,a}\}$:

$$\begin{aligned} E_6 = 8\hat{A}_{3,1} - 96\hat{A}_{3,2} + 24\hat{A}_{3,3} - 128\hat{A}_{3,4} + 192\hat{A}_{3,5} - 192\hat{A}_{3,6} \\ + 32\hat{A}_{3,7} - 64\hat{A}_{3,8}\,. \end{aligned} \tag{5.25}$$

The integration over the internal manifold will be performed below, after the remaining corrections are collected.

5.2.2 Corrections to the volume kinetic term

Correction to the volume kinetic term are affected by all possible contractions of the Riemann tensors. In order to determine their exact form, the contribution of all scalars $A_{4,a}$ need to be identified. By inserting (5.19) and (5.20) into the definitions of $A_{4,a}$ (5.13) one obtains

$$\begin{aligned}
&A_{4,1} \to -\tfrac{14}{3}\mathcal{A}\hat{A}_{3,1}\,, && A_{4,2} \to -\tfrac{1}{6}\mathcal{A}(2\hat{A}_{3,1} + 7\hat{A}_{3,2}),\\
&A_{4,3} \to -\tfrac{1}{9}\mathcal{A}(\mathcal{A}\hat{A}_{3,1} + 21\hat{A}_{3,3})\,, && A_{4,4} \to -\tfrac{1}{6}\mathcal{A}(-3\hat{A}_{3,2} + 7\hat{A}_{3,4})\,,\\
&A_{4,5} \to -\tfrac{1}{6}\mathcal{A}(\hat{A}_{3,3} + 7\hat{A}_{3,5})\,, && A_{4,6} \to -\tfrac{1}{9}\mathcal{A}(\hat{A}_{3,2} + \tfrac{3}{2}\hat{A}_{3,3} + \tfrac{21}{2}\hat{A}_{3,6})\,,\\
&A_{4,7} \to -\tfrac{1}{6}\mathcal{A}(7\hat{A}_{3,7} - \hat{A}_{3,3})\,, && A_{4,8} \to -\tfrac{1}{24}\mathcal{A}(2\hat{A}_{3,2} - \hat{A}_{3,3} + 28\hat{A}_{3,8})\,,\\
&A_{4,9} \to \tfrac{2}{3}\mathcal{A}\hat{A}_{3,4}\,, && A_{4,10} \to -\tfrac{2}{3}\mathcal{A}\hat{A}_{3,2}\,,\\
&A_{4,11} \to -\tfrac{1}{36}\mathcal{A}(-\hat{A}_{3,2} + 5\hat{A}_{3,4} - 12\hat{A}_{3,5})\,, && A_{4,12} \to -\tfrac{1}{9}\mathcal{A}(\hat{A}_{3,4} - 3\hat{A}_{3,6})\,, \qquad (5.26)\\
&A_{4,13} \to -\tfrac{1}{9}\mathcal{A}(\hat{A}_{3,5} + 3\hat{A}_{3,6})\,, && A_{4,14} \to -\tfrac{1}{18}\mathcal{A}(\hat{A}_{3,2} + 5\hat{A}_{3,5})\,,\\
&A_{4,15} \to -\tfrac{1}{18}\mathcal{A}(-\hat{A}_{3,4} + \hat{A}_{3,5} + 6\hat{A}_{3,6})\,, && A_{4,16} \to -\tfrac{1}{9}\mathcal{A}(\hat{A}_{3,2} + 3\hat{A}_{3,3})\,,\\
&A_{4,17} \to -\tfrac{1}{36}\mathcal{A}(\hat{A}_{3,3} + 4\hat{A}_{3,5} + 5\hat{A}_{3,6})\,, && A_{4,18} \to -\tfrac{1}{6}\mathcal{A}(\hat{A}_{3,6} + \hat{A}_{3,7})\,,\\
&A_{4,19} \to \tfrac{1}{54}\mathcal{A}(\hat{A}_{3,4} - \tfrac{1}{3}\hat{A}_{3,5} + \hat{A}_{3,6} - 4\hat{A}_{3,8}), && A_{4,20} \to -\tfrac{2}{9}\mathcal{A}\hat{A}_{3,3}\,,\\
&A_{4,21} \to -\tfrac{2}{9}\mathcal{A}\hat{A}_{3,6}\,, && A_{4,22} \to -\tfrac{2}{9}\mathcal{A}\hat{A}_{3,7}\,,\\
&A_{4,23} \to -\tfrac{1}{9}\mathcal{A}\hat{A}_{3,7}\,, && A_{4,24} \to -\tfrac{1}{36}\mathcal{A}(2\hat{A}_{3,6} - \hat{A}_{3,7} + 4\hat{A}_{3,8})\,,\\
&A_{4,25} \to -\tfrac{1}{36}\mathcal{A}(4\hat{A}_{3,5} + \hat{A}_{3,7} - 4\hat{A}_{3,8})\,, && A_{4,26} \to -\tfrac{1}{36}\mathcal{A}(4\hat{A}_{3,6} - \hat{A}_{3,7} + 4\hat{A}_{3,8})\,,
\end{aligned}$$

with $\mathcal{A} = v_6^{-1}(\partial_\mu \ln v_6)^2$, omitting terms containing higher orders in $(\partial_\mu v_6)$ or the Riemann tensors of the four-dimensional space-time, as described above. By inserting (5.26) in (5.17) the corrections to the volume kinetic term can be computed:

$$\begin{aligned}\mathcal{L}^{\pm}_{v_6} = \frac{8}{3} v_6^{-1}(\partial_\mu \ln v_6)^2 \Big[& -r^+ \hat{E}_6 + 8\, r^+ \hat{A}_{3,1} - 96\, r^+ \hat{A}_{3,2} + (16\, r^+ - 48\, r^{\pm})\hat{A}_{3,3} \\ & -128\, r^+ \hat{A}_{3,4} + 128\, r^+ \hat{A}_{4,5} - 128\, r^+ \hat{A}_{3,6} \Big] \,.\end{aligned} \tag{5.27}$$

5.3 Integration over the internal manifold

Eventually, to obtain the corrected form of the four dimensional effective action, the integral over the internal manifold Y has to be performed. The only terms in (5.23) and (5.27) well manageable are those proportional to $\hat{E}_6$, as the integral over $\hat{E}_6$ is proportional to the Euler characteristic χ of the internal manifold by the Gauss-Bonnet theorem (see e.g. [25]). But, as will be show in the next lines, all other terms in (5.27) can be eliminated by a redefinition of the ten-dimensional metric. To be precise, the goal will be to alter the composition of $\mathcal{L}^{\pm}_{R^4}$ in terms of the scalars $A_{4,a}$ so that the relevant corrections to the volume kinetic term (5.27) become proportional to $\hat{E}_6$ and the corrections to the Einstein term (5.23) remain proportional to $\hat{E}_6$:

$$\mathcal{L}^{\pm}_{\mathcal{R}} + X^{\mathcal{R}} \stackrel{!}{=} c\, v_6^{-1} \mathcal{R}\, \hat{E}_6 \,, \tag{5.28a}$$

$$\mathcal{L}^{\pm}_{v_6} + X^{v_6} \stackrel{!}{=} d\, v_6^{-1} (\partial_\mu \ln v_6)^2\, \hat{E}_6 \,, \tag{5.28b}$$

where $X^{\mathcal{R}}$ and X^{v_6} are terms induced by the redefinition of the metric and will be specified in a moment.

The relevant redefinition is

$$G_{AB} \to G_{AB} + X_{AB} \,, \tag{5.29}$$

with X_{AB} being a symmetric tensor build of three Riemann tensors. Under (5.29) the action transforms as [29] (see also appendix D)

$$\int d^{10}x\sqrt{G}R \to \int d^{10}x\sqrt{G}\left(R + \left(\frac{1}{2}RG_{AB} - R_{AB}\right)X^{AB} + \mathcal{O}(RX^2)\right) \,. \tag{5.30}$$

In ten dimensions there are 24 linear independent two-tensors constructed of three Riemann tensors and the metric [27]. The expansion of X_{AB} in a basis of these tensors is

$$X_{AB} = \quad x_1\, R_{AB} R^2 + x_2\, R R_A{}^M R_{BM}$$

$$
\begin{aligned}
&+ x_3\, R_{AB}R_{MN}R^{MN} + x_4\, R_{AM}R_{BN}R^{MN} \\
&+ x_5\, RR^{MN}R_{AMBN} + x_6\, R^{MN}R_M{}^P R_{NAPB} \\
&+ x_7\, R^{MN}R^P{}_A R_{MPNB} + x_8\, RR_{MNPA}R^{MNP}{}_B \\
&+ x_9\, R_{AB}R_{MNPQ}R^{MNPQ} + x_{10}\, R_{AM}R^{NPQM}R_{NPQB} \\
&+ x_{11}\, R^{MN}R_{PQMA}R^{PQ}{}_{NB} + x_{12}\, R_{MN}R^{MPNQ}R_{PAQB} \\
&+ x_{13}\, R_{MN}R^{PMQ}{}_A R_P{}^N{}_{QB} + x_{14}\, R^{MNPQ}R_{MNRA}R_{PQ}{}^R{}_B \\
&+ x_{15}\, R^{MNPQ}R_{MRPA}R_N{}^R{}_{QB} + x_{16}\, R^{MNPQ}R_{MNPR}R_{QA}{}^R{}_B \\
&+ x_{17}\, G_{AB}\, A_{3,1} + x_{18}\, G_{AB}\, A_{3,2} + x_{19}\, G_{AB}\, A_{3,3} + x_{20}\, G_{AB}\, A_{3,4} \\
&+ x_{21}\, G_{AB}\, A_{3,5} + x_{22}\, G_{AB}\, A_{3,6} + x_{23}\, G_{AB}\, A_{3,7} + x_{24}\, G_{AB}\, A_{3,8}\,.
\end{aligned}
\tag{5.31}
$$

As $R_{ijk\mu}$, $R_{\mu\nu\rho i}$ and $R_{ij\mu\nu}$ as well as $R_{i\mu}$ vanish, the block diagonal form of the metric will not be altered by (5.29), but $g_{\mu\nu} \to g_{\mu\nu} + X_{\mu\nu}$ and $\gamma_{ij} \to \gamma_{ij} + v_6^{-1/3} X_{ij}$. The form of $X_{\mu\nu}$ and X_{ij} can be read off (5.31). The terms induced by the redefinition of the metric are calculated easily

$$
\begin{aligned}
(\tfrac{1}{2}RG_{AB} - R_{AB})X^{AB} = {} & (\tfrac{1}{2}x_1 + 4x_{17})A_{4,1} + (\tfrac{1}{2}x_3 + \tfrac{1}{2}x_2 - x_1 + 4x_{18} + \tfrac{1}{2}x_5)A_{4,2} \\
& + (\tfrac{1}{2}x_9 + \tfrac{1}{2}x_8 + 4x_{19})A_{4,3} + (-\tfrac{1}{2}x_4 - \tfrac{1}{2}x_6 + x_2 + 4x_{20})A_{4,4} \\
& + (-x_5 + \tfrac{1}{2}x_7 + \tfrac{1}{2}x_{12} + 4x_{21})A_{4,5} \\
& + (-x_8 + 4x_{22} + \tfrac{1}{2}x_{13} + \tfrac{1}{2}x_{11} + \tfrac{1}{2}x_{10} + \tfrac{1}{2}x_{16})A_{4,6} \\
& + (\tfrac{1}{2}x_{14} + 4x_{23})A_{4,7} + (4x_{24} + \tfrac{1}{2}x_{15})A_{4,8} - x_4A_{4,9} \\
& - x_4A_{4,10} + (x_7 + x_6)A_{4,11} + x_{10}A_{4,12} - x_{11}A_{4,13} \\
& - x_{12}A_{4,14} - x_{13}A_{4,15} - x_9A_{4,16} - x_{16}A_{4,17} \\
& - x_{14}A_{4,18} - x_{15}A_{4,19}\,.
\end{aligned}
\tag{5.32}
$$

The corrections to the kinetic terms contributed in this way can be determined similarly to the calculations above by reducing to terms proportional to $\mathcal{R}$ and using (5.26), respectively:

$$
\begin{aligned}
X^{\mathcal{R}} = {} & \hat{A}_{3,1}(\tfrac{1}{2}x_14 + x_{17}) + \hat{A}_{3,2}(-x_1 + \tfrac{1}{2}x_2 + \tfrac{1}{2}x_3 + \tfrac{1}{2}x_5 + 4x_{18}) \\
& + \hat{A}_{3,3}(\tfrac{1}{2}x_8 + \tfrac{1}{2}x_9 + 4x_{19}) + \hat{A}_{3,4}(x_2 - \tfrac{1}{2}x_4 - \tfrac{1}{2}x_6 + 4x_{20}) \\
& + \hat{A}_{3,5}(-x_5\tfrac{1}{2}x_7 + \tfrac{1}{2}x_{12} + 4x_{21}) \\
& + \hat{A}_{3,6}(-x_8\tfrac{1}{2}x_{10} + \tfrac{1}{2}x_{11} + \tfrac{1}{2}x_{13} + \tfrac{1}{2}x_{16} + 4x_{22}) \\
& + \hat{A}_{3,7}(\tfrac{1}{2}x_{14} + 4x_{23}) + \hat{A}_{3,8}(\tfrac{1}{2}x_{15} + 4x_{24})\,,
\end{aligned}
\tag{5.33}
$$

$$
\begin{aligned}
72X^{v_6} = {} & \hat{A}_{3,1}(-12x_2 - 12x_3 - 10x_5 - c_7 - 4x_8 - 4x_9 - 144x_1 - x_{12} \\
& - 1344x_{17} - 96x_{18} - 32x_{19} - 8x_{21}) \\
& + \hat{A}_{3,2}(168x_1 - 48x_2 - 36x_3 - 18x_4 - 62x_5 - 16x_6 - 9x_7 + 8x_8
\end{aligned}
$$

$$
\begin{aligned}
&\quad + 8x_9 - 4x_{10} - 4x_{11} - 7x_{12} - 4x_{13} - 3x_{15} - 4x_{16} - 672x_{18} \\
&\quad + 144x_{20} - 88x_{21} - 32x_{22} - 24x_{24}) \\
&+ \hat{A}_{3,3}(-72x_8 - 72x_9 - 6x_{10} - 6x_{11} - 6x_{13} - 6x_{14} + \tfrac{3}{2}x_{15} \\
&\quad - 4x_{16} - 672x_{19} - 48x_{22} - 48x_{23} + 12x_{24}) \\
&+ \hat{A}_{3,4}(-84x_2 - 6x_4 + 32x_6 - 10x_7 - 8x_{10} - 4x_{13} - 2x_{15} - 336x_{20}) \\
&+ \hat{A}_{3,5}(84x_5 + 24x_6 - 18x_7 + 8x_{11} + 4x_{13} - 22x_{12} + 4x_{15} + 8x_{16} - 336x_{21}) \\
&+ \hat{A}_{3,6}(84x_8 - 18x_{10} - 18x_{11} - 18x_{13} + 12x_{14} - 3x_{15} - 32x_{16} - 336x_{22}) \\
&+ \hat{A}_{3,7}(-30x_{14} - 336x_{23}) \ + \hat{A}_{3,8}(-30x_{15} - 336x_{24})
\end{aligned} \tag{5.34}
$$

Inserting these expressions into (5.28) results in 16 equations (eight for each coefficient of $A_{3,a}$ ($a = 1, \ldots, 8$)), restricting the 24 parameters in the expansion of X_{AB}:

$$
\begin{aligned}
d &= 0 \\
x_1 &= 4\,c - 8\,r^{\pm} - 8\,x_{17} \\
x_2 &= -88\,c + 176\,r^{\pm} - x_3 - x_5 - 16\,x_{17} - 8\,x_{18} \\
x_4 &= 15248/7\,c + 2752/7\,r^{+} - 33184/7\,r^{\pm} - 86/7\,x_3 + 2\,x_5 - x_7 + 1312/7\,x_{17} \\
&\quad + 656/7\,x_{18} - 328/7\,x_{20} + 272/7\,x_{21} \\
x_6 &= -14688/7\,c - 2752/7\,r^{+} + 32064/7\,r^{\pm} + 72/7\,x_3 - 4\,x_5 + x_7 - 1536/7\,x_{17} \\
&\quad - 768/7\,x_{18} + 384/7\,x_{20} - 272/7\,x_{21} \\
x_8 &= -1152/7\,c - 688/7\,r^{+} + 2976/7\,r^{\pm} + 60/7\,x_3 - 384/7\,x_{17} - 192/7\,x_{18} \\
&\quad - 8\,x_{19} + 96/7\,x_{20} - 96/7\,x_{21} \\
x_9 &= 1320/7\,c + 688/7\,r^{+} - 3312/7\,r^{\pm} - 60/7\,x_3 + 384/7\,x_{17} \\
&\quad + 192/7\,x_{18} - 96/7\,x_{20} + 96/7\,x_{21} \\
x_{10} &= -122872/7\,c - 26352/7\,r^{+} + 274640/7\,r^{\pm} + 1002/7\,x_3 - 14\,x_5 + 7\,x_7 + x_{11} \\
&\quad - 12864/7\,x_{17} - 6432/7\,x_{18} + 3216/7\,x_{20} - 2824/7\,x_{21} \\
x_{12} &= 384\,c - 768\,r^{\pm} + 2\,x_5 - x_7 - 8\,x_{21} \\
x_{13} &= 119872/7\,c + 27008/7\,r^{+} - 272000/7\,r^{\pm} - 1152/7\,x_3 + 14\,x_5 - 7\,x_7 - 2\,x_{11} \\
&\quad + 13824/7\,x_{17} + 6912/7\,x_{18} - 3456/7\,x_{20} + 3064/7\,x_{21} \\
x_{14} &= 224\,c + 64\,r^{+} - 448\,r^{\pm} \\
x_{15} &= -448\,c - 128\,r^{+} + 896\,r^{\pm} \\
x_{16} &= -6912/7\,c + 21888/7\,r^{\pm} - 3456/7\,r^{+} + 360/7\,x_3 - 2304/7\,x_{17} - 1152/7\,x_{18} \\
&\quad + 576/7\,x_{20} - 576/7\,x_{21} \\
x_{22} &= 615/7\,c + 178/7\,r^{+} - 1650/7\,r^{\pm} - 45/28\,x_3 + 72/7\,x_{17} + 36/7\,x_{18} - 2\,x_{19} \\
&\quad - 18/7\,x_{20} + 18/7\,x_{21} \\
x_{23} &= -20\,c - 8\,r^{+} + 40\,r^{\pm} \\
x_{24} &= 40\,c + 16\,r^{+} - 80\,r^{\pm}
\end{aligned} \tag{5.35}
$$

Ten parameters ($x_3, x_5, x_7, x_{11}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}$ and c) remain arbitrary. Redefining the metric in this way results in absorbing all corrections to the volume kinetic term in the metric, as d has to be set zero. Thus, the R^4-terms of the Lagrangian after redefining the metric are given by

$$S^{\pm}_{R^4} = \int d^{10}\sqrt{|G|}c\, v_6^{-1}\,\mathcal{R}\,\hat{E}_6\,. \tag{5.36}$$

As the only y dependent terms is E_6 the integration over the internal manifold can easily be performed:

$$S^{\pm}_{R^4} = \int d^4x\sqrt{|g|}\,6(4\pi)^3 c\,\mathcal{R} \tag{5.37}$$

where χ denotes the Euler number of the internal manifold Y and the Gauss-Bonnet theorem

$$\int_Y d^6y\sqrt{|\gamma|}E_6 = 2^6\,6\pi^3\chi\,, \tag{5.38}$$

has been used. Thus, the complete action is given by

$$S^{\pm}_{\mathcal{R},v_6} = \int d^4x\sqrt{|g|}\left((e^{-2\phi}v_6 + 6(4\pi)^3 c\,\chi)\mathcal{R} - \tfrac{5}{6}e^{-2\phi}v_6(\partial_\mu \ln v_6)^2\right)\,. \tag{5.39}$$

5.4 Comparison with Calabi-Yau compactifications

As mentioned above, these calculations have already been carried out for the simpler case of the internal manifold Y being a Calabi-Yau manifold. The corrections to the Einstein-Hilbert term have been presented in [30]:

$$S_{\mathcal{R}} = \int d^4x\sqrt{|g|}\left(v_6e^{-2\phi} + e^{-2\phi}\frac{2\zeta(3)\chi}{(2\pi)^3} + \frac{\chi}{12\pi}\right)\mathcal{R}\,, \tag{5.40}$$

which is consistent with (5.39), as c can be chosen to be

$$c = e^{-2\phi}\frac{\zeta(3)}{24(2\pi)^6} + \frac{1}{18(4\pi)^4}\,. \tag{5.41}$$

For the volume kinetic term, only one-loop corrections has been calculated [15]

$$S_{v_6} = \int d^4x\sqrt{|g|}\frac{1}{6}(5v_6e^{-2\phi} + (16-3\mu_1)\frac{\chi}{12\pi})(\partial_\mu \ln v_6)(\partial^\mu \ln v_6)\,, \tag{5.42}$$

with $\mu_1^2 = 4$. In contrast to this result, the corrections to the volume kinetic term have to be absorbed by a redefinition of the metric in the case of a generic manifold as demonstrated above.

6 Summary and Conclusions

In this thesis two aspects of string theory compactifications have been devised. First, the orientifold projection of type IIA theory compactified on Calabi-Yau and manifolds with $SU(3)$-structure were examined. The description of the orientifold projection outlined in [12] for type IIB theory on a Calabi-Yau has been successfully transfered to these theories. The spectrum of the orientifold compactifications were explicitly calculated in [10] for Calabi-Yau compactifications and in [11] for compactifications on manifolds with $SU(3)$-structure. In this thesis, it has been shown, that the description of [12] can be adapted to type IIA theory. By demanding the projective superfields describing the hypermultiplet sector to be either parity odd or even, the truncation of the spectra calculated in [10, 11] have been generated. Furthermore, the Kähler potential for the $N = 1$ chiral multiplets obtained in this way has been calculated using the superconformal formalism and shown to match the result of [10, 11] as well.

Second, corrections to the four-dimensional action of type IIA/B theory induced by R^4-terms were calculated for compactifications on a generic Riemannian manifold. The compactification of these terms was already performed in [14, 15] for Calabi-Yau manifolds. In the case of Calabi-Yau compactifications, the Ricci-flatness of Calabi-Yau manifolds together with the symmetries of the Riemann tensor can be used to show, that the R^4-terms lead to corrections in the four-dimensional action proportional to the Euler characteristic of the internal manifold. It was demonstrated in section 5 that in the case of a compactification on a generic manifold, the corrections to the Einstein term in four dimensions assumes the same form. However, in contrast to Calabi-Yau compactifications, the corrections to the volume kinetic term have been absorbed by a redefinition of the metric in ten dimensions.

A $N = 1, 2$ supergravity

In this section the basic properties of supersymmetry required in the main text will be reviewed. After stating the supergravity algebra, its representations needed here will be discussed. A general references for this topic is [2].

Supersymmetry transformations extend the Poincaré algebra, generating space-time symmetry transformations, by introducing a number of fermionic transformation generators Q^I_α. The Indices α, $\beta = 1, 2$ are Weyl spinor indices and I, $J = 1, ..., N$ are isospin indices. Together with the momentum operators P_μ and the angular momentum and Lorenz boost operators $M_{\mu\nu}$ the supersymmetry generators form the supersymmetry algebra

$$\{Q^I_\alpha, \bar{Q}_{\dot{\beta}J}\} = 2\,\delta^I_J\,(\sigma^\mu)_{\alpha\dot{\beta}}\,P_\mu\,, \tag{A.1a}$$

$$\{Q^I_\alpha, Q^J_\beta\} = \epsilon_{\alpha\beta} Z^{IJ}\,, \tag{A.1b}$$

$$[M_{\mu\nu}, Q^I_\alpha] = i(\sigma_{\mu\nu})_\alpha{}^\beta\,Q^I_\beta\,, \tag{A.1c}$$

with all other (anti)commutators vanishing. Z_{IJ} are the central charges and

$$(\sigma^{\mu\nu})_\alpha{}^\beta \equiv \tfrac{1}{4}\left[(\sigma^\mu)_{\alpha\dot{\gamma}}(\bar{\sigma}^\nu)^{\dot{\gamma}\beta} - (\sigma^\nu)_{\alpha\dot{\gamma}}(\bar{\sigma}^\mu)^{\dot{\gamma}\beta}\right] \tag{A.1d}$$

with σ^μ being the Pauli matrices. The following discussion will be restricted to $N = 1, 2$ supersymmetry, as these are the only cases needed in the main text.

A.1 $N = 1, 2$ supermultiplets

The representations, that are important for the calculations in the main text are:

- $N = 1$ **Chiral multiplet:** Field content (v, F, Ψ_α).
  Here, v is a complex scalar, F a complex auxiliary scalar and Ψ_α a Weyl spinor.
- $N = 1$ **Tensor multiplet:** Field content $(x, t_{\mu\nu}, \Psi_\alpha)$.
  As the name suggests, the tensor multiplet contains among its bosonic components an anti-symmetric tensor $t_{\mu\nu}$. Furthermore, it contains a real scalar x and a Weyl spinor Ψ_α.
- $N = 2$ **Hypermultiplet:** Field content $(A^i, \Psi^i_\alpha, F^i)$ with $i = 1, 2$.
  The content of the $N = 2$ hypermultiplet is that of two $N = 1$ chiral multiplets. Namely, two complex scalars A^i two auxiliary scalars F^i and a doublet of spinors Ψ^i_α.
- $N = 2$ **Vector multiplet:** Field content $(A, V_\mu, \Psi^i_\alpha)$ with $i = 1, 2$.
  Here, A denotes a complex scalar, Ψ^i_α a doublet of spinors and V_μ a vector.

- $N = 2$ **Tensor multiplet:** Field content $(x, v, t_{\mu\nu}, \Psi^i_\alpha)$ with $i = 1, 2$.
  The content is the sum of a $N = 1$ chiral and tensor multiplet: A real and a complex scalar x and v as well as a tensor $t_{\mu\nu}$, a doublet of spinors Ψ^i_α and an auxiliary scalar F.

A.2 Generic $N = 1$ chiral multiplet actions

An important property of actions for a number of self-interacting $N = 1$ chiral multiplets is, that they can be described in terms of a single function K. The kinetic terms of the bosonic fields of n chiral $N = 1$ multiplets v^i $(i = 1, \ldots, n)$ can always be written in the form

$$S = \int d^4x \sqrt{|g|}\, \frac{\partial^2 K}{\partial v^i \partial \bar{v}^j} (\partial_\mu v^i)(\partial \bar{v}^j) \,, \tag{A.2}$$

with K being an arbitrary function of the fields v^i and their complex conjugates. The function K is called the Kähler potential for the chiral multiplets. Using the superfield formalism, the action of the complete multiplets can easily be given in terms of K. This formulation obviously simplifies the discussion of actions for $N = 1$ chiral multiplet.

A.3 Duality transformations

An important feature of supersymmetric theories is the duality between chiral and tensor multiplets in four dimensions [31]. $N = 1, 2$ chiral multiplets and $N = 1, 2$ tensor multiplets both carry the same physical degrees of freedom. As the connection between these two representations of supersymmetry is crucial for the calculations in sections 3 and 4, it will be reviewed in this section. Whereas Lagrangian for a number of chiral or tensor multiplets are completely equivalent in the free theory, the duality transformation generally can only be performed from tensor to chiral multiplets when interactions are present. The reverse is only possible for a restricted class of interactions. But as in the main text only the transformation from tensor to chiral multiplets is relevant, the discussion will be restricted to this direction.

In the following it will be shown, that the a generic action for a number of self-interacting $N = 2$ tensor multiples is dual to an action for $N = 2$ hypermultiplets. The duality will be demonstrated following [21] on the level of bosonic component fields, as the calculations in the main text are also restricted to these. The duality can be equally demonstrated for $N = 1$ multiplets [31].

The bosonic part of a generic action for n $N=2$ tensor multiples reads [21]

$$S = \int d^4x \Big[\mathcal{L}_{x^I x^J}\left(\tfrac{1}{4}(\partial_\mu x^I \partial^\mu x^J - H^J_\mu H^{\mu J}) + \partial_\mu v^I \partial^\mu v^J\right) \\ + \frac{1}{2} i (\mathcal{L}_{v^I x^J}\partial_\mu v^I - \mathcal{L}_{\bar{v}^I x^J}\partial_\mu \bar{v}^I) H^{\mu J}\Big] \qquad (I = 1, \dots, n)\,, \tag{A.3}$$

where $H^{\mu I} = \epsilon^{\mu\nu\rho\sigma}\partial_\nu t_{\rho\sigma}$ and the subscripts on $\mathcal{L}$ again denote partial derivatives with respect to the listed fields. As mentioned in section 3 the function $\mathcal{L}$ can not be chosen arbitrary, in order to give rise to an superconformal action. Namely, it has to be homogeneous of first degree, invariant under phase transformations of the v^I and obey the differential equations [21]

$$\mathcal{L}_{x^I x^J} + \mathcal{L}_{v^I \bar{v}^J} = 0\,, \tag{A.4a}$$

$$\mathcal{L}_{x^I v^J} - \mathcal{L}_{x^J v^I} = 0\,. \tag{A.4b}$$

In order to establish the connection to $N=2$ hypermultiplets, real Lagrangian multipliers y^I are introduced by adding $-\frac{1}{2} y_I \partial_\mu H^{\mu I}$ to the action. After this, $H^{\mu I}$ can be removed from the action in favor of y^I by means of the field equations for $H^{\mu I}$. This results in an action of the form [21]

$$S = \int d^4x \Big[\mathcal{L}_{x^I x^J}(\partial_\mu v^I \partial^\mu + \tfrac{1}{4}\partial_\mu x^I \partial^\mu x^J) \\ \tfrac{1}{4}\mathcal{L}^{x^I x^J}(\partial_\mu y_I + i(\partial_\mu v^K \mathcal{L}_{v^K x^I} - \mathcal{L}_{x^I \bar{v}^K}\partial_\mu \bar{v}^K)) \\ \times\, (\partial^\mu y_J + i(\partial^\mu v^L \mathcal{L}_{v^L x^J} - \mathcal{L}_{x^J \bar{v}^L}\partial^\mu \bar{v}^L))\Big]\,, \tag{A.5}$$

where $\mathcal{L}^{x^I x^J}$ denotes the inverse of $\mathcal{L}_{x^I x^J}$, i.e. $\mathcal{L}^{x^I x^J}\mathcal{L}_{x^J x^K} = \delta^I{}_K$. This action contains the same bosonic degrees of freedom as the $N=2$ hypermultiplet action. However it is not obvious from a component field point of view, how this action can be expressed in terms of a Kähler potential as described above. It can be shown, that the right set of fields describing the bosonic components of the $N=2$ hypermultiplets are v^I together with

$$\omega_I = \frac{1}{2}(\mathcal{L}_{x^I} + i y_I)\,. \tag{A.6}$$

Rewriting the action (A.5) in terms of these fields yields an action gives [21]

$$\mathcal{L} = \chi_{v^I \bar{v}^J}(\partial_\mu v^I)(\partial^\mu \bar{v}^J) + \chi_{\omega_I \bar{v}^J}(\partial_\mu \omega_I)(\partial^\mu \bar{v}^J) \\ + \chi_{v^I \bar{\omega}_J}(\partial_\mu v^I)(\partial^\mu \bar{\omega}_J) + \chi_{\omega_I \bar{\omega}_J}(\partial_\mu \omega_I)(\partial^\mu \bar{\omega}_J)\,, \tag{A.7}$$

with

$$\chi(v, \omega, \bar{v}, \bar{\omega}) = \mathcal{L}(v, \bar{v}, x) - (\omega_I + \bar{\omega}_I) x^I\,, \tag{A.8}$$

where x^I has to be regarded as a function of $(\omega, \bar{\omega})$, defined implicitly by (A.6).

Thus, the bosonic action of n $N = 2$ tensor multiplets can be cast into the form of the bosonic part of an $N = 2$ hypermultiplet action. The fact, that this duality holds for the entire multiplet can be most easily shown in a superfield calculation [31]. As in the calculations in the main text only the duality of the bosonic components is needed, the proof of the duality of the entire multiplets will not be given here, but can be found in [31].

B Complex geometry

In this appendix basic features of complex manifolds are reviewed, in order to provide the basics for Calabi-Yau manifold compactifications. A general reference for this topic is [25]. **Complex manifolds** are defined as even-dimensional manifolds with an atlas of charts that map to open sets in $\mathbb{C}^n$ and possess holomorphic transition functions.

On every complex manifold M one can define a tensor field J, the complex structure of M, which is given on every chart $(U_i, z^\mu = x^\mu + iy^\mu)$ by

$$J_p\left(\frac{\partial}{\partial x^\mu}\right) = \frac{\partial}{\partial x^\mu}\ , \quad J_p\left(\frac{\partial}{\partial y^\mu}\right) = -\frac{\partial}{\partial y^\mu}\ . \tag{B.1}$$

Obviously, J squares to minus one

$$J_p^2 = -\mathrm{id}_{T_pM}\ , \tag{B.2}$$

implying, that J_p has eigenvalues $\pm i$. Thus, J can be used to divide T_pM into two disjoint vector subspaces

$$T_pM = T_pM^+ \oplus T_pM^-\ , \quad \text{with} \quad T_pM^\pm = \{X \in T_pM | J_pX = \pm iX\}\ . \tag{B.3}$$

On the other hand, J can be used to define complex manifolds: A $2n$ dimensional differentiable manifold M together with an tensor field J satisfying (B.2), the **almost complex structure** of M, is called an **almost complex manifold**. As the name suggest, these manifolds are not in general complex, because there are not necessary subsets of a coordinate basis spanning $T_p^\pm M$. An almost complex manifold can be shown to be complex, if the almost complex structure is integrable, that is the Lie bracket is closed under J:

$$[X, Y] \in T_p^\pm \quad \text{for all} \quad X, Y \in T_p^\pm \text{ and all } p \in M\ . \tag{B.4}$$

The next important objects on (almost) complex manifolds are their metrics. A metric g of a complex manifold is said to be **hermitian** if it satisfies

$$g_p(J_pX, J_pY) = g_p(X, Y) \tag{B.5}$$

for all points $p \in M$ and any $X, Y \in T_pM$. The pair (M, g) is called a **hermitian manifold**.

Definition: Kähler manifold
A complex manifold is called Kähler, if its Kähler form

$$\Omega_p(X, Y) := g_p(J_pX, Y) \tag{B.6}$$

is closed $d\Omega = 0$ or equivalently the complex structure is covariantly constant

$$\nabla_\mu J = 0\,. \tag{B.7}$$

It can be shown, that this implies that the metric can locally be completely specified in terms of a single function K

$$g_{i\bar{j}} = \partial_i \partial_{\bar{j}} K\,. \tag{B.8}$$

The function K is called the Kähler potential of the Kähler metric g.

As already demonstrated, the complex structure can be used to split the tangent bundle into $TM^{\pm}$. This decomposition can be used to split the spaces of q-forms on a complex manifold: Let M be a complex manifold of dimension $2m$ and $V_i \in T_pM$ $(i = 1, \ldots, q)$. A q-form ω is said to be of bidegree (r, s), with $r + s = q$, or simply a (r, s)-form, if $\omega(V_1, \ldots, V_q) = 0$ unless r of the V_i are in T_pM^+ and s of them are in T_pM^-. The components of a (r, s)-form are label as

$$\omega_{i_1,\ldots,i_r,\bar{i}_{r+1},\ldots,\bar{i}_{r+s}} = \omega\left(\frac{\partial}{\partial z^{i_1}}, \ldots, \frac{\partial}{\partial z^{i_r}}, \frac{\partial}{\partial \bar{z}^{i_{r+1}}}, \ldots, \frac{\partial}{\partial \bar{z}^{i_{r+s}}}\right)\,. \tag{B.9}$$

C Details on type IIA compactifications

In this appendix, some formulas are collected for the sake of completeness, that were omitted in the main text as they are not required for the calculations in section 4 and 5.

The matrices $\mathcal{N}$ and G^J in (2.9) containing the informations about the couplings of the field strength F^Λ is given by [17]

$$\mathrm{Re}\mathcal{N} = \begin{pmatrix} -\frac{1}{3}\mathcal{K}_{ABC}b^A b^B b^C & \frac{1}{2}\mathcal{K}_{ABC}b^B b^C \\ -\frac{1}{2}\mathcal{K}_{ABC}b^B b^C & -\mathcal{K}_{ABC}b^C \end{pmatrix} , \tag{C.1}$$

$$\mathrm{Im}\mathcal{N} = -\frac{\mathcal{K}}{6}\begin{pmatrix} 1 + G^J_{AB}b^A b^B & -4G^J_{AB}b^B \\ -4G^J_{AB}b^B & 4G^J_{AB} \end{pmatrix} , \tag{C.2}$$

with

$$G^J_{AB} = \frac{3}{2\mathcal{K}}\int_Y \omega_A \wedge *\omega_B \tag{C.3}$$

$$\mathcal{K}_{ABC} = \int_Y \omega_A \wedge \omega_B \wedge_B , \qquad \mathcal{K} = \int_Y J \wedge J \wedge J . \tag{C.4}$$

D Redefinition of the metric by $G_{AB} \to G_{AB} + X_{AB}$

In this appendix it will be shown, that the redefinition of the metric of ten-dimensional space-time by

$$G_{AB} \to \tilde{G}_{AB} := G_{AB} + X_{AB} \,, \tag{D.1}$$

with X_{AB} being a tensor build of three Riemann tensors, results in the modification of the action by [29]

$$\int dx^{10} \sqrt{|G|} R \to \int dx^{10} \sqrt{G} \left(R + \left(\tfrac{1}{2} R G_{AB} - R_{AB} \right) X^{AB} + \mathcal{O}(RX^2) \right) \,. \tag{D.2}$$

To proof this relation, the Ricci scalar $\tilde{R}$ related to $\tilde{G}_{AB}$ as well as the determinant of the metric $\tilde{G}_{AB}$ has to be calculated. As the focus lies on terms of the Lagrangian, that are of the order R^4 maximum, that is of order eight or lower in derivatives of the metric G_{AB}, all terms, that will lead to expressions of higher order will be omitted in the following. All indices will be lowered and raised only with G_{AB} and its inverse.

At first, the determinant of the redefined metric will be computed:

$$\begin{aligned} |G| \equiv -\det \tilde{G}_{AB} &= -\det(G_{AB} + X_{AB}) = -\det(G_{AB}) \cdot \det(\mathbb{1} + G^{AC} X_{CB}) \\ &= -\det(G_{AB}) \cdot \exp(\mathrm{trace}(\log(\mathbb{1} + G^{AC} X_{CB}))) \\ &= -\det(G_{AB}) \cdot (1 + X^A{}_A - \tfrac{1}{2} X^{AB} X_{AB} + \cdots) \,. \end{aligned} \tag{D.3}$$

Therefore, its square root is given by

$$\sqrt{|\tilde{G}_{AB}|} = \sqrt{|G_{AB}|} \cdot (1 + \tfrac{1}{2} G^{AB} X_{AB} + \cdots) \,. \tag{D.4}$$

Second, the effect on the Ricci-tensor will be determined. To this end, it is necessary to know the inverse metric $\tilde{G}^{AB}$

$$\tilde{G}^{AB} = (G + X)^{-1\,AB} = G^{AB} - X^{AB} + \ldots \tag{D.5}$$

and the Christoffel symbols

$$\begin{aligned} \tilde{\Gamma}^A{}_{BC} &= \tfrac{1}{2} \tilde{G}^{AD} (\partial_C \tilde{G}_{BD} + \partial_B \tilde{G}_{DC} - \partial_D \tilde{G}_{BC}) \\ &= \Gamma^A{}_{BC} - \tfrac{1}{2} X^{AD} (\partial_C G_{BD} + \partial_B G_{DC} - \partial_D G_{BC}) + \cdots \\ &= \Gamma^A{}_{BC} - X^A{}_D \Gamma^D{}_{BC} + \tfrac{1}{2} G^{AD} (\partial_B X_{CD} + \partial_C x_{BD} - \partial_D X_{BC}) + \cdots \,. \end{aligned} \tag{D.6}$$

With the help of these equations the Ricci scalar $\tilde{R}$ can be calculated by contracting the Riemann tensor of the redefined metric

$$\tilde{R} = \tilde{G}^{AC} \tilde{G}^{BD} \tilde{R}_{ABCD}$$

$$
\begin{aligned}
&= (G^{AC} - X^{AC})(G^{BD} - X^{BD} R_{ABCD}) \\
&\quad + \tfrac{1}{2} G^{AC} G^{BD} (\partial^2_{BC} X_{AD} + \partial^2_{AD} X_{BC} - \partial^2_{AC} X_{BD} - \partial^2_{BD} X_{AC}) \\
&\quad - X_{EF} (\Gamma^E{}_{BC} \Gamma^F{}_{AD} - \Gamma^E{}_{AC} \Gamma^F{}_{BD}) \\
&\quad + \tfrac{1}{2} G^{AC} G^{BD} (\partial_B X_{EC} + \partial_C X_{EB} - \partial_E X_{BC}) \Gamma^E{}_{AD} \\
&\quad + \tfrac{1}{2} G^{AC} G^{BD} (\partial_A X_{ED} + \partial_D X_{EA} - \partial_E X_{AD}) \Gamma^E{}_{BC} \\
&\quad - \tfrac{1}{2} G^{AC} G^{BD} (\partial_A X_{EC} + \partial_C X_{EA} - \partial_E X_{AC}) \Gamma^E{}_{BD} \\
&\quad - \tfrac{1}{2} G^{AC} G^{BD} (\partial_B X_{ED} + \partial_D X_{EB} - \partial_E X_{BD}) \Gamma^E{}_{AC} + \ldots \\
&= R - R_{AB} X^{AB} + G^{AC} G^{BD} (\nabla_B \nabla_C X_{AD} - \nabla_A \nabla_C X_{BD}) + \ldots \, .
\end{aligned} \tag{D.7}
$$

In the first step the definitions of the Riemann tensor (5.2) and the expressions for the Christoffel symbols (D.6) and the inverse metric (D.5) have been inserted. Terms of higher order than R^4 were omitted in the calculation. The third term in the last line is a total derivative and therefore does not contribute to the action. Merging (D.7) and (D.4) gives the form of the action after the redefinition of the metric

$$
\begin{aligned}
S &= \int dx^{10} \sqrt{|\tilde{G}|} \tilde{R} \\
&= \int dx^{10} \sqrt{|G|} \left((1 + \tfrac{1}{2} G^{AB} X_{AB}) \cdot (R - R_{AB} X^{AB}) + \mathcal{O}(R X^2) \right) \\
&= \int dx^{10} \sqrt{G} \left(R + \left(\tfrac{1}{2} R G_{AB} - R_{AB} \right) X^{AB} + \mathcal{O}(R X^2) \right) \, .
\end{aligned} \tag{D.8}
$$

This proofs that the action transforms under the redefinition of the metric (D.1) as claimed in (D.2) and (5.30).

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Erklärung gemäß Diplomprüfungsordnung:

Ich versichere, diese Arbeit selbstständig und ausschließlich unter Verwendung der angegebenen Quellen und Hilfsmittel verfasst zu haben.

Hamburg, den 10. Februar 2010